SECTION 1.1

Introduction to Algebra: Variables and Mathematical Models

Objectives

1. Evaluate algebraic expressions.
2. Translate English phrases into algebraic expressions.
3. Determine whether a number is a solution of an equation.
4. Translate English sentences into algebraic equations.
5. Evaluate formulas.

Feeling attractive with a suntan that gives you a "healthy glow"? Think again. Direct sunlight is known to promote skin cancer. Although sunscreens protect you from burning, dermatologists are concerned with the long-term damage that results from the sun even without sunburn.

Algebraic Expressions

Let's see what this man's "healthy glow" has to do with algebra. The biggest difference between arithmetic and algebra is the use of *variables* in algebra. A **variable** is a letter that represents a variety of different numbers. For example, we can let x represent the number of minutes that a person can stay in the sun without burning with no sunscreen. With a number 6 sunscreen, exposure time without burning is six times as long, or 6 times x. This can be written $6 \cdot x$, but it is usually expressed as $6x$. Placing a number and a letter next to one another indicates multiplication.

Notice that $6x$ combines the number 6 and the variable x using the operation of multiplication. A combination of variables and numbers using the operations of addition, subtraction, multiplication, or division, as well as powers or roots, is called an **algebraic expression**. Here are some examples of algebraic expressions:

$x + 6$	$x - 6$	$6x$	$\frac{x}{6}$	$3x + 5$	$\sqrt{x} + 7.$
The variable x increased by 6	The variable x decreased by 6	6 times the variable x	The variable x divided by 6	5 more than 3 times the variable x	7 more than the square root of the variable x

1 Evaluate algebraic expressions.

Evaluating Algebraic Expressions

We can replace a variable that appears in an algebraic expression by a number. We are **substituting** the number for the variable. The process is called **evaluating the expression**. For example, we can evaluate $6x$ (from the sunscreen example) for $x = 15$. We substitute 15 for x. We obtain $6 \cdot 15$, or 90. This means that if you can stay in the sun for 15 minutes without burning when you don't put on any lotion, then with a number 6 lotion, you can sunbathe for 90 minutes without burning.

Many algebraic expressions involve more than one operation. The order in which we add, subtract, multiply, and divide is important. In Section 1.8, we will discuss the rules for the order in which operations should be done. For now, follow this order:

A First Look at Order of Operations

1. Perform all operations within grouping symbols, such as parentheses.
2. Do all multiplications in the order in which they occur from left to right.
3. Do all additions and subtractions in the order in which they occcur from left to right.

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Study Tip

Study the step-by-step solutions in the worked examples. Reading the solutions with great care will prepare you for success with the exercises in the Exercise Sets.

EXAMPLE 1 Evaluating Expressions

Evaluate each algebraic expression for $x = 5$:

a. $3 + 4x$ **b.** $4(x + 3)$.

Solution

a. We begin by substituting 5 for x in $3 + 4x$. Then we follow the order of operations: Multiply first, and then add.

$$3 + 4x$$

Replace x with 5.

$$\begin{aligned} &= 3 + 4 \cdot 5 && \\ &= 3 + 20 && \text{Perform the multiplication: } 4 \cdot 5 = 20. \\ &= 23 && \text{Perform the addition.} \end{aligned}$$

b. We begin by substituting 5 for x in $4(x + 3)$. Then we follow the order of operations: Perform the addition in parentheses first, and then multiply.

$$4(x + 3)$$

Replace x with 5.

$$\begin{aligned} &= 4(5 + 3) && \\ &= 4(8) && \text{Perform the addition inside the parentheses: } 5 + 3 = 8. \text{ } 4(8) \text{ can also be written as } 4 \cdot 8. \\ &= 32 && \text{Multiply.} \end{aligned}$$

Study Tip

You learn best by doing. To be sure you understand the worked examples, try each Check Point. Check your answer in the answer section before continuing your reading. Expect to read this book with pencil and paper handy to work the Check Points.

✓ **CHECK POINT 1** Evaluate each expression for $x = 10$:

a. $6 + 2x$ **b.** $2(x + 6)$.

Example 2 illustrates that algebraic expressions can contain more than one variable.

EXAMPLE 2 Evaluating Expressions

Evaluate each algebraic expression for $x = 6$ and $y = 4$:

a. $5x - 3y$ **b.** $\dfrac{3x + 5y + 2}{2x - y}$.

Solution

a.

$$5x - 3y \qquad \text{This is the given algebraic expression.}$$

Replace x with 6. Replace y with 4.

$$\begin{aligned} &= 5 \cdot 6 - 3 \cdot 4 && \text{We are evaluating the expression for } x = 6 \text{ and } y = 4. \\ &= 30 - 12 && \text{Multiply: } 5 \cdot 6 = 30 \text{ and } 3 \cdot 4 = 12. \\ &= 18 && \text{Subtract.} \end{aligned}$$

b. $\dfrac{3x + 5y + 2}{2x - y}$ This is the given algebraic expression.

Replace x with 6. Replace y with 4.

$= \dfrac{3 \cdot 6 + 5 \cdot 4 + 2}{2 \cdot 6 - 4}$ We are evaluating the expression for $x = 6$ and $y = 4$.

$= \dfrac{18 + 20 + 2}{12 - 4}$ Multiply: $3 \cdot 6 = 18$, $5 \cdot 4 = 20$, and $2 \cdot 6 = 12$.

$= \dfrac{40}{8}$ Add in the numerator. Subtract in the denominator.

$= 5$ Simplify by dividing 40 by 8.

✓ **CHECK POINT 2** Evaluate each algebraic expression for $x = 3$ and $y = 8$:

a. $7x + 2y$ **b.** $\dfrac{6x - y}{2y - x - 8}$.

2 Translate English phrases into algebraic expressions.

Translating to Algebraic Expressions

Problem solving in algebra often requires the ability to translate word phrases into algebraic expressions. **Table 1.1** lists some key words associated with the operations of addition, subtraction, multiplication, and division.

Table 1.1 Key Words for Addition, Subtraction, Multiplication, and Division

Operation	Addition (+)	Subtraction (−)	Multiplication (·)	Division (÷)
Key Words	plus	minus	times	divide
	sum	difference	product	quotient
	more than	less than	twice	ratio
	increased by	decreased by	multiplied by	divided by

EXAMPLE 3 Translating English Phrases into Algebraic Expressions

Write each English phrase as an algebraic expression. Let the variable x represent the number.

a. the sum of a number and 7

b. ten less than a number

c. twice a number, decreased by 6

d. the product of 8 and a number

e. three more than the quotient of a number and 11

Solution

a. The sum of a number and 7

$x + 7$

The algebraic expression for "the sum of a number and 7" is $x + 7$.

b. Ten less than | a number

$x - 10$

The algebraic expression for "ten less than a number" is $x - 10$.

c. Twice a number, | decreased by 6

$2x - 6$

The algebraic expression for "twice a number, decreased by 6" is $2x - 6$.

d. The product of | 8 and a number

$8 \cdot x$

The algebraic expression for "the product of 8 and a number" is $8 \cdot x$, or $8x$.

e. Three more than | the quotient of a number and 11

$\frac{x}{11} + 3$

The algebraic expression for "three more than the quotient of a number and 11" is $\frac{x}{11} + 3$.

Study Tip

Pay close attention to order when translating phrases involving subtraction.

Phrase	Translation
A number decreased by 7	$x - 7$
A number subtracted from 7	$7 - x$
Seven less than a number	$x - 7$
Seven less a number	$7 - x$

Think carefully about what is expressed in English before you translate into the language of algebra.

✓ **CHECK POINT 3** Write each English phrase as an algebraic expression. Let the variable x represent the number.

a. the product of 6 and a number
b. a number added to 4
c. three times a number, increased by 5
d. twice a number subtracted from 12
e. the quotient of 15 and a number

3 Determine whether a number is a solution of an equation.

Equations

An **equation** is a statement that two algebraic expressions are equal. **An equation always contains the equality symbol =.** Here are some examples of equations:

$$6x + 16 = 46 \qquad 4y + 2 = 2y + 6 \qquad 2(z + 1) = 5(z - 2).$$

Solutions of an equation are values of the variable that make the equation a true statement. To determine whether a number is a solution, substitute that number for the variable and evaluate each side of the equation. If the values on both sides of the equation are the same, the number is a solution.

EXAMPLE 4 Determining Whether Numbers Are Solutions of Equations

Determine whether the given number is a solution of the equation.

a. $6x + 16 = 46; 5$

b. $2(z + 1) = 5(z - 2); 7$

Solution

a.

$6x + 16 = 46$ This is the given equation.

Is 5 a solution?

$6 \cdot 5 + 16 \overset{?}{=} 46$ To determine whether 5 is a solution, substitute 5 for x. The question mark over the equal sign indicates that we do not yet know if the statement is true.

$30 + 16 \overset{?}{=} 46$ Multiply: $6 \cdot 5 = 30$.

$46 = 46$ Add: $30 + 16 = 46$.

This statement is true.

Because the values on both sides of the equation are the same, the number 5 is a solution of the equation.

b.

$2(z + 1) = 5(z - 2)$ This is the given equation.

Is 7 a solution?

$2(7 + 1) \overset{?}{=} 5(7 - 2)$ To determine whether 7 is a solution, substitute 7 for z.

$2(8) \overset{?}{=} 5(5)$ Perform operations in parentheses: $7 + 1 = 8$ and $7 - 2 = 5$.

$16 = 25$ Multiply: $2 \cdot 8 = 16$ and $5 \cdot 5 = 25$.

This statement is false.

Because the values on both sides of the equation are not the same, the number 7 is not a solution of the equation.

✓ **CHECK POINT 4** Determine whether the given number is a solution of the equation.

a. $9x - 3 = 42; 6$

b. $2(y + 3) = 5y - 3; 3$

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4 Translate English sentences into algebraic equations.

Translating to Equations

Earlier in the section, we translated English phrases into algebraic expressions. Now we will translate English sentences into equations. You'll find that there are a number of different words and phrases for an equation's equality symbol.

equals
gives
yields

$=$

is the same as
is/was/will be

EXAMPLE 5 Translating English Sentences into Algebraic Equations

Write each sentence as an equation. Let the variable x represent the number.

a. The product of 6 and a number is 30.

b. Seven less than 3 times a number gives 17.

Study Tip

Do not be concerned if you don't know how to find the solutions of the equations in Example 5. We'll discuss how to solve these equations in Chapter 2.

Solution

a.

$$\overset{\text{The product of 6 and a number}}{6x} \quad \overset{\text{is}}{=} \quad \overset{\text{30.}}{30}$$

The equation for "the product of 6 and a number is 30" is $6x = 30$.

b.

$$\overset{\text{3 times a number}}{3x}\ \overset{\text{Seven less than}}{-\ 7} \quad \overset{\text{gives}}{=} \quad \overset{\text{17.}}{17}$$

The equation for "seven less than 3 times a number gives 17" is $3x - 7 = 17$.

✓ **CHECK POINT 5** Write each sentence as an equation. Let the variable x represent the number.

a. The quotient of a number and 6 is 5.

b. Seven decreased by twice a number yields 1.

Study Tip

Commas make a difference. In English, sentences and phrases can take on different meanings depending on the way words are grouped with commas. Some examples:

These are meant to be amusing.

- What's the latest dope?
  What's the latest, dope?
- Population of Amsterdam broken down by age and sex
  Population of Amsterdam, broken down by age and sex

Algebraically, this is the important item.

- The product of 6 and a number increased by 5 is 30: $6(x + 5) = 30$.
  The product of 6 and a number, increased by 5, is 30: $6x + 5 = 30$.

5 Evaluate formulas.

Formulas and Mathematical Models

One aim of algebra is to provide a compact, symbolic description of the world. These descriptions involve the use of *formulas.* A **formula** is an equation that expresses a relationship between two or more variables. For example, one variety of crickets chirps faster as the temperature rises. You can calculate the temperature by counting the number of times a cricket chirps per minute and applying the following formula:

$$T = 0.3n + 40.$$

In the formula, T is the temperature, in degrees Fahrenheit, and n is the number of cricket chirps per minute. We can use this formula to determine the temperature if you are sitting on your porch and count 80 chirps per minute. Here is how to do so:

$T = 0.3n + 40$ This is the given formula.

$T = 0.3(80) + 40$ Substitute 80 for n.

$T = 24 + 40$ Multiply: $0.3(80) = 24$.

$T = 64.$ Add.

When there are 80 cricket chirps per minute, the temperature is 64 degrees.

The process of finding formulas to describe real-world phenomena is called **mathematical modeling**. Such formulas, together with the meaning assigned to the variables, are called **mathematical models**. We often say that these formulas model, or describe, the relationship among the variables.

In creating mathematical models, we strive for both accuracy and simplicity. For example, the formula $T = 0.3n + 40$ is relatively simple to use. However, you should not get upset if you count 80 cricket chirps and the actual temperature is 62 degrees, rather than 64 degrees, as predicted by the formula. Many mathematical models give an approximate, rather than an exact, description of the relationship between variables.

Sometimes a mathematical model gives an estimate that is not a good approximation or is extended to include values of the variable that do not make sense. In these cases, we say that **model breakdown** has occurred. Here is an example:

Use the mathematical model $T = 0.3n + 40$ with $n = 1200$ (1200 cricket chirps per minute).

$$T = 0.3(1200) + 40 = 360 + 40 = 400$$

At 400° F, forget about 1200 chirps per minute! At this temperature, the cricket would "cook" and, alas, all chirping would cease.

EXAMPLE 6 Age at Marriage and the Probability of Divorce

Divorce rates are considerably higher for couples who marry in their teens. The line graphs in **Figure 1.1** show the percentages of marriages ending in divorce based on the wife's age at marriage.

FIGURE 1.1

Source: B. E. Pruitt et al., *Human Sexuality*, Prentice Hall, 2007

Here are two mathematical models that approximate the data displayed by the line graphs:

Wife is under 18 at time of marriage.

Wife is over 25 at time of marriage.

$$d = 4n + 5 \qquad d = 2.3n + 1.5.$$

In each model, the variable n is the number of years after marriage and the variable d is the percentage of marriages ending in divorce.

a. Use the appropriate formula to determine the percentage of marriages ending in divorce after 10 years when the wife is over 25 at the time of marriage.

b. Use the appropriate line graph in **Figure 1.1** to determine the percentage of marriages ending in divorce after 10 years when the wife is over 25 at the time of marriage.

c. Does the value given by the mathematical model underestimate or overestimate the actual percentage of marriages ending in divorce after 10 years as shown by the graph? By how much?

Solution

a. Because the wife is over 25 at the time of marriage, we use the formula on the right, $d = 2.3n + 1.5$. To find the percentage of marriages ending in divorce after 10 years, we substitute 10 for n and evaluate the formula.

$d = 2.3n + 1.5$ This is one of the two given mathematical models.

$d = 2.3(10) + 1.5$ Replace n with 10.

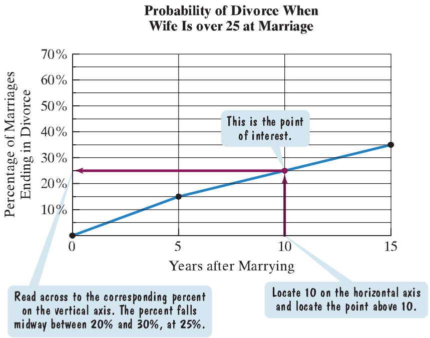
$d = 23 + 1.5$ Multiply: $2.3(10) = 23$.

$d = 24.5$ Add.

The model indicates that 24.5% of marriages end in divorce after 10 years when the wife is over 25 at the time of marriage.

b. Now let's use the line graph that shows the percentage of marriages ending in divorce when the wife is over 25 at the time of marriage. The graph is shown again in **Figure 1.2**. To find the percentage of marriages ending in divorce after 10 years:

- Locate 10 on the horizontal axis and locate the point above 10.
- Read across to the corresponding percent on the vertical axis.



This is the point of interest.

Read across to the corresponding percent on the vertical axis. The percent falls midway between 20% and 30%, at 25%.

Locate 10 on the horizontal axis and locate the point above 10.

FIGURE 1.2

The actual data displayed by the graph indicate that 25% of these marriages end in divorce after 10 years.

c. Here's a summary of what we found in parts (a) and (b).

Percentage of Marriages Ending in Divorce after 10 years (wife over 25 at Marriage)
Mathematical model: 24.5%
Actual data displayed by graph: 25.0%

The value obtained by evaluating the mathematical model, 24.5%, is close to, but slightly less than, the actual percentage of divorces, 25.0%. The difference between these percents is $25.0\% - 24.5\%$, or 0.5%. The value given by the mathematical model underestimates the actual percent by only 0.5%, providing a fairly accurate description of the data.

✓ CHECK POINT 6

a. Use the appropriate formula from Example 6 to determine the percentage of marriages ending in divorce after 15 years when the wife is under 18 at the time of marriage.

b. Use the appropriate line graph in **Figure 1.1** to determine the percentage of marriages ending in divorce after 15 years when the wife is under 18 at the time of marriage.

c. Does the value given by the mathematical model underestimate or overestimate the actual percentage of marriages ending in divorce after 15 years as shown by the graph? By how much?

1.1 EXERCISE SET

MyMathLab MathXL PRACTICE WATCH DOWNLOAD READ REVIEW

Study Tip

Do homework soon. It's okay to take a short break after class, but start reviewing and working the assigned homework in the Exercise Sets as soon as possible.

Practice Exercises

In Exercises 1–14, evaluate each expression for $x = 4$.

1. $x + 8$
2. $x + 10$
3. $12 - x$
4. $16 - x$
5. $5x$
6. $6x$
7. $\frac{28}{x}$
8. $\frac{36}{x}$
9. $5 + 3x$
10. $3 + 5x$
11. $2(x + 5)$
12. $5(x + 3)$
13. $\frac{12x - 8}{2x}$
14. $\frac{5x + 52}{3x}$

In Exercises 15–24, evaluate each expression for $x = 7$ and $y = 5$.

15. $2x + y$
16. $3x + y$
17. $2(x + y)$
18. $3(x + y)$
19. $4x - 3y$
20. $5x - 4y$
21. $\frac{21}{x} + \frac{35}{y}$
22. $\frac{50}{y} - \frac{14}{x}$
23. $\frac{2x - y + 6}{2y - x}$
24. $\frac{2y - x + 24}{2x - y}$

In Exercises 25–42, write each English phrase as an algebraic expression. Let the variable x represent the number.

25. four more than a number
26. six more than a number
27. four less than a number
28. six less than a number
29. the sum of a number and 4
30. the sum of a number and 6
31. nine subtracted from a number
32. three subtracted from a number
33. nine decreased by a number
34. three decreased by a number
35. three times a number, decreased by 5
36. five times a number, decreased by 3
37. one less than the product of 12 and a number
38. three less than the product of 13 and a number
39. the sum of 10 divided by a number and that number divided by 10
40. the sum of 20 divided by a number and that number divided by 20
41. six more than the quotient of a number and 30
42. four more than the quotient of 30 and a number

In Exercises 43–58, determine whether the given number is a solution of the equation.

43. $x + 14 = 20; 6$
44. $x + 17 = 22; 5$
45. $30 - y = 10; 20$
46. $50 - y = 20; 30$
47. $4z = 20; 10$

48. $5z = 30; 8$

49. $\frac{r}{6} = 8; 48$

50. $\frac{r}{9} = 7; 63$

51. $4m + 3 = 23; 6$

52. $3m + 4 = 19; 6$

53. $5a - 4 = 2a + 5; 3$

54. $5a - 3 = 2a + 6; 3$

55. $6(p - 4) = 3p; 8$

56. $4(p + 3) = 6p; 6$

57. $2(w + 1) = 3(w - 1); 7$

58. $3(w + 2) = 4(w - 3); 10$

In Exercises 59–74, write each sentence as an equation. Let the variable x represent the number.

59. Four times a number is 28.

60. Five times a number is 35.

61. The quotient of 14 and a number is $\frac{1}{2}$.

62. The quotient of a number and 8 is $\frac{1}{4}$.

63. The difference between 20 and a number is 5.

64. The difference between 40 and a number is 10.

65. The sum of twice a number and 6 is 16.

66. The sum of twice a number and 9 is 29.

67. Five less than 3 times a number gives 7.

68. Three less than 4 times a number gives 29.

69. The product of 4 and a number, increased by 5, is 33.

70. The product of 6 and a number, increased by 3, is 33.

71. The product of 4 and a number increased by 5 is 33.

72. The product of 6 and a number increased by 3 is 33.

73. Five times a number is equal to 24 decreased by the number.

74. Four times a number is equal to 25 decreased by the number.

Practice PLUS

75. Evaluate $\frac{x - y}{4}$ when x is 2 more than 7 times y and $y = 5$.

76. Evaluate $\frac{x - y}{3}$ when x is 2 more than 5 times y and $y = 4$.

77. Evaluate $4x + 3(y + 5)$ when x is 1 less than the quotient of y and 4 and $y = 12$.

78. Evaluate $3x + 4(y + 6)$ when x is 1 less than the quotient of y and 3, and $y = 15$.

79. **a.** Evaluate $2(x + 3y)$ for $x = 4$ and $y = 1$.
  b. Is the number you obtained in part (a) a solution of $5z - 30 = 40$?

80. **a.** Evaluate $3(2x + y)$ for $x = 1$ and $y = 5$.
  b. Is the number you obtained in part (a) a solution of $4z - 30 = 54$?

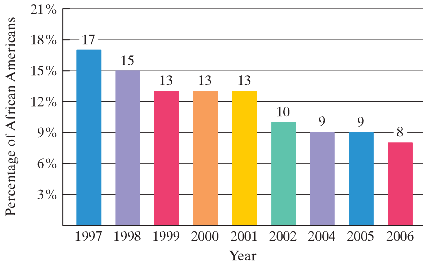
81. **a.** Evaluate $6x - 2y$ for $x = 3$ and $y = 6$.
  b. Is the number you obtained in part (a) a solution of $7w = 45 - 2w$?

82. **a.** Evaluate $5x - 14y$ for $x = 3$ and $y = \frac{1}{2}$.
  b. Is the number you obtained in part (a) a solution of $4w = 54 - 5w$?

Application Exercises

In 2006, 60 years after Jackie Robinson broke the sports color barrier, only 8% of Major League Baseball players were African American. The bar graph shows the decline in the percentage of African-American Major League Baseball players from 1997 through 2006.

Percentage of African-American Major League Baseball Players



Source: University of Central Florida's Institute for Diversity and Ethics in Sports (Data for 2003 unavailable.)

Here is a mathematical model that approximates the data displayed by the bar graph:

$$p = 16 - n.$$

Number of years after 1997

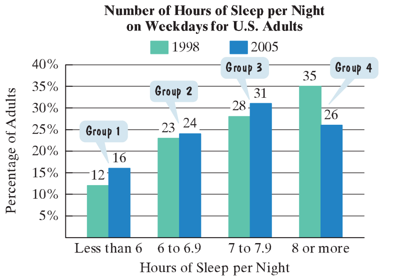
Percentage of African-American Major League Baseball players

Use this formula to solve Exercises 83–84.

83. **a.** Use the formula to find the percentage of African-American players 5 years after 1997, or in 2002. Does the mathematical model underestimate or overestimate the actual percent shown by the bar graph for 2002? By how much?
  b. Use the formula to find the percentage of African-American players in 2004. How well does the model describe the percent in the bar graph for this year?

84. **a.** Use the formula to find the percentage of African-American players 8 years after 1997, or in 2005. Does the mathematical model underestimate or overestimate the actual percent shown by the bar graph for 2005? By how much?
  b. Use the formula to find the percentage of African-American players in 2000. How well does the model describe the percent in the bar graph for this year?

Although lack of sleep affects the way you feel and perform, fewer Americans are getting enough sleep. The graph compares hours of sleep in 1998 and 2005. For each year, it shows the number of hours of sleep per night on weekdays and the percentage of U.S. adults sleeping for this number of hours.



Source: National Sleep Foundation poll of 1506 adults (percents do not add up to 100% due to rounding.)

Here are two mathematical models that approximate the data displayed by the graph:

$$1998\text{:}\quad \underbrace{p}_{\text{Percentage of U.S. adults in the group}} = 7.4\overbrace{h}^{\text{Group number – 1, 2, 3, or 4 – for hours of sleep per night on weekdays}} + 6 \qquad 2005\text{:}\quad p = 3.7h + 15$$

Use these formulas to solve Exercises 85–86.

85. **a.** Use the formula for 1998 to find the percentage of U.S. adults in group 4—that is, the percentage who got 8 or more hours of sleep per night on weekdays in 1998. Does the mathematical model underestimate or overestimate the percent shown for 1998 for this group? By how much?

b. Use the formula for 2005 to find the percentage of U.S. adults who slept 8 or more hours on weekdays in 2005. Does this overestimate or underestimate the percent shown for 2005 for this group? By how much?

86. **a.** Use the formula for 1998 to find the percentage of U.S. adults in group 3—that is, the percentage who got between 7 and 7.9 hours of sleep per night on weekdays in 1998. Does the mathematical model underestimate or overestimate the percent shown for 1998 for this group? By how much?

b. Use the formula for 2005 to find the percentage of U.S. adults who slept between 7 and 7.9 hours per night on weekdays in 2005. Does this overestimate or underestimate the percent shown for 2005 for this group? By how much?

A bowler's handicap, H, is often found using the following formula:

$$\underbrace{H}_{\text{Bowler's handicap}} = 0.8(200 - \underbrace{A}_{\text{Bowler's average score}}).$$

A bowler's final score for a game is the score for that game increased by the handicap.

Use this information to solve Exercises 87–88.

87. **a.** If your average bowling score is 145, what is your handicap?

b. What would your final score be if you bowled 120 in a game?

88. **a.** If your average bowling score is 165, what is your handicap?

b. What would your final score be if you bowled 140 in a game?

Writing in Mathematics

Writing about mathematics will help you to learn mathematics. For all writing exercises in this book, use complete sentences to respond to the questions. Some writing exercises can be answered in a sentence; others require a paragraph or two. You can decide how much you need to write as long as your writing clearly and directly answers the question in the exercise. Standard references such as a dictionary and a thesaurus should be helpful.

89. What is a variable?

90. What is an algebraic expression?

91. Explain how to evaluate $2 + 5x$ for $x = 3$.

92. If x represents the number, explain the difference between translating the following phrases:

a number decreased by 5

a number subtracted from 5.

93. What is an equation?

94. How do you tell the difference between an algebraic expression and an equation?

95. How do you determine whether a given number is a solution of an equation?

96. What is a mathematical model?

97. The bar graph for Exercises 83–84 shows a decline in the percentage of African-American Major League Baseball players from 1997 through 2006. What explanations can you offer for this trend?

98. In Exercises 87–88, we used the formula $H = 0.8(200 - A)$ to find a bowler's handicap, H, where the variable A represents the bowler's average score. Describe what happens to the handicap when the average score is 200.

Critical Thinking Exercises

Make Sense? *In Exercises 99–102, determine whether each statement "makes sense" or "does not make sense" and explain your reasoning.*

99. As I read this book, I write questions in the margins that I might ask in class.

100. I'm solving a problem that requires me to determine if 5 is a solution of $4x + 7$.

101. The model $p = 16 - n$ describes the percentage of African-American Major League Baseball players n years after 1997, so I can use it to estimate the percentage of African-American players in 1997.

102. Because there are four quarters in a dollar, I can use the formula $q = 4d$ to determine the number of quarters, q, in d dollars.

In Exercises 103–106, determine whether each statement is true or false. If the statement is false, make the necessary change(s) to produce a true statement.

103. The algebraic expression for "3 less than a number" is the same as the algebraic expression for "a number decreased by 3."

104. Some algebraic expressions contain the equality symbol, =.

105. The algebraic expressions $3 + 2x$ and $(3 + 2)x$ do not mean the same thing.

106. The algebraic expression for "the quotient of a number and 6" is the same as the algebraic expression for "the quotient of 6 and a number."

In Exercises 107–108, define variables and write a formula that describes the relationship in each table.

107.

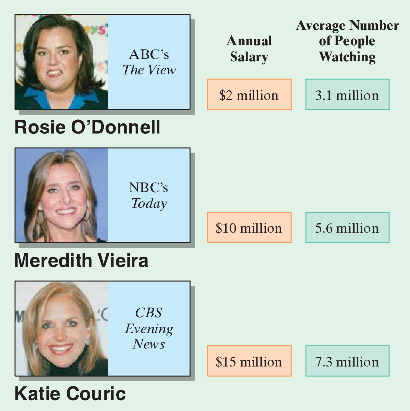
Number of Hours Worked	Salary
3	\$60
4	\$80
5	\$100
6	\$120

108.

Number of Workers	Number of Televisions Built
3	30
4	40
5	50
6	60

Technology Exercise

109. In 2006, a month into their new jobs, *Entertainment Weekly* magazine wanted to know which of three TV ladies was the "best buy." The data:



		Annual Salary	Average Number of People Watching
Rosie O'Donnell	ABC's *The View*	\$2 million	3.1 million
Meredith Vieira	NBC's *Today*	\$10 million	5.6 million
Katie Couric	*CBS Evening News*	\$15 million	7.3 million

Source: Entertainment Weekly, November 3, 2006

Here's the formula used by *Entertainment Weekly*:

Price per viewer

$$p = \frac{s}{w}$$

Annual salary, in millions

Number of people watching, in millions

In the words of the magazine, "the lower the price per viewer, the better the bargain."

a. Use a calculator to find the price per viewer, correct to the nearest cent, for each TV lady.

b. Using the magazine's criterion, which of the three women was the best buy?

Preview Exercises

Exercises 110–112 will help you prepare for the material covered in the next section. In each exercise, use the given formula to perform the indicated operation with the two fractions.

110. $\frac{a}{b} \cdot \frac{c}{d} = \frac{a \cdot c}{b \cdot d}$; $\frac{3}{7} \cdot \frac{2}{5}$

111. $\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{c} = \frac{a \cdot d}{b \cdot c}$; $\frac{2}{3} \div \frac{7}{5}$

112. $\frac{a}{b} - \frac{c}{b} = \frac{a - c}{b}$; $\frac{9}{17} - \frac{5}{17}$

SECTION 1.2

Objectives

1. Convert between mixed numbers and improper fractions.
2. Write the prime factorization of a composite number.
3. Reduce or simplify fractions.
4. Multiply fractions.
5. Divide fractions.
6. Add and subtract fractions with identical denominators.
7. Add and subtract fractions with unlike denominators.
8. Solve problems involving fractions in algebra.

Fractions in Algebra

Had a good workout lately? If so, could you tell from your heart rate if you were overdoing it or not pushing yourself hard enough?

Couch-Potato Exercise

$$\underset{\text{Heart rate, in beats per minute}}{H} = \frac{2}{5}(220 - \underset{\text{Age}}{a})$$

Working It

$$\underset{\text{Heart rate, in beats per minute}}{H} = \frac{9}{10}(220 - \underset{\text{Age}}{a})$$

The fractions $\frac{2}{5}$ and $\frac{9}{10}$ provide the difference between these formulas. Recall that in a fraction, the number that is written above the fraction bar is called the **numerator**. The number below the fraction bar is called the **denominator**.

Numerator

Denominator

$$\frac{2}{5} \qquad \frac{9}{10}$$

Fraction bar

Numerator

Denominator

The numerators and denominators of these fractions, 2, 5, 9, and 10, are examples of *natural numbers*. The **natural numbers** are the numbers that we use for counting.

Natural numbers 1, 2, 3, 4, 5, . . .

The three dots after the 5 indicate that the list continues in the same manner without ending.

Fractions appear throughout algebra. The first part of this section provides a review of the arithmetic of fractions. Later in the section, we focus on fractions in algebra.

1 Convert between mixed numbers and improper fractions.

Mixed Numbers and Improper Fractions

A **mixed number** consists of the sum of a natural number and a fraction, expressed without the use of an addition sign. Here is an example of a mixed number:

$$3\frac{4}{5}.$$

The natural number is 3 and the fraction is $\frac{4}{5}$. $3\frac{4}{5}$ means $3 + \frac{4}{5}$.

An **improper fraction** is a fraction whose numerator is greater than its denominator. An example of an improper fraction is $\frac{19}{5}$.

The mixed number $3\frac{4}{5}$ can be converted to the improper fraction $\frac{19}{5}$ using the following procedure:

Converting a Mixed Number to an Improper Fraction

1. Multiply the denominator of the fraction by the natural number and add the numerator to this product.
2. Place the result from step 1 over the denominator of the original mixed number.

EXAMPLE 1 Converting from Mixed Number to Improper Fraction

Convert $3\frac{4}{5}$ to an improper fraction.

Solution

$$3\frac{4}{5} = \frac{5 \cdot 3 + 4}{5}$$

Multiply the denominator by the natural number and add the numerator.

Place the result over the mixed number's denominator.

$$= \frac{15 + 4}{5} = \frac{19}{5}$$

✓ CHECK POINT 1 Convert $2\frac{5}{8}$ to an improper fraction.

An improper fraction can be converted to a mixed number using the following procedure:

Converting an Improper Fraction to a Mixed Number

1. Divide the denominator into the numerator. Record the quotient (the result of the division) and the remainder.
2. Write the mixed number using the following form:

$$\text{quotient}\,\frac{\text{remainder}}{\text{original denominator}}.$$

natural number part

fraction part

EXAMPLE 2 Converting from an Improper Fraction to a Mixed Number

Convert $\frac{42}{5}$ to a mixed number.

Solution We use two steps to convert $\frac{42}{5}$ to a mixed number.

Step 1. Divide the denominator into the numerator.

$$\begin{array}{rl} 8 & \text{quotient} \\ 5\overline{)42} & \\ \underline{40} & \\ 2 & \text{remainder} \end{array}$$

Step 2. Write the mixed number using quotient $\frac{\textbf{remainder}}{\textbf{original denominator}}$. Thus,

$$\frac{42}{5} = \underset{\text{quotient}}{8}\frac{\overset{\text{remainder}}{2}}{\underset{\text{original denominator}}{5}}.$$

Study Tip

In applied problems, answers are usually expressed as mixed numbers, which many people find more meaningful than improper fractions. Improper fractions are often easier to work with when performing operations with fractions.

✓ **CHECK POINT 2** Convert $\frac{5}{3}$ to a mixed number.

---

2 Write the prime factorization of a composite number.

Factors and Prime Factorizations

Fractions can be simplified by first factoring the natural numbers that make up the numerator and the denominator. To **factor** a natural number means to write it as two or more natural numbers being multiplied. For example, 21 can be factored as $7 \cdot 3$. In the statement $7 \cdot 3 = 21$, 7 and 3 are called the **factors** and 21 is the **product**.

$$\underset{\text{7 is a factor of 21.}}{7} \cdot \underset{\text{3 is a factor of 21.}}{3} = \overset{\text{The product of 7 and 3 is 21.}}{21}$$

Are 7 and 3 the only factors of 21? The answer is no because 21 can also be factored as $1 \cdot 21$. Thus, 1 and 21 are also factors of 21. The factors of 21 are 1, 3, 7, and 21.

Unlike the number 21, some natural numbers have only two factors: the number itself and 1. For example, the number 7 has only two factors: 7 (the number itself) and 1. The only way to factor 7 is $1 \cdot 7$ or, equivalently, $7 \cdot 1$. For this reason, 7 is called a *prime number*.

Prime Numbers

A **prime number** is a natural number greater than 1 that has only itself and 1 as factors.

The first ten prime numbers are

2, 3, 5, 7, 11, 13, 17, 19, 23, and 29.

Can you see why the natural number 15 is not in this list? In addition to having 15 and 1 as factors $(15 = 1 \cdot 15)$, it also has factors of 3 and $5 (15 = 3 \cdot 5)$. The number 15 is an example of a *composite number*.

Study Tip

The number 1 is the only natural number that is neither a prime number nor a composite number.

Composite Numbers

A **composite number** is a natural number greater than 1 that is not a prime number.

Every composite number can be expressed as the product of prime numbers. For example, the composite number 45 can be expressed as

$$45 = 3 \cdot 3 \cdot 5.$$

This product contains only prime numbers: 3 and 5.

Expressing a composite number as the product of prime numbers is called the **prime factorization** of that composite number. The prime factorization of 45 is $3 \cdot 3 \cdot 5$. The order in which we write these factors does not matter. This means that

$$45 = 3 \cdot 3 \cdot 5 \quad \text{or} \quad 45 = 5 \cdot 3 \cdot 3 \quad \text{or} \quad 45 = 3 \cdot 5 \cdot 3.$$

To find the prime factorization of a composite number, begin by selecting any two numbers, excluding 1 and the number itself, whose product is the number to be factored. If one or both of the factors are not prime numbers, continue by factoring each composite number. Stop when all numbers in the factorization are prime.

EXAMPLE 3 Prime Factorization of a Composite Number

Find the prime factorization of 100.

Solution Begin by selecting any two numbers, excluding 1 and 100, whose product is 100. Here is one possibility:

$$100 = 4 \cdot 25.$$

Because the factors 4 and 25 are not prime, we factor each of these composite numbers.

$$\begin{aligned} 100 &= 4 \cdot 25 && \text{This is our first factorization.} \\ &= 2 \cdot 2 \cdot 5 \cdot 5 && \text{Factor 4 and 25.} \end{aligned}$$

Notice that 2 and 5 are both prime. The prime factorization of 100 is $2 \cdot 2 \cdot 5 \cdot 5$.

✓ **CHECK POINT 3** Find the prime factorization of 36.

---

3 Reduce or simplify fractions.

Reducing Fractions

Two fractions are **equivalent** if they represent the same value. Writing a fraction as an equivalent fraction with a smaller denominator is called **reducing a fraction**. A fraction is **reduced to its lowest terms** when the numerator and denominator have no common factors other than 1.

FIGURE 1.3

Look at the rectangle in **Figure 1.3**. Can you see that it is divided into 6 equal parts? Of these 6 parts, 4 of the parts are red. Thus, $\frac{4}{6}$ of the rectangle is red.

The rectangle in **Figure 1.3** is also divided into 3 equal stacks and 2 of the stacks are red. Thus, $\frac{2}{3}$ of the rectangle is red. Because both $\frac{4}{6}$ and $\frac{2}{3}$ of the rectangle are red, we can conclude that $\frac{4}{6}$ and $\frac{2}{3}$ are equivalent fractions.

How can we show that $\frac{4}{6} = \frac{2}{3}$ without using **Figure 1.3**? Prime factorizations of 4 and 6 play an important role in the process. So does the **Fundamental Principle of Fractions**.

Fundamental Principle of Fractions

In words: The value of a fraction does not change if both the numerator and the denominator are divided (or multiplied) by the same nonzero number.

In algebraic language: If $\frac{a}{b}$ is a fraction and c is a nonzero number, then

$$\frac{a \cdot c}{b \cdot c} = \frac{a}{b}.$$

We use prime factorizations and the Fundamental Principle to reduce $\frac{4}{6}$ to its lowest terms as follows:

$$\frac{4}{6} = \frac{2 \cdot 2}{3 \cdot 2} = \frac{2}{3}$$

Write prime factorizations of 4 and 6.

Divide the numerator and the denominator by the common prime factor, 2.

Here is a procedure for writing a fraction in lowest terms:

Reducing a Fraction to its Lowest Terms

1. Write the prime factorizations of the numerator and the denominator.
2. Divide the numerator and the denominator by the greatest common factor, the product of all factors common to both.

Division lines can be used to show dividing out common factors from a fraction's numerator and denominator:

$$\frac{4}{6} = \frac{2 \cdot \cancel{2}}{3 \cdot \cancel{2}} = \frac{2}{3}.$$

Study Tip

When reducing a fraction to its lowest terms, only *factors* that are common to the numerator and the denominator can be divided out. **If you have not factored** and expressed the numerator and denominator in terms of multiplication, **do not divide out**.

Correct:

$$\frac{2 \cdot \cancel{2}}{3 \cdot \cancel{2}} = \frac{2}{3}$$

Incorrect:

$$\frac{2 + \cancel{2}}{3 + \cancel{2}} = \frac{2}{3}$$

Note that $\frac{2+2}{3+2} = \frac{4}{5}$, not $\frac{2}{3}$.

EXAMPLE 4 Reducing Fractions

Reduce each fraction to its lowest terms:

a. $\frac{6}{14}$ **b.** $\frac{15}{75}$ **c.** $\frac{25}{11}$ **d.** $\frac{11}{33}$.

Solution For each fraction, begin with the prime factorization of the numerator and the denominator.

a. $\frac{6}{14} = \frac{3 \cdot \cancel{2}}{7 \cdot \cancel{2}} = \frac{3}{7}$ 2 is the greatest common factor of 6 and 14. Divide the numerator and the denominator by 2.

Including 1 as a factor is helpful when all other factors can be divided out.

b. $\frac{15}{75} = \frac{3 \cdot 5}{3 \cdot 25} = \frac{1 \cdot \cancel{3} \cdot \cancel{5}}{\cancel{3} \cdot \cancel{5} \cdot 5} = \frac{1}{5}$ $3 \cdot 5$, or 15, is the greatest common factor of 15 and 75. Divide the numerator and the denominator by $3 \cdot 5$.

c. $\frac{25}{11} = \frac{5 \cdot 5}{1 \cdot 11}$

Because 11 and 25 share no common factor (other than 1), $\frac{11}{25}$ is already reduced to its lowest terms.

d. $\frac{11}{33} = \frac{1 \cdot \cancel{11}}{3 \cdot \cancel{11}} = \frac{1}{3}$ 11 is the greatest common factor of 11 and 33. Divide the numerator and denominator by 11.

When reducing fractions, it may not always be necessary to write prime factorizations. In some cases, you can use inspection to find the greatest common factor of the numerator and the denominator. For example, when reducing $\frac{15}{75}$, you can use 15 rather than $3 \cdot 5$:

$$\frac{15}{75} = \frac{1 \cdot \cancel{15}}{5 \cdot \cancel{15}} = \frac{1}{5}.$$

✓ CHECK POINT 4 Reduce each fraction to its lowest terms:

a. $\frac{10}{15}$ **b.** $\frac{42}{24}$ **c.** $\frac{13}{15}$ **d.** $\frac{9}{45}$.

---

4 Multiply fractions.

Multiplying Fractions

The result of multiplying two fractions is called their **product**.

Multiplying Fractions

In words: The product of two or more fractions is the product of their numerators divided by the product of their denominators.

In algebraic language: If $\frac{a}{b}$ and $\frac{c}{d}$ are fractions, then

$$\frac{a}{b} \cdot \frac{c}{d} = \frac{a \cdot c}{b \cdot d}.$$

Here is an example that illustrates the rule in the previous box:

$$\frac{3}{8} \cdot \frac{5}{11} = \frac{3 \cdot 5}{8 \cdot 11} = \frac{15}{88}.$$

The product of $\frac{3}{8}$ and $\frac{5}{11}$ is $\frac{15}{88}$.

Multiply numerators and multiply denominators.

EXAMPLE 5 Multiplying Fractions

Multiply. If possible, reduce the product to its lowest terms:

a. $\frac{3}{7} \cdot \frac{2}{5}$ **b.** $5 \cdot \frac{7}{12}$ **c.** $\frac{2}{3} \cdot \frac{9}{4}$ **d.** $\left(3\frac{2}{3}\right)\left(1\frac{1}{4}\right)$.

Solution

a. $\frac{3}{7} \cdot \frac{2}{5} = \frac{3 \cdot 2}{7 \cdot 5} = \frac{6}{35}$ Multiply numerators and multiply denominators.

b. $5 \cdot \frac{7}{12} = \frac{5}{1} \cdot \frac{7}{12} = \frac{5 \cdot 7}{1 \cdot 12} = \frac{35}{12}$ or $2\frac{11}{12}$ Write 5 as . Then multiply numerators and multiply denominators.

c. $\frac{2}{3} \cdot \frac{9}{4} = \frac{2 \cdot 9}{3 \cdot 4} = \frac{18}{12} = \frac{3 \cdot \cancel{6}}{2 \cdot \cancel{6}} = \frac{3}{2}$ or $1\frac{1}{2}$

Simplify $\frac{18}{12}$; 6 is the greatest common factor of 18 and 12.

d. $\left(3\frac{2}{3}\right)\left(1\frac{1}{4}\right) = \frac{11}{3} \cdot \frac{5}{4} = \frac{11 \cdot 5}{3 \cdot 4} = \frac{55}{12}$ or $4\frac{7}{12}$

✓ **CHECK POINT 5** Multiply. If possible, reduce the product to its lowest terms:

a. $\frac{4}{11} \cdot \frac{2}{3}$ **b.** $6 \cdot \frac{3}{5}$

c. $\frac{3}{7} \cdot \frac{2}{3}$ **d.** $\left(3\frac{2}{5}\right)\left(1\frac{1}{2}\right)$.

Study Tip

You can divide numerators and denominators by common factors before performing multiplication. Then multiply the remaining factors in the numerators and multiply the remaining factors in the denominators. For example,

$$\frac{7}{15} \cdot \frac{20}{21} = \frac{\cancel{7} \cdot 1}{\cancel{5} \cdot 3} \cdot \frac{\cancel{5} \cdot 4}{\cancel{7} \cdot 3} = \frac{1 \cdot 4}{3 \cdot 3} = \frac{4}{9}.$$

7 is the greatest common factor of 7 and 21.

5 is the greatest common factor of 15 and 20.

The divisions involving the common factors, 7 and 5, are often shown as follows:

$$\frac{7}{15} \cdot \frac{20}{21} = \frac{\overset{1}{\cancel{7}}}{\underset{3}{\cancel{15}}} \cdot \frac{\overset{4}{\cancel{20}}}{\underset{3}{\cancel{21}}} = \frac{1 \cdot 4}{3 \cdot 3} = \frac{4}{9}.$$

Divide by 7.

Divide by 5.

5 Divide fractions.

Dividing Fractions

The result of dividing two fractions is called their **quotient**. A geometric figure is useful for developing a process for determining the quotient of two fractions.

Consider the division

$$\frac{4}{5} \div \frac{1}{10}.$$

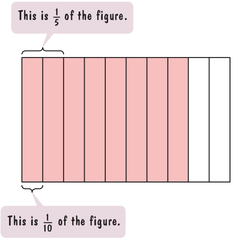


FIGURE 1.4

We want to know how many $\frac{1}{10}$'s are in $\frac{4}{5}$. We can use **Figure 1.4** to find this quotient. The rectangle is divided into fifths. The dashed lines further divide the rectangle into tenths.

Figure 1.4 shows that $\frac{4}{5}$ of the rectangle is red. How many $\frac{1}{10}$'s of the rectangle does this include? Can you see that this includes eight of the $\frac{1}{10}$ pieces? Thus, there are eight $\frac{1}{10}$'s in $\frac{4}{5}$:

$$\frac{4}{5} \div \frac{1}{10} = 8.$$

We can obtain the quotient 8 in the following way:

$$\frac{4}{5} \div \frac{1}{10} = \frac{4}{5} \cdot \frac{10}{1} = \frac{4 \cdot 10}{5 \cdot 1} = \frac{40}{5} = 8.$$

Change the division to multiplication.

Invert the divisor, $\frac{1}{10}$.

By inverting the divisor, $\frac{1}{10}$, and obtaining $\frac{10}{1}$, we are writing the divisor's *reciprocal*. Two fractions are **reciprocals** of each other if their product is 1. Thus, $\frac{1}{10}$ and $\frac{10}{1}$ are reciprocals because $\frac{1}{10} \cdot \frac{10}{1} = 1$.

Generalizing from the result shown on the previous page and using the word *reciprocal*, we obtain the following rule for dividing fractions:

Dividing Fractions

In words: The quotient of two fractions is the first fraction multiplied by the reciprocal of the second fraction.

In algebraic language: If $\frac{a}{b}$ and $\frac{c}{d}$ are fractions and $\frac{c}{d}$ is not 0, then

$$\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{c}.$$

Change division to multiplication.

Invert $\frac{c}{d}$ and write its reciprocal.

EXAMPLE 6 Dividing Fractions

Divide:

a. $\frac{2}{3} \div \frac{7}{15}$ **b.** $\frac{3}{4} \div 5$ **c.** $4\frac{3}{4} \div 1\frac{1}{2}$.

Solution

a. $$\frac{2}{3} \div \frac{7}{15} = \frac{2}{3} \cdot \frac{15}{7} = \frac{2 \cdot 15}{3 \cdot 7} = \frac{30}{21} = \frac{10 \cdot \cancel{3}}{7 \cdot \cancel{3}} = \frac{10}{7} \text{ or } 1\frac{3}{7}$$

Change division to multiplication.

Invert $\frac{7}{15}$ and write its reciprocal.

Simplify: 3 is the greatest common factor of 30 and 21.

b. $$\frac{3}{4} \div 5 = \frac{3}{4} \div \frac{5}{1} = \frac{3}{4} \cdot \frac{1}{5} = \frac{3 \cdot 1}{4 \cdot 5} = \frac{3}{20}$$

Change division to multiplication.

Invert $\frac{5}{1}$ and write its reciprocal.

c. $$4\frac{3}{4} \div 1\frac{1}{2} = \frac{19}{4} \div \frac{3}{2} = \frac{19}{4} \cdot \frac{2}{3} = \frac{19 \cdot 2}{4 \cdot 3} = \frac{38}{12} = \frac{19 \cdot \cancel{2}}{6 \cdot \cancel{2}} = \frac{19}{6} \text{ or } 3\frac{1}{6}$$

✓ **CHECK POINT 6** Divide:

a. $\frac{5}{4} \div \frac{3}{8}$ **b.** $\frac{2}{3} \div 3$ **c.** $3\frac{3}{8} \div 2\frac{1}{4}$.

---

6 Add and subtract fractions with identical denominators.

Adding and Subtracting Fractions with Identical Denominators

The result of adding two fractions is called their **sum**. The result of subtracting two fractions is called their **difference**. A geometric figure is useful for developing a process for determining the sum or difference of two fractions with identical denominators.

Consider the addition

$$\frac{3}{7} + \frac{2}{7}.$$

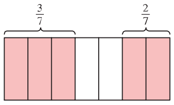


FIGURE 1.5

We can use **Figure 1.5** to find this sum. The rectangle is divided into sevenths. On the left, $\frac{3}{7}$ of the rectangle is red. On the right, $\frac{2}{7}$ of the rectangle is red. Including both the left and the right, a total of $\frac{5}{7}$ of the rectangle is red. Thus,

$$\frac{3}{7} + \frac{2}{7} = \frac{5}{7}.$$

We can obtain the sum $\frac{5}{7}$ in the following way:

$$\frac{3}{7}+\frac{2}{7}=\frac{3+2}{7}=\frac{5}{7}.$$

Add numerators and put this result over the common denominator.

Generalizing from this result gives us the following rule:

Adding and Subtracting Fractions with Identical Denominators

In words: The sum or difference of two fractions with identical denominators is the sum or difference of their numerators over the common denominator.

In algebraic language: If $\frac{a}{b}$ and $\frac{c}{b}$ are fractions, then

$$\frac{a}{b}+\frac{c}{b}=\frac{a+c}{b} \quad \text{and} \quad \frac{a}{b}-\frac{c}{b}=\frac{a-c}{b}.$$

EXAMPLE 7 Adding and Subtracting Fractions with Identical Denominators

Perform the indicated operations:

a. $\frac{3}{11}+\frac{4}{11}$ **b.** $\frac{11}{12}-\frac{5}{12}$ **c.** $5\frac{1}{4}-2\frac{3}{4}$.

Solution

a. $\frac{3}{11}+\frac{4}{11}=\frac{3+4}{11}=\frac{7}{11}$

b. $\frac{11}{12}-\frac{5}{12}=\frac{11-5}{12}=\frac{6}{12}=\frac{1\cdot\cancel{6}}{2\cdot\cancel{6}}=\frac{1}{2}$

c. $5\frac{1}{4}-2\frac{3}{4}=\frac{21}{4}-\frac{11}{4}=\frac{21-11}{4}=\frac{10}{4}=\frac{\cancel{2}\cdot 5}{\cancel{2}\cdot 2}=\frac{5}{2}$ or $2\frac{1}{2}$

✓ CHECK POINT 7 Perform the indicated operations:

a. $\frac{2}{11}+\frac{3}{11}$ **b.** $\frac{5}{6}-\frac{1}{6}$ **c.** $3\frac{3}{8}-1\frac{1}{8}$.

---

7 Add and subtract fractions with unlike denominators.

Adding and Subtracting Fractions with Unlike Denominators

How do we add or subtract fractions with different denominators? We must first rewrite them as equivalent fractions with the same denominator. We do this by using the Fundamental Principle of Fractions: The value of a fraction does not change if the numerator and the denominator are multiplied by the same nonzero number. Thus, if $\frac{a}{b}$ is a fraction and c is a nonzero number, then

$$\frac{a}{b}=\frac{a\cdot c}{b\cdot c}.$$

EXAMPLE 8 Writing an Equivalent Fraction

Write $\frac{3}{4}$ as an equivalent fraction with a denominator of 16.

Solution To obtain a denominator of 16, we must multiply the denominator of the given fraction, $\frac{3}{4}$, by 4. So that we do not change the value of the fraction, we also multiply the numerator by 4.

$$\frac{3}{4} = \frac{3 \cdot 4}{4 \cdot 4} = \frac{12}{16}$$

✓ **CHECK POINT 8** Write $\frac{2}{3}$ as an equivalent fraction with a denominator of 21.

Equivalent fractions can be used to add fractions with different denominators, such as $\frac{1}{2}$ and $\frac{1}{3}$. **Figure 1.6** indicates that the sum of half the whole figure and one-third of the whole figure results in 5 parts out of 6, or $\frac{5}{6}$, of the figure. Thus,

$$\frac{1}{2} + \frac{1}{3} = \frac{5}{6}.$$

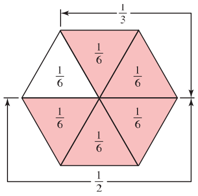


FIGURE 1.6 $\frac{1}{2} + \frac{1}{3} = \frac{5}{6}$

We can obtain the sum $\frac{5}{6}$ if we rewrite each fraction as an equivalent fraction with a denominator of 6.

$$\begin{aligned} \frac{1}{2} + \frac{1}{3} &= \frac{1 \cdot 3}{2 \cdot 3} + \frac{1 \cdot 2}{3 \cdot 2} && \text{Rewrite each fraction as an equivalent fraction with a denominator of 6.} \\ &= \frac{3}{6} + \frac{2}{6} && \text{Perform the multiplications. We now have a common denominator.} \\ &= \frac{3 + 2}{6} && \text{Add the numerators and place this sum over the common denominator.} \\ &= \frac{5}{6} && \text{Perform the addition.} \end{aligned}$$

When adding $\frac{1}{2}$ and $\frac{1}{3}$, there are many common denominators that we can use, such as 6, 12, 18, and so on. The given denominators, 2 and 3, divide into all of these numbers. However, the denominator 6 is the smallest number that 2 and 3 divide into. For this reason, 6 is called the *least common denominator*, abbreviated LCD.

Adding and Subtracting Fractions with Unlike Denominators

1. Rewrite the fractions as equivalent fractions with the least common denominator.
2. Add or subtract the numerators, putting this result over the common denominator.

EXAMPLE 9 Adding and Subtracting Fractions with Unlike Denominators

Perform the indicated operation:

a. $\frac{1}{5} + \frac{3}{4}$ **b.** $\frac{3}{4} - \frac{1}{6}$ **c.** $2\frac{7}{15} - 1\frac{4}{5}$.

Solution

a. Just by looking, can you tell that the smallest number divisible by both 5 and 4 is 20? Thus, the least common denominator for the denominators 5 and 4 is 20. We rewrite both fractions as equivalent fractions with the least common denominator, 20.

Discover for Yourself

Try Example 9(a), $\frac{1}{5} + \frac{3}{4}$, using a common denominator of 40. Because both 5 and 4 divide into 40, 40 is a common denominator, although not the *least* common denominator. Describe what happens. What is the advantage of using the least common denominator?

$$\frac{1}{5} + \frac{3}{4} = \frac{1 \cdot 4}{5 \cdot 4} + \frac{3 \cdot 5}{4 \cdot 5}$$

To obtain denominators of 20, multiply the numerator and denominator of the first fraction by 4 and the second fraction by 5.

$$= \frac{4}{20} + \frac{15}{20}$$

Perform the multiplications.

$$= \frac{4 + 15}{20}$$

Add the numerators and put this sum over the least common denominator.

$$= \frac{19}{20}$$

Perform the addition.

b. By looking at $\frac{3}{4} - \frac{1}{6}$, can you tell that the smallest number divisible by both 4 and 6 is 12? Thus, the least common denominator for the denominators 4 and 6 is 12. We rewrite both fractions as equivalent fractions with the least common denominator, 12.

$$\frac{3}{4} - \frac{1}{6} = \frac{3 \cdot 3}{4 \cdot 3} - \frac{1 \cdot 2}{6 \cdot 2}$$

To obtain denominators of 12, multiply the numerator and denominator of the first fraction by 3 and the second fraction by 2.

$$= \frac{9}{12} - \frac{2}{12}$$

Perform the multiplications.

$$= \frac{9 - 2}{12}$$

Subtract the numerators and put this difference over the least common denominator.

$$= \frac{7}{12}$$

Perform the subtraction.

c. $2\frac{7}{15} - 1\frac{4}{5} = \frac{37}{15} - \frac{9}{5}$ Convert each mixed number to an improper fraction.

The smallest number divisible by both 15 and 5 is 15. Thus, the least common denominator for the denominators 15 and 5 is 15. Because the first fraction already has a denominator of 15, we only have to rewrite the second fraction.

$$= \frac{37}{15} - \frac{9 \cdot 3}{5 \cdot 3}$$

To obtain denominators of 15, multiply the numerator and denominator of the second fraction by 3.

$$= \frac{37}{15} - \frac{27}{15}$$

Perform the multiplications.

$$= \frac{37 - 27}{15}$$

Subtract the numerators and put this difference over the common denominator.

$$= \frac{10}{15}$$

Perform the subtraction.

$$= \frac{2 \cdot \cancel{5}}{3 \cdot \cancel{5}} = \frac{2}{3}$$

Reduce to lowest terms.

✓ **CHECK POINT 9** Perform the indicated operation:

a. $\frac{1}{2} + \frac{3}{5}$ **b.** $\frac{4}{3} - \frac{3}{4}$ **c.** $3\frac{1}{6} - 1\frac{11}{12}$.

EXAMPLE 10 Using Prime Factorizations to Find the LCD

Perform the indicated operation: $\frac{1}{15} + \frac{7}{24}$.

Solution We need to first find the least common denominator. Using inspection, it is difficult to determine the smallest number divisible by both 15 and 24. We will use their prime factorizations to find the least common denominator:

$$15 = 5 \cdot 3 \quad \text{and} \quad 24 = 8 \cdot 3 = 2 \cdot 2 \cdot 2 \cdot 3.$$

The different prime factors are 5, 3, and 2. The least common denominator is obtained by using the greatest number of times each factor appears in any prime factorization. Because 5 and 3 appear as prime factors and 2 is a factor of 24 three times, the least common denominator is

$$5 \cdot 3 \cdot 2 \cdot 2 \cdot 2 = 5 \cdot 3 \cdot 8 = 120.$$

Now we can rewrite both fractions as equivalent fractions with the least common denominator, 120.

$$\begin{aligned} \frac{1}{15} + \frac{7}{24} &= \frac{1 \cdot 8}{15 \cdot 8} + \frac{7 \cdot 5}{24 \cdot 5} && \text{To obtain denominators of 120, multiply the numerator and denominator of the first fraction by 8 and the second fraction by 5.} \\ &= \frac{8}{120} + \frac{35}{120} && \text{Perform the multiplications.} \\ &= \frac{8+35}{120} && \text{Add the numerators and put this sum over the least common denominator.} \\ &= \frac{43}{120} && \text{Perform the addition.} \end{aligned}$$

✓ **CHECK POINT 10** Perform the indicated operation: $\frac{3}{10} + \frac{7}{12}$.

8 Solve problems involving fractions in algebra.

Fractions in Algebra

Fractions appear throughout algebra. Operations with fractions can be used to determine whether a particular fraction is a solution of an equation.

EXAMPLE 11 Determining Whether Fractions Are Solutions of Equations

Determine whether the given number is a solution of the equation.

a. $x + \frac{1}{4}x = 7; 6\frac{2}{5}$ **b.** $\frac{1}{7} - w = \frac{1}{2}w; \frac{2}{21}$

Solution

a. To determine whether $6\frac{2}{5}$ is a solution of $x + \frac{1}{4}x = 7$, we begin by converting $6\frac{2}{5}$ from a mixed number to an improper fraction.

$$6\frac{2}{5} = \frac{5 \cdot 6 + 2}{5} = \frac{30+2}{5} = \frac{32}{5}$$

Now we substitute $\frac{32}{5}$ for x.

$$x + \frac{1}{4}x = 7 \qquad \text{This is the given equation.}$$

Is $\frac{32}{5}$ a solution?

$$\begin{aligned} \frac{32}{5} + \frac{1}{4} \cdot \frac{32}{5} &\stackrel{?}{=} 7 && \text{Substitute } \frac{32}{5} \text{ for } x. \\ \frac{32}{5} + \frac{8}{5} &\stackrel{?}{=} 7 && \text{Multiply: } \frac{1}{\overset{1}{\cancel{4}}} \cdot \frac{\overset{8}{\cancel{32}}}{5} = \frac{1 \cdot 8}{5} = \frac{8}{5}. \\ \frac{40}{5} &\stackrel{?}{=} 7 && \text{Add: } \frac{32}{5} + \frac{8}{5} = \frac{32+8}{5} = \frac{40}{5}. \\ 8 &= 7 && \text{Simplify: } \frac{40}{5} = 8. \end{aligned}$$

This statement is false.

Because the values on both sides of the equation are not the same, the fraction $\frac{32}{5}$, or equivalently $6\frac{2}{5}$, is not a solution of the equation.

b.

$$\frac{1}{7} - w = \frac{1}{2}w$$

This is the given equation.

Is $\frac{2}{21}$ a solution?

$$\frac{1}{7} - \frac{2}{21} \stackrel{?}{=} \frac{1}{2} \cdot \frac{2}{21}$$

Substitute $\frac{2}{21}$ for w.

$$\frac{1}{7} - \frac{2}{21} \stackrel{?}{=} \frac{1}{21}$$

Multiply: $\frac{1}{\underset{1}{\cancel{2}}} \cdot \frac{\overset{1}{\cancel{2}}}{21} = \frac{1 \cdot 1}{1 \cdot 21} = \frac{1}{21}$.

$$\frac{1 \cdot 3}{7 \cdot 3} - \frac{2}{21} \stackrel{?}{=} \frac{1}{21}$$

Turn to the subtraction. The LCD is 21, so multiply the numerator and denominator of the first fraction by 3.

$$\frac{3}{21} - \frac{2}{21} \stackrel{?}{=} \frac{1}{21}$$

Perform the multiplications.

This statement is true.

$$\frac{1}{21} = \frac{1}{21}$$

Subtract: $\frac{3}{21} - \frac{2}{21} = \frac{3-2}{21} = \frac{1}{21}$.

Because the values on both sides of the equation are the same, the fraction $\frac{2}{21}$ is a solution of the equation.

✓ **CHECK POINT 11** Determine whether the given number is a solution of the equation.

a. $x - \frac{2}{9}x = 1; 1\frac{2}{7}$

b. $\frac{1}{5} - w = \frac{1}{3}w; \frac{3}{20}$

In Section 1.1, we translated phrases into algebraic expressions and sentences into equations. When these phrases and sentences involve fractions, the word *of* frequently appears. **When used with fractions, the word *of* represents multiplication.** For example, the phrase "$\frac{2}{5}$ of a number" can be represented by the algebraic expression $\frac{2}{5} \cdot x$, or $\frac{2}{5}x$.

EXAMPLE 12 Algebraic Representations of Phrases and Sentences with Fractions

Translate from English to an algebraic expression or equation, whichever is appropriate. Let the variable x represent the number.

a. $\frac{1}{3}$ of a number increased by 5 is half of that number.

b. $\frac{1}{4}$ of a number, decreased by 7

Solution Part (a) is a sentence, so we will translate into an equation. Part (b) is a phrase, so we will translate into an algebraic expression.

a.

$$\overset{\frac{1}{3}}{\frac{1}{3}} \quad \overset{\text{of}}{\cdot} \quad \overset{\text{a number increased by 5}}{(x + 5)} \quad \overset{\text{is}}{=} \quad \overset{\text{half}}{\frac{1}{2}} \quad \overset{\text{of}}{\cdot} \quad \overset{\text{that number.}}{x}$$

The equation for "$\frac{1}{3}$ of a number increased by 5 is half of that number" is $\frac{1}{3}(x + 5) = \frac{1}{2}x$.

b.

$$\begin{array}{ccccc} \boxed{\tfrac{1}{4}} & \boxed{\text{of}} & \boxed{\text{a number}}\,, & & \boxed{\text{decreased by 7}} \\ \frac{1}{4} & \cdot & x & - & 7 \end{array}$$

The algebraic expression for "$\frac{1}{4}$ of a number, decreased by 7" is $\frac{1}{4}x - 7$. ■

✓ **CHECK POINT 12** Translate from English to an algebraic expression or equation, whichever is appropriate. Let the variable x represent the number.

a. $\frac{2}{3}$ of a number decreased by 6

b. $\frac{3}{4}$ of a number, decreased by 2, is $\frac{1}{5}$ of that number.

Many formulas and mathematical models contain fractions. For example, consider temperatures on the Celsius scale and on the Fahrenheit scale, as shown in **Figure 1.7**. The formula $C = \frac{5}{9}(F - 32)$ expresses the relationship between Fahrenheit temperature, F, and Celsius temperature, C.

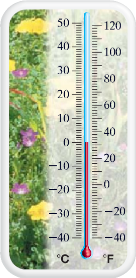


FIGURE 1.7 The Celsius scale is on the left and the Fahrenheit scale is on the right.

The Formula	What the Formula Tells Us
$C = \frac{5}{9}(F - 32)$	If 32 is subtracted from the Fahrenheit temperature, $F - 32$, and this difference is multiplied by , the resulting product, $(F - 32)$, gives the Celsius temperature.

EXAMPLE 13 Evaluating a Formula Containing a Fraction

The temperature on a warm summer day is 86°F. Use the formula $C = \frac{5}{9}(F - 32)$ to find the equivalent temperature on the Celsius scale.

Solution Because the temperature is 86°F, we substitute 86 for F in the given formula. Then we determine the value of C.

$$C = \frac{5}{9}(F - 32) \quad \text{This is the given formula.}$$

$$C = \frac{5}{9}(86 - 32) \quad \text{Replace } F \text{ with } 86.$$

$$C = \frac{5}{9}(54) \quad \text{Work inside parentheses first: } 86 - 32 = 54.$$

$$C = 30 \quad \text{Multiply: } \frac{5}{9}(54) = \frac{5}{\underset{1}{\cancel{9}}} \cdot \frac{\overset{6}{\cancel{54}}}{1} = \frac{5 \cdot 6}{1 \cdot 1} = \frac{30}{1} = 30.$$

Thus, 86°F is equivalent to 30°C. ■

✓ **CHECK POINT 13** The temperature on a warm spring day is 77°F. Use the formula $C = \frac{5}{9}(F - 32)$ to find the equivalent temperature on the Celsius scale.

1.2 EXERCISE SET

MyMathLab MathXL PRACTICE WATCH DOWNLOAD READ REVIEW

Study Tip

Don't panic at the length of the Exercise Sets. You are not expected to work all, or even most, of the problems. Your instructor will provide guidance on which exercises to work by assigning those problems that are consistent with the goals and objectives of your course.

Practice Exercises

In Exercises 1–6, convert each mixed number to an improper fraction.

1. $2\frac{3}{8}$

2. $2\frac{7}{9}$

3. $7\frac{3}{5}$

4. $6\frac{2}{5}$

5. $8\frac{7}{16}$

6. $9\frac{5}{16}$

In Exercises 7–12, convert each improper fraction to a mixed number.

7. $\frac{23}{5}$

8. $\frac{47}{8}$

9. $\frac{76}{9}$

10. $\frac{59}{9}$

11. $\frac{711}{20}$

12. $\frac{788}{25}$

In Exercises 13–28, identify each natural number as prime or composite. If the number is composite, find its prime factorization.

13. 22

14. 15

15. 20

16. 75

17. 37

18. 23

19. 36

20. 100

21. 140

22. 110

23. 79

24. 83

25. 81

26. 64

27. 240

28. 360

In Exercises 29–40, simplify each fraction by reducing it to its lowest terms.

29. $\frac{10}{16}$

30. $\frac{8}{14}$

31. $\frac{15}{18}$

32. $\frac{18}{45}$

33. $\frac{35}{50}$

34. $\frac{45}{50}$

35. $\frac{32}{80}$

36. $\frac{75}{80}$

37. $\frac{44}{50}$

38. $\frac{38}{50}$

39. $\frac{120}{86}$

40. $\frac{116}{86}$

In Exercises 41–90, perform the indicated operation. Where possible, reduce the answer to its lowest terms.

41. $\frac{2}{5} \cdot \frac{1}{3}$

42. $\frac{3}{7} \cdot \frac{1}{4}$

43. $\frac{3}{8} \cdot \frac{7}{11}$

44. $\frac{5}{8} \cdot \frac{3}{11}$

45. $9 \cdot \frac{4}{7}$

46. $8 \cdot \frac{3}{7}$

47. $\frac{1}{10} \cdot \frac{5}{6}$

48. $\frac{1}{8} \cdot \frac{2}{3}$

49. $\frac{5}{4} \cdot \frac{6}{7}$

50. $\frac{7}{4} \cdot \frac{6}{11}$

51. $\left(3\frac{3}{4}\right)\left(1\frac{3}{5}\right)$

52. $\left(2\frac{4}{5}\right)\left(1\frac{1}{4}\right)$

53. $\frac{5}{4} \div \frac{4}{3}$

54. $\frac{7}{8} \div \frac{2}{3}$

55. $\frac{18}{5} \div 2$

56. $\frac{12}{7} \div 3$

57. $2 \div \frac{18}{5}$

58. $3 \div \frac{12}{7}$

59. $\frac{3}{4} \div \frac{1}{4}$

60. $\frac{3}{7} \div \frac{1}{7}$

61. $\frac{7}{6} \div \frac{5}{3}$

62. $\frac{7}{4} \div \frac{3}{8}$

63. $\frac{1}{14} \div \frac{1}{7}$

64. $\frac{1}{8} \div \frac{1}{4}$

65. $6\frac{3}{5} \div 1\frac{1}{10}$

66. $1\frac{3}{4} \div 2\frac{5}{8}$

67. $\frac{2}{11} + \frac{4}{11}$

68. $\frac{5}{13} + \frac{2}{13}$

69. $\frac{7}{12} + \frac{1}{12}$

70. $\frac{5}{16} + \frac{1}{16}$

71. $\frac{5}{8} + \frac{5}{8}$

72. $\frac{3}{8} + \frac{3}{8}$

73. $\frac{7}{12} - \frac{5}{12}$

74. $\frac{13}{18} - \frac{5}{18}$

75. $\frac{16}{7} - \frac{2}{7}$

76. $\frac{17}{5} - \frac{2}{5}$

77. $\frac{1}{2} + \frac{1}{5}$

78. $\frac{1}{3} + \frac{1}{5}$

79. $\frac{3}{4} + \frac{3}{20}$

80. $\frac{2}{5} + \frac{2}{15}$

81. $\frac{3}{8} + \frac{5}{12}$

82. $\frac{3}{10} + \frac{2}{15}$

83. $\frac{11}{18} - \frac{2}{9}$

84. $\frac{17}{18} - \frac{4}{9}$

85. $\frac{4}{3} - \frac{3}{4}$

86. $\frac{3}{2} - \frac{2}{3}$

87. $\frac{7}{10} - \frac{3}{16}$

88. $\frac{7}{30} - \frac{5}{24}$

89. $3\frac{3}{4} - 2\frac{1}{3}$

90. $3\frac{2}{3} - 2\frac{1}{2}$

In Exercises 91–102, determine whether the given number is a solution of the equation.

91. $\frac{7}{2}x = 28; 8$

92. $\frac{5}{3}x = 30; 18$

93. $w - \frac{2}{3} = \frac{3}{4}; 1\frac{5}{12}$

94. $w - \frac{3}{4} = \frac{7}{4}; 2\frac{1}{2}$

95. $20 - \frac{1}{3}z = \frac{1}{2}z; 12$

96. $12 - \frac{1}{4}z = \frac{1}{2}z; 20$

97. $\frac{2}{9}y + \frac{1}{3}y = \frac{3}{7}; \frac{27}{35}$

98. $\frac{2}{3}y + \frac{5}{6}y = 2; 1\frac{1}{3}$

99. $\frac{1}{3}(x - 2) = \frac{1}{5}(x + 4) + 3; 26$

100. $\frac{1}{2}(x - 2) + 3 = \frac{3}{8}(3x - 4); 4$

101. $(y \div 6) + \frac{2}{3} = (y \div 2) - \frac{7}{9}; 4\frac{1}{3}$

102. $(y \div 6) + \frac{1}{3} = (y \div 2) - \frac{5}{9}; 2\frac{2}{3}$

In Exercises 103–114, translate from English to an algebraic expression or equation, whichever is appropriate. Let the variable x represent the number.

103. $\frac{1}{5}$ of a number

104. $\frac{1}{6}$ of a number

105. A number decreased by $\frac{1}{4}$ of itself

106. A number decreased by $\frac{1}{3}$ of itself

107. A number decreased by $\frac{1}{4}$ is half of that number.

108. A number decreased by $\frac{1}{3}$ is half of that number.

109. The sum of $\frac{1}{7}$ of a number and $\frac{1}{8}$ of that number gives 12.

110. The sum of $\frac{1}{9}$ of a number and $\frac{1}{10}$ of that number gives 15.

111. The product of $\frac{2}{3}$ and a number increased by 6

112. The product of $\frac{3}{4}$ and a number increased by 9

113. The product of $\frac{2}{3}$ and a number, increased by 6, is 3 less than the number.

114. The product of $\frac{3}{4}$ and a number, increased by 9, is 2 less than the number.

Practice PLUS

In Exercises 115–118, perform the indicated operation. Write the answer as an algebraic expression.

115. $\frac{3}{4}\cdot\frac{a}{5}$ **116.** $\frac{2}{3}\div\frac{a}{7}$

117. $\frac{11}{x}+\frac{9}{x}$ **118.** $\frac{10}{y}-\frac{6}{y}$

In Exercises 119–120, perform the indicated operations. Begin by performing operations in parentheses.

119. $\left(\frac{1}{2}-\frac{1}{3}\right)\div\frac{5}{8}$ **120.** $\left(\frac{1}{2}+\frac{1}{4}\right)\div\left(\frac{1}{2}+\frac{1}{3}\right)$

In Exercises 121–122, determine whether the given number is a solution of the equation.

121. $\frac{1}{5}(x+2)=\frac{1}{2}\left(x-\frac{1}{5}\right); \frac{5}{8}$

122. $12-3(x-2)=4x-(x+3); 3\frac{1}{2}$

Application Exercises

The formula

$$C=\frac{5}{9}(F-32)$$

expresses the relationship between Fahrenheit temperature, F, and Celsius temperature, C. In Exercises 123–124, use the formula to convert the given Fahrenheit temperature to its equivalent temperature on the Celsius scale.

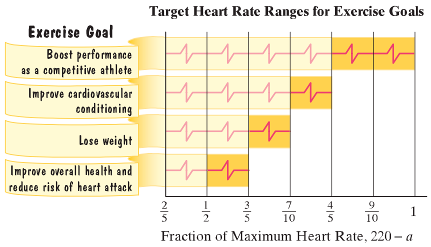
123. 68°F **124.** 41°F

The maximum heart rate, in beats per minute, that you should achieve during exercise is 220 minus your age:

$$220-a.$$

This algebraic expression gives maximum heart rate in terms of age, a.

The bar graph at the top of the next column shows the target heart rate ranges for four types of exercise goals. The lower and upper limits of these ranges are fractions of the maximum heart rate, 220 – a. Exercises 125–128 are based on the information in the graph.


Target Heart Rate Ranges for Exercise Goals

125. If your exercise goal is to improve cardiovascular conditioning, the graph shows the following range for target heart rate, H, in beats per minute:

Lower limit of range $H=\frac{7}{10}(220-a)$

Upper limit of range $H=\frac{4}{5}(220-a).$

a. What is the lower limit of the heart range, in beats per minute, for a 20-year-old with this exercise goal?

b. What is the upper limit of the heart range, in beats per minute, for a 20-year-old with this exercise goal?

126. If your exercise goal is to improve overall health, the graph shows the following range for target heart rate, H, in beats per minute:

Lower limit of range $H=\frac{1}{2}(220-a)$

Upper limit of range $H=\frac{3}{5}(220-a).$

a. What is the lower limit of the heart range, in beats per minute, for a 30-year-old with this exercise goal?

b. What is the upper limit of the heart range, in beats per minute, for a 30-year-old with this exercise goal?

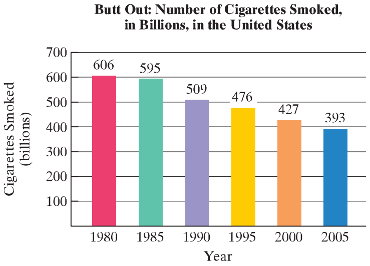
127. a. Write a formula that models the heart rate, H, in beats per minute, for a person who is a years old and would like to achieve $\frac{9}{10}$ of maximum heart rate during exercise.

b. Use your formula from part (a) to find the heart rate during exercise for a 40-year-old with this goal.

128. a. Write a formula that models the heart rate, H, in beats per minute, for a person who is a years old and would like to achieve $\frac{7}{8}$ of maximum heart rate during exercise.

b. Use your formula from part (a) to find the heart rate during exercise for a 20-year-old with this goal.

The graph shows the number of cigarettes smoked, in billions, in the United States for selected years from 1980 through 2005.



Source: Orzechowski and Walker

Here is a mathematical model that approximates the data displayed in the bar graph:

$$c = 615 - \frac{47}{5}n.$$

Number of cigarettes smoked, in billions

Number of years after 1980

Use this formula to solve Exercises 129–130.

129. Use the formula to find the number of cigarettes smoked, in billions, in 1995. Does the mathematical model underestimate or overestimate the actual number shown in the bar graph for 1995? By how much?

130. Use the formula to find the number of cigarettes smoked, in billions, in 2000. How well does the mathematical model describe the actual number shown in the bar graph for 2000?

Writing in Mathematics

131. Explain how to convert a mixed number to an improper fraction and give an example.

132. Explain how to convert an improper fraction to a mixed number and give an example.

133. Describe the difference between a prime number and a composite number.

134. What is meant by the prime factorization of a composite number?

135. What is the Fundamental Principle of Fractions?

136. Explain how to reduce a fraction to its lowest terms. Give an example with your explanation.

137. Explain how to multiply fractions and give an example.

138. Explain how to divide fractions and give an example.

139. Describe how to add or subtract fractions with identical denominators. Provide an example with your description.

140. Explain how to add fractions with different denominators. Use $\frac{5}{6} + \frac{1}{2}$ as an example.

Critical Thinking Exercises

Make Sense? *In Exercises 141–144, determine whether each statement "makes sense" or "does not make sense" and explain your reasoning.*

141. I find it easier to multiply $\frac{1}{5}$ and $\frac{3}{4}$ than to add them.

142. Fractions frustrated me in arithmetic, so I'm glad I won't have to use them in algebra.

143. I need to be able to perform operations with fractions to determine whether $\frac{3}{2}$ is a solution of $8x = 12\left(x - \frac{1}{2}\right)$.

144. I saved money by buying a computer for $\frac{3}{2}$ of its original price.

In Exercises 145–148, determine whether each statement is true or false. If the statement is false, make the necessary change(s) to produce a true statement.

145. $\frac{1}{2} + \frac{1}{5} = \frac{2}{7}$

146. $\frac{1}{2} \div 4 = 2$

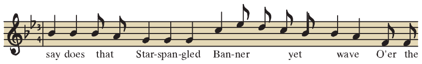
147. Every fraction has infinitely many equivalent fractions.

148. $\frac{3+7}{30} = \frac{\overset{1}{\cancel{3}}+7}{\underset{10}{\cancel{30}}} = \frac{8}{10} = \frac{4}{5}$

149. Shown below is a short excerpt from "The Star-Spangled Banner." The time is $\frac{3}{4}$, which means that each measure must contain notes that add up to $\frac{3}{4}$. The values of the different notes tell musicians how long to hold each note.

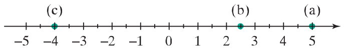
𝅝 $= 1$ 𝅗𝅥 $= \frac{1}{2}$ ♩ $= \frac{1}{4}$ ♪ $= \frac{1}{8}$

Use vertical lines to divide this line of "The Star-Spangled Banner" into measures.



Preview Exercises

Exercises 150–152 will help you prepare for the material covered in the next section. Consider the following "infinite ruler" that shows numbers that lie to the left and to the right of zero.



150. What number is represented by point (a)?

151. What number is represented by point (b)? Express the number as an improper fraction.

152. What number is represented by point (c)?

SECTION 1.3

Objectives

1. Define the sets that make up the real numbers.
2. Graph numbers on a number line.
3. Express rational numbers as decimals.
4. Classify numbers as belonging to one or more sets of the real numbers.
5. Understand and use inequality symbols.
6. Find the absolute value of a real number.

The Real Numbers

The U.N. building is designed with three golden rectangles.

The United Nations Building in New York was designed to represent its mission of promoting world harmony. Viewed from the front, the building looks like three rectangles stacked upon each other. In each rectangle, the ratio of the width to height is $\sqrt{5} + 1$ to 2, approximately 1.618 to 1. The ancient Greeks believed that such a rectangle, called a **golden rectangle**, was the most visually pleasing of all rectangles.

The ratio 1.618 to 1 is approximate because $\sqrt{5}$ is an irrational number, a special kind of real number. Irrational? Real? Let's make sense of all this by describing the kinds of numbers you will encounter in this course.

1 Define the sets that make up the real numbers.

Natural Numbers and Whole Numbers

Before we describe the set of real numbers, let's be sure you are familiar with some basic ideas about sets. A **set** is a collection of objects whose contents can be clearly determined. The objects in a set are called the **elements** of the set. For example, the set of numbers used for counting can be represented by

$$\{1, 2, 3, 4, 5, \dots\}.$$

The braces, { }, indicate that we are representing a set. This form of representing a set uses commas to separate the elements of the set. Remember that the three dots after the 5 indicate that there is no final element and that the listing goes on forever.

We have seen that the set of numbers used for counting is called the set of **natural numbers**. When we combine the number 0 with the natural numbers, we obtain the set of **whole numbers**.

Natural Numbers and Whole Numbers

The set of **natural numbers** is $\{1, 2, 3, 4, 5, \dots\}$.
The set of **whole numbers** is $\{0, 1, 2, 3, 4, 5, \dots\}$.

Integers and the Number Line

The whole numbers do not allow us to describe certain everyday situations. For example, if the balance in your checking account is \$30 and you write a check for \$35, your checking account is overdrawn by \$5. We can write this as -5, read *negative* 5. The set consisting of the natural numbers, 0, and the negatives of the natural numbers is called the set of **integers**.

Integers

The set of **integers** is

$$\{\underbrace{\ldots, -4, -3, -2, -1}_{\text{Negative integers}},\ 0,\ \underbrace{1, 2, 3, 4, \ldots}_{\text{Positive integers}}\}.$$

Notice that the term **positive integers** is another name for the natural numbers. The positive integers can be written in two ways:

1. Use a "+" sign. For example, +4 is "positive four."
2. Do not write any sign. For example, 4 is assumed to be "positive four."

EXAMPLE 1 Practical Examples of Negative Integers

Write a negative integer that describes each of the following situations:

a. A debt of \$10

b. The shore surrounding the Dead Sea is 1312 feet below sea level.

Solution

a. A debt of \$10 can be expressed by the negative integer −10 (negative ten).

b. The shore surrounding the Dead Sea is 1312 feet below sea level, expressed as −1312.

✓ **CHECK POINT 1** Write a negative integer that describes each of the following situations:

a. A debt of \$500

b. Death Valley, the lowest point in North America, is 282 feet below sea level.

---

2 Graph numbers on a number line.

The **number line** is a graph we use to visualize the set of integers, as well as other sets of numbers. The number line is shown in **Figure 1.8**.

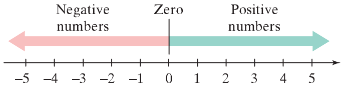


FIGURE 1.8 The number line

The number line extends indefinitely in both directions. Zero separates the positive numbers from the negative numbers on the number line. The positive integers are located to the right of 0, and the negative integers are located to the left of 0. Zero is neither positive nor negative. For every positive integer on a number line, there is a corresponding negative integer on the opposite side of 0.

Integers are graphed on a number line by placing a dot at the correct location for each number.

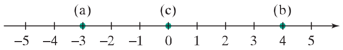
EXAMPLE 2 Graphing Integers on a Number Line

Graph:

a. −3 b. 4 c. 0.

Solution

Place a dot at the correct location for each integer.



✓ **CHECK POINT 2** Graph: **a.** -4 **b.** 0 **c.** 3.

Rational Numbers

If two integers are added, subtracted, or multiplied, the result is always another integer. This, however, is not always the case with division. For example, 10 divided by 5 is the integer 2. By contrast, 5 divided by 10 is $\frac{1}{2}$, and $\frac{1}{2}$ is not an integer. To permit divisions such as $\frac{5}{10}$, we enlarge the set of integers, calling the new collection the *rational numbers*. The set of **rational numbers** consists of all the numbers that can be expressed as a quotient of two integers, with the denominator not 0.

The Rational Numbers

The set of **rational numbers** is the set of all numbers that can be expressed in the form $\frac{a}{b}$, where a and b are integers and b is not equal to 0, written $b \neq 0$. The integer a is called the **numerator** and the integer b is called the **denominator**.

Study Tip

In Section 1.7, you will learn that a negative number divided by a positive number gives a negative result. Thus, $\frac{-3}{4}$ can also be written as $-\frac{3}{4}$.

Here are two examples of rational numbers:

- $\frac{1}{2}$ $a = 1$ $b = 2$
- $\frac{-3}{4}$ $a = -3$ $b = 4$

Is the integer 5 another example of a rational number? Yes. The integer 5 can be written with a denominator of 1.

$$5 = \frac{5}{1} \quad a = 5 \quad b = 1$$

All integers are also rational numbers because they can be written with a denominator of 1.

How can we express a negative mixed number, such as $-2\frac{3}{4}$, in the form $\frac{a}{b}$? Copy the negative sign and then follow the procedure discussed in the previous section:

$$-2\frac{3}{4} = -\frac{4 \cdot 2 + 3}{4} = -\frac{8 + 3}{4} = -\frac{11}{4} = \frac{-11}{4}. \quad a = -11 \quad b = 4$$

Copy the negative sign from step to step and convert $2\frac{3}{4}$ to an improper fraction.

Rational numbers are graphed on a number line by placing a dot at the correct location for each number.

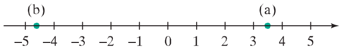
EXAMPLE 3 Graphing Rational Numbers on a Number Line

Graph: **a.** $\frac{7}{2}$ **b.** -4.6.

Solution Place a dot at the correct location for each rational number.

a. Because $\frac{7}{2} = 3\frac{1}{2}$, its graph is midway between 3 and 4.

b. Because $-4.6 = -4\frac{6}{10}$, its graph is $\frac{6}{10}$, or $\frac{3}{5}$, of a unit to the left of -4.



✓ **CHECK POINT 3** Graph: **a.** $\frac{9}{2}$ **b.** -1.2.

3 Express rational numbers as decimals.

Every rational number can be expressed as a fraction and as a decimal. To express the fraction $\frac{a}{b}$ as a decimal, divide the denominator, b, into the numerator, a.

EXAMPLE 4 Expressing Rational Numbers as Decimals

Express each rational number as a decimal:

a. $\frac{5}{8}$ **b.** $\frac{7}{11}$.

Solution In each case, divide the denominator into the numerator.

a.

$$\begin{array}{r} 0.625 \\ 8\overline{)5.000} \\ \underline{4\,8} \\ 20 \\ \underline{16} \\ 40 \\ \underline{40} \\ 0 \end{array}$$

b.

$$\begin{array}{r} 0.6363\ldots \\ 11\overline{)7.0000\ \ldots} \\ \underline{6\,6} \\ 40 \\ \underline{33} \\ 70 \\ \underline{66} \\ 40 \\ \underline{33} \\ 70 \\ \vdots \end{array}$$

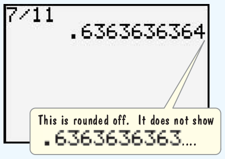
Using Technology

Calculators and Decimals

Given a rational number, $\frac{a}{b}$, you can express it as a decimal by entering

a $\boxed{\div}$ b

into a calculator. In the case of a repeating decimal, the calculator rounds off in the final decimal place displayed.



This is rounded off. It does not show .6363636363....

In Example 4, the decimal for $\frac{5}{8}$, namely 0.625, stops and is called a **terminating decimal**. Other examples of terminating decimals are

$$\tfrac{1}{4} = 0.25, \qquad \tfrac{2}{5} = 0.4, \qquad \text{and} \qquad \tfrac{7}{8} = 0.875.$$

By contrast, the division process for $\frac{7}{11}$ results in $0.6363\ldots$, with the digits 63 repeating over and over indefinitely. To indicate this, write a bar over the digits that repeat. Thus,

$$\tfrac{7}{11} = 0.\overline{63}.$$

The decimal for $\frac{7}{11}$, $0.\overline{63}$, is called a **repeating decimal**. Other examples of repeating decimals are

$$\tfrac{1}{3} = 0.333\ldots = 0.\overline{3} \qquad \text{and} \qquad \tfrac{2}{3} = 0.666\ldots = 0.\overline{6}.$$

Rational Numbers and Decimals

Any rational number can be expressed as a decimal. The resulting decimal will either terminate (stop), or it will have a digit that repeats or a block of digits that repeat.

☑ **CHECK POINT 4** Express each rational number as a decimal:

a. $\frac{3}{8}$ **b.** $\frac{5}{11}$. $\overline{45}$

Irrational Numbers

Can you think of a number that, when written in decimal form, neither terminates nor repeats? An example of such a number is $\sqrt{2}$ (read: "the square root of 2"). The number $\sqrt{2}$ is a number that can be multiplied by itself to obtain 2. No terminating or repeating decimal can be multiplied by itself to get 2. However, some approximations come close to 2.

- 1.4 is an approximation of $\sqrt{2}$:

$$1.4 \times 1.4 = 1.96.$$

- 1.41 is an approximation of $\sqrt{2}$:

$$1.41 \times 1.41 = 1.9881.$$

- 1.4142 is an approximation of $\sqrt{2}$:

$$1.4142 \times 1.4142 = 1.99996164.$$

Can you see how each approximation in the list is getting better? This is because the products are getting closer and closer to 2.

The number $\sqrt{2}$, whose decimal representation does not come to an end and does not have a block of repeating digits, is an example of an **irrational number**.

The Irrational Numbers

Any number that can be represented on the number line that is not a rational number is called an **irrational number**. Thus, the set of irrational numbers is the set of numbers whose decimal representations are neither terminating nor repeating.

Perhaps the best known of all the irrational numbers is π (pi). This irrational number represents the distance around a circle (its circumference) divided by the diameter of the circle. In the *Star Trek* episode "Wolf in the Fold," Spock foils an evil computer by telling it to "compute the last digit in the value of π." Because π is an irrational number, there is no last digit in its decimal representation:

$$\pi = 3.14159265358979323846264338327 95 \ldots .$$

Because irrational numbers cannot be represented by decimals that come to an end, mathematicians use symbols such as $\sqrt{2}$, $\sqrt{3}$, and π to represent these numbers. However, **not all square roots are irrational**. For example, $\sqrt{25} = 5$ because 5 multiplied by itself is 25. Thus, $\sqrt{25}$ is a natural number, a whole number, an integer, and a rational number $\left(\sqrt{25} = \frac{5}{1}\right)$.

Using Technology

You can obtain decimal approximations for irrational numbers using a calculator. For example, to approximate $\sqrt{2}$, use the following keystrokes:

Scientific Calculator	Graphing Calculator
$2\ \boxed{\sqrt{}}$ or $2\ \boxed{\begin{matrix}\text{2ND}\\ \text{INV}\end{matrix}}\ \boxed{x^2}$	$\boxed{\sqrt{}}\ 2\ \boxed{\text{ENTER}}$ or $\boxed{\begin{matrix}\text{2ND}\\ \text{INV}\end{matrix}}\ \boxed{x^2}\ 2\ \boxed{\text{ENTER}}$

Some graphing calculators show an open parenthesis after displaying $\sqrt{}$. In this case, enter a closed parenthesis, $\boxed{)}$, after 2.

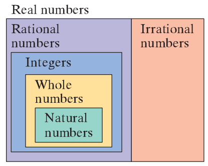
The display may read 1.41421356237, although your calculator may show more or fewer digits. Between which two integers would you graph $\sqrt{2}$ on a number line?

4 Classify numbers as belonging to one or more sets of the real numbers.

The Set of Real Numbers

All numbers that can be represented by points on the number line are called **real numbers**. Thus, the set of real numbers is formed by combining the rational numbers and the irrational numbers. Every real number is either rational or irrational.

The sets that make up the real numbers are summarized in **Table 1.2**. Notice the use of the symbol $\approx$ in the examples of irrational numbers. The symbol $\approx$ means "is approximately equal to."



This diagram shows that every real number is rational or irrational.

Table 1.2 The Sets that Make Up the Real Numbers

Name	Description	Examples
Natural numbers	$\{1, 2, 3, 4, 5, \ldots\}$ These numbers are used for counting.	2, 3, 5, 17
Whole numbers	$\{0, 1, 2, 3, 4, 5, \ldots\}$ The set of whole numbers is formed by adding 0 to the set of natural numbers.	0, 2, 3, 5, 17
Integers	$\{\ldots, -5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5, \ldots\}$ The set of integers is formed by adding negatives of the natural numbers to the set of whole numbers.	$-17, -5, -3, -2, 0, 2, 3, 5, 17$
Rational numbers	The set of rational numbers is the set of all numbers that can be expressed in the form $\frac{a}{b}$, where a and b are integers and b is not equal to 0, written $b \neq 0$. Rational numbers can be expressed as terminating or repeating decimals.	$-17 = \frac{-17}{1}, -5 = \frac{-5}{1}, -3, -2,$ $0, 2, 3, 5, 17,$ $\frac{2}{5} = 0.4,$ $\frac{-2}{3} = -0.6666\cdots = -0.\overline{6}$
Irrational numbers	The set of irrational numbers is the set of all numbers whose decimal representations are neither terminating nor repeating. Irrational numbers cannot be expressed as a quotient of integers.	$\sqrt{2} \approx 1.414214$ $-\sqrt{3} \approx -1.73205$ $\pi \approx 3.142$ $-\frac{\pi}{2} \approx -1.571$

EXAMPLE 5 Classifying Real Numbers

Consider the following set of numbers:

$$\left\{-7, -\frac{3}{4}, 0, 0.\overline{6}, \sqrt{5}, \pi, 7.3, \sqrt{81}\right\}.$$

List the numbers in the set that are

a. natural numbers.
b. whole numbers.
c. integers.
d. rational numbers.
e. irrational numbers.
f. real numbers.

Solution

a. Natural numbers: The natural numbers are the numbers used for counting. The only natural number in the set is $\sqrt{81}$ because $\sqrt{81} = 9$. (9 multiplied by itself is 81.)

b. Whole numbers: The whole numbers consist of the natural numbers and 0. The elements of the set that are whole numbers are 0 and $\sqrt{81}$.

c. Integers: The integers consist of the natural numbers, 0, and the negatives of the natural numbers. The elements of the set that are integers are $\sqrt{81}$, 0, and -7.

d. Rational numbers: All numbers in the set that can be expressed as the quotient of integers are rational numbers. These include $-7 \left(-7 = \frac{-7}{1}\right)$, $-\frac{3}{4}$, $0 \left(0 = \frac{0}{1}\right)$, and $\sqrt{81}\left(\sqrt{81} = \frac{9}{1}\right)$. Furthermore, all numbers in the set that are terminating or repeating decimals are also rational numbers. These include $0.\overline{6}$ and 7.3.

e. Irrational numbers: The irrational numbers in the set are $\sqrt{5}$ $\left(\sqrt{5} \approx 2.236\right)$ and π $(\pi \approx 3.14)$. Both $\sqrt{5}$ and π are only approximately equal to 2.236 and 3.14, respectively. In decimal form, $\sqrt{5}$ and π neither terminate nor have blocks of repeating digits.

f. Real numbers: All the numbers in the given set are real numbers. ■

☑ **CHECK POINT 5** Consider the following set of numbers:

$$\left\{-9, -1.3, 0, 0.\overline{3}, \frac{\pi}{2}, \sqrt{9}, \sqrt{10}\right\}.$$

List the numbers in the set that are

a. natural numbers. **b.** whole numbers.
c. integers. **d.** rational numbers. $\overline{3}$
e. irrational numbers. **f.** real numbers. $\overline{3}$

---

5 Understand and use inequality symbols.

Ordering the Real Numbers

On the real number line, the real numbers increase from left to right. The lesser of two real numbers is the one farther to the left on a number line. The greater of two real numbers is the one farther to the right on a number line.

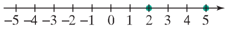


FIGURE 1.9

Look at the number line in **Figure 1.9**. The integers 2 and 5 are graphed. Observe that 2 is to the left of 5 on the number line. This means that 2 is less than 5.

$2 < 5$: 2 is less than 5 because 2 is to the *left* of 5 on the number line.

In **Figure 1.9**, we can also observe that 5 is to the right of 2 on the number line. This means that 5 is greater than 2.

$5 > 2$: 5 is greater than 2 because 5 is to the *right* of 2 on the number line.

The symbols $<$ and $>$ are called **inequality symbols**. These symbols always point to the lesser of the two real numbers when the inequality is true.

2 is less than 5. $2 < 5$ The symbol points to 2, the lesser number.
5 is greater than 2. $5 > 2$ The symbol points to 2, the lesser number.

EXAMPLE 6 Using Inequality Symbols

Insert either $<$ or $>$ in the shaded area between each pair of numbers to make a true statement:

a. 3 □ 17 **b.** −4.5 □ 1.2 **c.** −5 □ −83 **d.** $\frac{4}{5}$ □ $\frac{2}{3}$.

Solution In each case, mentally compare the graph of the first number to the graph of the second number. If the first number is to the left of the second number, insert the symbol $<$ for "is less than." If the first number is to the right of the second number, insert the symbol $>$ for "is greater than."

a. Compare the graphs of 3 and 17 on the number line. Because 3 is to the left of 17, this means that 3 is less than 17: $3 < 17$.

b. Compare the graphs of −4.5 and 1.2. Because −4.5 is to the left of 1.2, this means that −4.5 is less than 1.2: $-4.5 < 1.2$.

c. Compare the graphs of −5 and −83. Because −5 is to the right of −83, this means that −5 is greater than −83: $-5 > -83$.

d. Compare the graphs of $\frac{4}{5}$ and $\frac{2}{3}$. To do so, convert to decimal notation or use a common denominator. Using decimal notation, $\frac{4}{5} = 0.8$ and $\frac{2}{3} = 0.\overline{6}$. Because 0.8 is to the right of $0.\overline{6}$, this means that $\frac{4}{5}$ is greater than $\frac{2}{3}$: $\frac{4}{5} > \frac{2}{3}$.

✓ **CHECK POINT 6** Insert either $<$ or $>$ in the shaded area between each pair of numbers to make a true statement:

a. 14 ▭ 5 **b.** -5.4 ▭ 2.3 **c.** -19 ▭ -6 **d.** $\frac{1}{4}$ ▭ $\frac{1}{2}$.

The symbols $<$ and $>$ may be combined with an equal sign, as shown in the table.

This inequality is true if either the < part or the = part is true.

This inequality is true if either the > part or the = part is true.

Symbols	Meaning	Examples	Explanation
$a \le b$	a is less than or equal to b.	$3 \le 7$ $7 \le 7$	Because $3 < 7$ Because $7 = 7$
$b \ge a$	b is greater than or equal to a.	$7 \ge 3$ $-5 \ge -5$	Because $7 > 3$ Because $-5 = -5$

When using the symbol $\le$ (is less than or equal to), the inequality is a true statement if either the $<$ part or the $=$ part is true. When using the symbol $\ge$ (is greater than or equal to), the inequality is a true statement if either the $>$ part or the $=$ part is true.

EXAMPLE 7 Using Inequality Symbols

Determine whether each inequality is true or false:

a. $-7 \le 4$ **b.** $-7 \le -7$ **c.** $-9 \ge 6$.

Solution

a. $-7 \le 4$ is true because $-7 < 4$ is true.

b. $-7 \le -7$ is true because $-7 = -7$ is true.

c. $-9 \ge 6$ is false because neither $-9 > 6$ nor $-9 = 6$ is true.

✓ **CHECK POINT 7** Determine whether each inequality is true or false:

a. $-2 \le 3$ **b.** $-2 \ge -2$ **c.** $-4 \ge 1$.

---

6 Find the absolute value of a real number.

Absolute Value

Absolute value describes distance from 0 on a number line. If a represents a real number, the symbol $|a|$ represents its absolute value, read "the absolute value of a." For example,

$$|-5| = 5.$$

The absolute value of −5 is 5 because −5 is 5 units from 0 on a number line.

Absolute Value

The **absolute value** of a real number a, denoted by $|a|$, is the distance from 0 to a on a number line. Because absolute value describes a distance, it is never negative.

EXAMPLE 8 Finding Absolute Value

Find the absolute value:

a. $|-3|$ **b.** $|5|$ **c.** $|0|$.

Solution The solution is illustrated in **Figure 1.10**.

a. $|-3| = 3$ The absolute value of -3 is 3 because -3 is 3 units from 0.

b. $|5| = 5$ 5 is 5 units from 0.

c. $|0| = 0$ 0 is 0 units from itself.

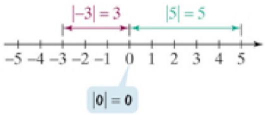


FIGURE 1.10 Absolute value describes distance from 0 on a number line.

Example 8 illustrates that the absolute value of a positive real number or 0 is the number itself. The absolute value of a negative real number, such as -3, is the number without the negative sign. Zero is the only real number whose absolute value is 0: $|0| = 0$. **The absolute value of any real number other than 0 is always positive.**

✓ **CHECK POINT 8** Find the absolute value:

a. $|-4|$ **b.** $|6|$ **c.** $|-\sqrt{2}|$.

1.3 EXERCISE SET

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Practice Exercises

In Exercises 1–8, write a positive or negative integer that describes each situation.

1. Meteorology: 20° below zero

2. Navigation: 65 feet above sea level

3. Health: A gain of 8 pounds

4. Economics: A loss of $12,500.00

5. Banking: A withdrawal of $3000.00

6. Physics: An automobile slowing down at a rate of 3 meters per second each second

7. Economics: A budget deficit of 4 billion dollars

8. Football: A 14-yard loss

In Exercises 9–20, start by drawing a number line that shows integers from −5 to 5. Then graph each real number on your number line.

9. 2 **10.** 5 **11.** -5 **12.** -2 **13.** $3\frac{1}{2}$ **14.** $2\frac{1}{4}$

15. $\frac{11}{3}$ **16.** $\frac{7}{3}$ **17.** -1.8 **18.** -3.4 **19.** $-\frac{16}{5}$ **20.** $-\frac{11}{5}$

In Exercises 21–32, express each rational number as a decimal.

21. $\frac{3}{4}$ **22.** $\frac{3}{5}$ **23.** $\frac{7}{20}$

24. $\frac{3}{20}$ **25.** $\frac{7}{8}$ **26.** $\frac{5}{16}$

27. $\frac{9}{11}$ **28.** $\frac{3}{11}$ **29.** $-\frac{1}{2}$

30. $-\frac{1}{4}$ **31.** $-\frac{5}{6}$ **32.** $-\frac{7}{6}$

In Exercises 33–36, list all numbers from the given set that are: **a.** *natural numbers,* **b.** *whole numbers,* **c.** *integers,* **d.** *rational numbers,* **e.** *irrational numbers,* **f.** *real numbers.*

33. $\{-9, -\frac{4}{5}, 0, 0.25, \sqrt{3}, 9.2, \sqrt{100}\}$

34. $\{-7, -0.\overline{6}, 0, \sqrt{49}, \sqrt{50}\}$

35. $\left\{-11, -\frac{5}{6}, 0, 0.75, \sqrt{5}, \pi, \sqrt{64}\right\}$

36. $\{-5, -0.\overline{3}, 0, \sqrt{2}, \sqrt{4}\}$

37. Give an example of a whole number that is not a natural number.

38. Give an example of an integer that is not a whole number.

39. Give an example of a rational number that is not an integer.

40. Give an example of a rational number that is not a natural number.

41. Give an example of a number that is an integer, a whole number, and a natural number.

42. Give an example of a number that is a rational number, an integer, and a real number.

43. Give an example of a number that is an irrational number and a real number.

44. Give an example of a number that is a real number, but not an irrational number.

In Exercises 45–62, insert either $<$ or $>$ in the shaded area between each pair of numbers to make a true statement.

45. $\frac{1}{2} \; \square \; 2$

46. $4 \; \square \; -3$

47. $3 \; \square \; -\frac{5}{2}$

48. $3 \; \square \; \frac{3}{2}$

49. $-4 \; \square \; -6$

50. $-\frac{5}{2} \; \square \; -\frac{5}{3}$

51. $-2.5 \; \square \; 1.5$

52. $-1.25 \; \square \; -0.5$

53. $-\frac{3}{4} \; \square \; -\frac{5}{4}$

54. $0 \; \square \; -\frac{1}{2}$

55. $-4.5 \; \square \; 3$

56. $-5.5 \; \square \; 2.5$

57. $\sqrt{2} \; \square \; 1.5$

58. $\sqrt{3} \; \square \; 2$

59. $0.\overline{3} \; \square \; 0.3$

60. $0.6 \; \square \; 0.\overline{6}$

61. $-\pi \; \square \; -3.5$

62. $-\frac{\pi}{2} \; \square \; -2.3$

In Exercises 63–70, determine whether each inequality is true or false.

63. $-5 \geq -13$

64. $-5 \leq -8$

65. $-9 \geq -9$

66. $-14 \leq -14$

67. $0 \geq -6$

68. $0 \geq -13$

69. $-17 \geq 6$

70. $-14 \geq 8$

In Exercises 71–78, find each absolute value.

71. $|6|$

72. $|3|$

73. $|-7|$

74. $|-9|$

75. $\left|\frac{5}{6}\right|$

76. $\left|\frac{4}{5}\right|$

77. $|-\sqrt{11}|$

78. $|-\sqrt{29}|$

Practice PLUS

In Exercises 79–86, insert either $<$, $>$, or $=$ in the shaded area to make a true statement.

79. $|-6| \; \square \; |-3|$

80. $|-20| \; \square \; |-50|$

81. $\left|\frac{3}{5}\right| \; \square \; |-0.6|$

82. $\left|\frac{5}{2}\right| \; \square \; |-2.5|$

83. $\frac{30}{40} - \frac{3}{4} \; \square \; \frac{14}{15} \cdot \frac{15}{14}$

84. $\frac{17}{18} \cdot \frac{18}{17} \; \square \; \frac{50}{60} - \frac{5}{6}$

85. $\frac{8}{13} \div \frac{8}{13} \; \square \; |-1|$

86. $|-2| \; \square \; \frac{4}{17} \div \frac{4}{17}$

Application Exercises

In Exercises 87–94, determine whether natural numbers, whole numbers, integers, rational numbers, or all real numbers are appropriate for each situation.

87. Shoe sizes of students on campus

88. Heights of students on campus

89. Temperatures in weather reports

90. Class sizes of algebra courses

91. Values of d given by the formula $d = \sqrt{1.5h}$, where d is the distance, in miles, that you can see to the horizon from a height of h feet

92. Values of C given by the formula $C = 2\pi r$, where C is the circumference of a circle with radius r

93. The number of pets a person has

94. The number of siblings a person has

95. The table shows the record low temperatures for five U.S. states.

State	Record Low (°F)	Date
Florida	−2	Feb. 13, 1899
Georgia	−17	Jan. 27, 1940
Hawaii	12	May 17, 1979
Louisiana	−16	Feb. 13, 1899
Rhode Island	−25	Feb. 5, 1996

Source: National Climatic Data Center

a. Graph the five record low temperatures on a number line.

b. Write the names of the states in order from the coldest record low to the warmest record low.

96. The table shows the record low temperatures for five U.S. states.

State	Record Low (°F)	Date
Virginia	−30	Jan. 22, 1985
Washington	−48	Dec. 30, 1968
West Virginia	−37	Dec. 30, 1917
Wisconsin	−55	Feb. 4, 1996
Wyoming	−66	Feb. 9, 1933

Source: National Climatic Data Center

a. Graph the five record low temperatures on a number line.

b. Write the names of the states in order from the coldest record low to the warmest record low.

Writing in Mathematics

97. What is a set?

98. What are the natural numbers?

99. What are the whole numbers?

100. What are the integers?

101. How does the set of integers differ from the set of whole numbers?

102. Describe how to graph a number on the number line.

103. What is a rational number?

104. Explain how to express $\frac{3}{8}$ as a decimal.

105. Describe the difference between a rational number and an irrational number.

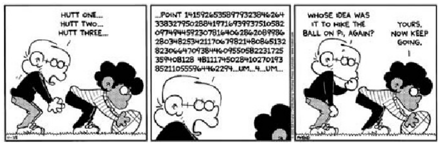
106. If you are given two different real numbers, explain how to determine which one is the lesser.

107. Describe what is meant by the absolute value of a number. Give an example with your explanation.

Critical Thinking Exercises

Make Sense? *In Exercises 108–111, determine whether each statement "makes sense" or "does not make sense" and explain your reasoning.*

108. The humor in this joke is based on the fact that the football will never be hiked.



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109. *Titanic* came to rest 12,500 feet below sea level and *Bismarck* came to rest 15,617 feet below sea level, so *Bismarck's* resting place is higher than *Titanic's*.

110. I expressed a rational number as a decimal and the decimal neither terminated nor repeated.

111. I evaluated the formula $d = \sqrt{1.5h}$ for a value of h that resulted in a rational number for d.

In Exercises 112–117, determine whether each statement is true or false. If the statement is false, make the necessary change(s) to produce a true statement.

112. Every rational number is an integer.

113. Some whole numbers are not integers.

114. Some rational numbers are not positive.

115. Irrational numbers cannot be negative

116. Some real numbers are not rational numbers.

117. Some integers are not rational numbers.

In Exercises 118–119, write each phrase as an algebraic expression.

118. a loss of $\frac{1}{3}$ of an investment of d dollars

119. a loss of half of an investment of d dollars

Technology Exercises

In Exercises 120–123, use a calculator to find a decimal approximation for each irrational number, correct to three decimal places. Between which two integers should you graph each of these numbers on the number line?

120. $\sqrt{3}$

121. $-\sqrt{12}$

122. $1 - \sqrt{2}$

123. $2 - \sqrt{5}$

Preview Exercises

Exercises 124–126 will help you prepare for the material covered in the next section. In each exercise, evaluate both expressions for $x = 4$. What do you observe?

124. $3(x + 5)$; $3x + 15$

125. $3x + 5x$; $8x$

126. $9x - 2x$; $7x$

SECTION

1.4

Basic Rules of Algebra

Objectives

1. Understand and use the vocabulary of algebraic expressions.
2. Use commutative properties.
3. Use associative properties.
4. Use distributive properties.
5. Combine like terms.
6. Simplify algebraic expressions.

Starting as a link among U.S. research scientists, more than one billion people worldwide now use the Internet. Some random Internet factoids:

- Number of searches performed per day on Google: 200 million
- Peak time for sex-related searches: 11 P.M.
- Fraction of people who use the word "password" as their password: $\frac{1}{8}$

- Vanity searchers: Fraction of people who typed their own name into a search engine: $\frac{1}{4}$
- Speed at which an email travels from San Francisco to New York: 30,000 miles per second

(*Source*: Paul Grobman, *Vital Statistics*, Plume, 2005)

In this section, we move from these quirky tidbits to mathematical models that describe the remarkable growth of the Internet among U.S. men and women. To use these models efficiently (you'll work with them in the Exercise Set), they should be simplified using basic rules of algebra. Before turning to these rules, we open the section with a closer look at algebraic expressions.

1 Understand and use the vocabulary of algebraic expressions.

The Vocabulary of Algebraic Expressions

We have seen that an algebraic expression combines numbers and variables. Here is an example of an algebraic expression:

$$7x + 3.$$

The **terms** of an algebraic expression are those parts that are separated by addition. For example, the algebraic expression $7x + 3$ contains two terms, namely $7x$ and 3. Notice that a term is a number, a variable, or a number multiplied by one or more variables.

The numerical part of a term is called its **coefficient**. In the term $7x$, the 7 is the coefficient. If a term containing one or more variables is written without a coefficient, the coefficient is understood to be 1. Thus, x means $1x$ and ab means $1ab$.

A term that consists of just a number is called a **constant term**. The constant term of $7x + 3$ is 3.

The parts of each term that are multiplied are called the **factors of the term**. The factors of the term $7x$ are 7 and x.

Like terms are terms that have exactly the same variable factors. Here are two examples of like terms:

$7x$ and $3x$ These terms have the same variable factor, x.

$4y$ and $9y$. These terms have the same variable factor, y.

By contrast, here are some examples of terms that are not like terms. These terms do not have the same variable factor.

$7x$ and 3 The variable factor of the first term is x. The second term has no variable factor.

$7x$ and $3y$ The variable factor of the first term is x. The variable factor of the second term is y.

Constant terms are like terms. Thus, the constant terms 7 and -12 are like terms.

EXAMPLE 1 Using the Vocabulary of Algebraic Expressions

Use the algebraic expression

$$4x + 7 + 5x$$

to answer the following questions:

a. How many terms are there in the algebraic expression?
b. What is the coefficient of the first term?
c. What is the constant term?
d. What are the like terms in the algebraic expression?

Solution

a. Because terms are separated by addition, the algebraic expression $4x + 7 + 5x$ contains three terms.

$$\underbrace{4x}_{\text{First term}} + \underbrace{7}_{\text{Second term}} + \underbrace{5x}_{\text{Third term}}$$

b. The coefficient of the first term, $4x$, is 4.

c. The constant term in $4x + 7 + 5x$ is 7.

d. The like terms in $4x + 7 + 5x$ are $4x$ and $5x$. These terms have the same variable factor, x.

✓ **CHECK POINT 1** Use the algebraic expression $6x + 2x + 11$ to answer each of the four questions in Example 1.

Equivalent Algebraic Expressions

In Example 1, we considered the algebraic expression

$$4x + 7 + 5x.$$

Let's compare this expression with a second algebraic expression

$$9x + 7.$$

Evaluate each expression for some choice of x. We will select $x = 2$.

$$4x + 7 + 5x$$

Replace x with 2.

$$= 4 \cdot 2 + 7 + 5 \cdot 2$$
$$= 8 + 7 + 10$$
$$= 25$$

$$9x + 7$$

Replace x with 2.

$$= 9 \cdot 2 + 7$$
$$= 18 + 7$$
$$= 25$$

Both algebraic expressions have the same value when $x = 2$. Regardless of what number you select for x, the algebraic expressions $4x + 7 + 5x$ and $9x + 7$ will have the same value. These expressions are called *equivalent algebraic expressions*. Two algebraic expressions that have the same value for all replacements are called **equivalent algebraic expressions**. Because $4x + 7 + 5x$ and $9x + 7$ are equivalent algebraic expressions, we write

$$4x + 7 + 5x = 9x + 7.$$

Properties of Real Numbers and Algebraic Expressions

We now turn to basic properties, or rules, that you know from past experiences in working with whole numbers and fractions. These properties will be extended to include all real numbers and algebraic expressions. We will give each property a name so that we can refer to it throughout the study of algebra.

2 Use commutative properties.

The Commutative Properties

The addition or multiplication of two real numbers can be done in any order. For example, $3 + 5 = 5 + 3$ and $3 \cdot 5 = 5 \cdot 3$. Changing the order does not change the answer of a sum or a product. These facts are called **commutative properties**.

Study Tip

The commutative property does not hold for subtraction or division.

$$6 - 1 \neq 1 - 6$$
$$8 \div 4 \neq 4 \div 8$$

The Commutative Properties

Let a and b represent real numbers, variables, or algebraic expressions.

Commutative Property of Addition

$$a + b = b + a$$

Changing order when adding does not affect the sum.

Commutative Property of Multiplication

$$ab = ba$$

Changing order when multiplying does not affect the product.

EXAMPLE 2 Using the Commutative Properties

Use the commutative properties to write an algebraic expression equivalent to each of the following:

a. $y + 6$ **b.** $5x$.

Solution

a. By the commutative property of addition, an algebraic expression equivalent to $y + 6$ is $6 + y$. Thus,

$$y + 6 = 6 + y.$$

b. By the commutative property of multiplication, an algebraic expression equivalent to $5x$ is $x5$. Thus,

$$5x = x5.$$

✓ **CHECK POINT 2** Use the commutative properties to write an algebraic expression equivalent to each of the following:

a. $x + 14$ **b.** $7y$.

EXAMPLE 3 Using the Commutative Properties

Write an algebraic expression equivalent to $13x + 8$ using

a. the commutative property of addition.

b. the commutative property of multiplication.

Solution

a. By the commutative property of addition, we change the order of the terms being added. This means that an algebraic expression equivalent to $13x + 8$ is $8 + 13x$:

$$13x + 8 = 8 + 13x.$$

b. By the commutative property of multiplication, we change the order of the factors being multiplied. This means that an algebraic expression equivalent to $13x + 8$ is $x13 + 8$:

$$13x + 8 = x13 + 8.$$

✓ **CHECK POINT 3** Write an algebraic expression equivalent to $5x + 17$ using

a. the commutative property of addition.

b. the commutative property of multiplication.

Blitzer Bonus

Commutative Words and Sentences

The commutative property states that a change in order produces no change in the answer. The words and sentences listed here suggest a characteristic of the commutative property; they read the same from left to right and from right to left!

- dad
- repaper
- never odd or even
- Go deliver a dare, vile dog!
- May a moody baby doom a yam?
- Madam, in Eden I'm Adam.
- Ma is a nun, as I am.
- A man, a plan, a canal: Panama
- Are we not drawn onward, we few, drawn onward to new era?

3 Use associative properties.

The Associative Properties

Parentheses indicate groupings. As we have seen, we perform operations within the parentheses first. For example,

$$(2 + 5) + 10 = 7 + 10 = 17$$

and

$$2 + (5 + 10) = 2 + 15 = 17.$$

In general, the way in which three numbers are grouped does not change their sum. It also does not change their product. These facts are called the **associative properties**.

Study Tip

The associative property does not hold for subtraction or division.

$$(6 - 3) - 1 \neq 6 - (3 - 1)$$

$$(8 \div 4) \div 2 \neq 8 \div (4 \div 2)$$

The Associative Properties

Let a, b, and c represent real numbers, variables, or algebraic expressions.

Associative Property of Addition

$$(a + b) + c = a + (b + c)$$

Changing grouping when adding does not affect the sum.

Associative Property of Multiplication

$$(ab)c = a(bc)$$

Changing grouping when multiplying does not affect the product.

The associative properties can be used to simplify some algebraic expressions by removing the parentheses.

EXAMPLE 4 Simplifying Using the Associative Properties

Simplify:

a. $3 + (8 + x)$ **b.** $8(4x)$.

Solution

a. $3 + (8 + x)$ This is the given algebraic expression.

$= (3 + 8) + x$ Use the associative property of addition to group the first two numbers.

$= 11 + x$ Add within parentheses.

Using the commutative property of addition, this simplified algebraic expression can also be written as $x + 11$.

b. $8(4x)$ This is the given algebraic expression.

$= (8 \cdot 4)x$ Use the associative property of multiplication to group the first two numbers.

$= 32x$ Multiply within parentheses.

We can use the commutative property of multiplication and write this simplified algebraic expression as $x32$ or $x \cdot 32$. However, it is customary to express a term with its coefficient on the left. Thus, we use $32x$ as the simplified form of the algebraic expression.

✓ **CHECK POINT 4** Simplify:

a. $8 + (12 + x)$ **b.** $6(5x)$.

The next example involves the use of both basic properties to simplify an algebraic expression.

EXAMPLE 5 Using the Commutative and Associative Properties

Simplify: $7 + (x + 2)$.

Solution

$7 + (x + 2)$	This is the given algebraic expression.
$= 7 + (2 + x)$	Use the commutative property to change the order of the addition.
$= (7 + 2) + x$	Use the associative property to group the first two numbers.
$= 9 + x$	Add within parentheses.

Using the commutative property of addition, an equivalent algebraic expression is $x + 9$.

Study Tip

Commutative: changes *order*
Associative: changes *grouping*

☑ **CHECK POINT 5** Simplify: $8 + (x + 4)$.

4 Use distributive properties.

The Distributive Properties

The **distributive property** involves both multiplication and addition. The property shows how to multiply the sum of two numbers by a third number. Consider, for example, $4(7 + 3)$, which can be calculated in two ways. One way is to perform the addition within the grouping symbols and then multiply:

$$4(7 + 3) = 4(10) = 40.$$

The other way is to *distribute* the multiplication by 4 over the addition by first multiplying each number within the parentheses by 4 and then adding:

$$4 \cdot 7 + 4 \cdot 3 = 28 + 12 = 40.$$

The result in both cases is 40. Thus,

$$4(7 + 3) = 4 \cdot 7 + 4 \cdot 3. \quad \text{Multiplication distributes over addition.}$$

The distributive property allows us to rewrite the product of a number and a sum as the sum of two products.

The Distributive Property

Let a, b, and c represent real numbers, variables, or algebraic expressions.

$$a(b + c) = ab + ac$$

Multiplication distributes over addition.

Study Tip

Do not confuse the distributive property with the associative property of multiplication.

Distributive:

$$4(5 + x) = 4 \cdot 5 + 4x$$
$$= 20 + 4x$$

Associative:

$$4(5 \cdot x) = (4 \cdot 5)x$$
$$= 20x$$

EXAMPLE 6 Using the Distributive Property

Multiply: $6(x + 4)$.

Solution Multiply *each term* inside the parentheses, x and 4, by the multiplier outside, 6.

$6(x + 4) = 6x + 6 \cdot 4$	Use the distributive property to remove parentheses.
$= 6x + 24$	Multiply: $6 \cdot 4 = 24$.

✓ **CHECK POINT 6** Multiply: $5(x + 3)$.

EXAMPLE 7 Using the Distributive Property

Multiply: $5(3y + 7)$.

Solution Multiply *each term* inside the parentheses, $3y$ and 7, by the multiplier outside, 5.

$$5(3y + 7) = 5 \cdot 3y + 5 \cdot 7$$
Use the distributive property to remove parentheses.

$$= 15y + 35$$
Multiply. Use the associative property of multiplication to find $5 \cdot 3y$: $5 \cdot 3y = (5 \cdot 3)y = 15y$.

✓ **CHECK POINT 7** Multiply: $6(4y + 7)$.

Study Tip

When using the distributive property to remove parentheses, be sure to multiply *each term* inside the parentheses by the multiplier outside.

Incorrect!

$$\xcancel{\begin{aligned} 5(3y + 7) &= 5 \cdot 3y + 7 \\ &= 15y + 7 \end{aligned}}$$

7 must also be multiplied by 5.

Table 1.3 shows a number of other forms of the distributive property.

Table 1.3 Other Forms of the Distributive Property

Property	Meaning	Example
$a(b - c) = ab - ac$	Multiplication distributes over subtraction.	$5(4x - 3) = 5 \cdot 4x - 5 \cdot 3$ $= 20x - 15$
$a(b + c + d) = ab + ac + ad$	Multiplication distributes over three or more terms in parentheses.	$4(x + 10 + 3y)$ $= 4x + 4 \cdot 10 + 4 \cdot 3y$ $= 4x + 40 + 12y$
$(b + c)a = ba + ca$	Multiplication on the right distributes over addition (or subtraction).	$(x + 7)9 = x \cdot 9 + 7 \cdot 9$ $= 9x + 63$

5 Combine like terms.

Combining Like Terms

The distributive property

$$a(b + c) = ab + ac$$

lets us add and subtract like terms. To do this, we will usually apply the property in the form

$$ax + bx = (a + b)x$$

and then combine a and b. For example,

$$3x + 7x = (3 + 7)x = 10x.$$

This process is called **combining like terms**.

EXAMPLE 8 Combining Like Terms

Combine like terms:

a. $4x + 15x$ **b.** $7a - 2a$.

Solution

a. $4x + 15x$	These are like terms because 4x and 15x have identical variable factors.
$= (4 + 15)x$	Apply the distributive property.
$= 19x$	Add within parentheses.
b. $7a - 2a$	These are like terms because 7a and 2a have identical variable factors.
$= (7 - 2)a$	Apply the distributive property.
$= 5a$	Subtract within parentheses.

✓ **CHECK POINT 8** Combine like terms:

a. $7x + 3x$ **b.** $9a - 4a$.

When combining like terms, you may find yourself leaving out the details of the distributive property. For example, you may simply write

$$7x + 3x = 10x.$$

It might be useful to think along these lines: Seven things plus three of the (same) things give ten of those things. To add like terms, add the coefficients and copy the common variable.

Combining Like Terms Mentally

1. Add or subtract the coefficients of the terms.
2. Use the result of step 1 as the coefficient of the term's variable factor.

When an expression contains three or more terms, use the commutative and associative properties to group like terms. Then combine the like terms.

EXAMPLE 9 Grouping and Combining Like Terms

Simplify:

a. $7x + 5 + 3x + 8$ **b.** $4x + 7y + 2x + 3y$.

Solution

a. $7x + 5 + 3x + 8$	
$= (7x + 3x) + (5 + 8)$	Rearrange terms and group the like terms using the commutative and associative properties. This step is often done mentally.
$= 10x + 13$	Combine like terms: 7x + 3x = 10x. Combine constant terms: 5 + 8 = 13.
b. $4x + 7y + 2x + 3y$	
$= (4x + 2x) + (7y + 3y)$	Group like terms.
$= 6x + 10y$	Combine like terms by adding coefficients and keeping the variable factor.

✓ **CHECK POINT 9** Simplify:

a. $8x + 7 + 10x + 3$ **b.** $9x + 6y + 5x + 2y$.

Study Tip

Combining like terms should remind you of adding and subtracting numbers of like objects.

7 apples + 3 apples = 10 apples — $7a + 3a = 10a$

9 feet − 5 feet = 4 feet — $9f - 5f = 4f$

6 apples + 10 feet = ? — $6a$ and $10f$ are not like terms and cannot be added.

6 Simplify algebraic expressions.

Simplifying Algebraic Expressions

An algebraic expression is **simplified** when parentheses have been removed and like terms have been combined.

Simplifying Algebraic Expressions

1. Use the distributive property to remove parentheses.
2. Rearrange terms and group like terms using the commutative and associative properties. This step may be done mentally.
3. Combine like terms by combining the coefficients of the terms and keeping the same variable factor.

EXAMPLE 10 Simplifying an Algebraic Expression

Simplify: $5(3x + 7) + 6x$.

Solution

$$\begin{aligned} &5(3x + 7) + 6x \\ &= 5 \cdot 3x + 5 \cdot 7 + 6x && \text{Use the distributive property to remove the parentheses.} \\ &= 15x + 35 + 6x && \text{Multiply.} \\ &= (15x + 6x) + 35 && \text{Group like terms.} \\ &= 21x + 35 && \text{Combine like terms.} \end{aligned}$$

✓ **CHECK POINT 10** Simplify: $7(2x + 3) + 11x$.

EXAMPLE 11 Simplifying an Algebraic Expression

Simplify: $6(2x + 4y) + 10(4x + 3y)$.

Solution

$$\begin{aligned} &6(2x + 4y) + 10(4x + 3y) \\ &= 6 \cdot 2x + 6 \cdot 4y + 10 \cdot 4x + 10 \cdot 3y && \text{Use the distributive property to remove the parentheses.} \\ &= 12x + 24y + 40x + 30y && \text{Multiply.} \\ &= (12x + 40x) + (24y + 30y) && \text{Group like terms.} \\ &= 52x + 54y && \text{Combine like terms.} \end{aligned}$$

✓ **CHECK POINT 11** Simplify: $7(4x + 3y) + 2(5x + y)$.

Study Tip

Use the distributive property to remove parentheses when the terms inside parentheses are not like terms:

$$4(3x + 5y) = 4 \cdot 3x + 4 \cdot 5y = 12x + 20y.$$

3x and 5y are not like terms.

It is not necessary to use the distributive property to remove parentheses when the terms inside parentheses are like terms:

$$4(3x + 5x) = 4(8x) = 32x.$$

$3x$ and $5x$ are like terms and can be combined: $3x + 5x = 8x$.

We mentally applied the associative property: $4(8x) = (4 \cdot 8)x = 32x$.

1.4 EXERCISE SET

MyMathLab PRACTICE WATCH DOWNLOAD READ REVIEW

Practice Exercises

In Exercises 1–6, an algebraic expression is given. Use each expression to answer the following questions.

a. *How many terms are there in the algebraic expression?*
b. *What is the numerical coefficient of the first term?*
c. *What is the constant term?*
d. *Does the algebraic expression contain like terms? If so, what are the like terms?*

1. $3x + 5$
2. $9x + 4$
3. $x + 2 + 5x$
4. $x + 6 + 7x$
5. $4y + 1 + 3x$
6. $8y + 1 + 10x$

In Exercises 7–14, use the commutative property of addition to write an equivalent algebraic expression.

7. $y + 4$
8. $x + 7$
9. $5 + 3x$
10. $4 + 9x$
11. $4x + 5y$
12. $10x + 9y$
13. $5(x + 3)$
14. $6(x + 4)$

In Exercises 15–22, use the commutative property of multiplication to write an equivalent algebraic expression.

15. $9x$
16. $8x$
17. $x + y6$
18. $x + y7$
19. $7x + 23$
20. $13x + 11$
21. $5(x + 3)$
22. $6(x + 4)$

In Exercises 23–26, use an associative property to rewrite each algebraic expression. Once the grouping has been changed, simplify the resulting algebraic expression.

23. $7 + (5 + x)$
24. $9 + (3 + x)$
25. $7(4x)$
26. $8(5x)$

In Exercises 27–46, use a form of the distributive property to rewrite each algebraic expression without parentheses.

27. $3(x + 5)$
28. $4(x + 6)$
29. $8(2x + 3)$
30. $9(2x + 5)$
31. $\frac{1}{3}(12 + 6r)$
32. $\frac{1}{4}(12 + 8r)$
33. $5(x + y)$
34. $7(x + y)$
35. $3(x - 2)$
36. $4(x - 5)$
37. $2(4x - 5)$
38. $6(3x - 2)$
39. $\frac{1}{2}(5x - 12)$
40. $\frac{1}{3}(7x - 21)$
41. $(2x + 7)4$
42. $(5x + 3)6$
43. $6(x + 3 + 2y)$
44. $7(2x + 4 + y)$
45. $5(3x - 2 + 4y)$
46. $4(5x - 3 + 7y)$

In Exercises 47–64, simplify each algebraic expression.

47. $7x + 10x$
48. $5x + 13x$
49. $11a - 3a$
50. $14b - 5b$
51. $3 + (x + 11)$
52. $7 + (x + 10)$
53. $5y + 3 + 6y$
54. $8y + 7 + 10y$
55. $2x + 5 + 7x - 4$
56. $7x + 8 + 2x - 3$
57. $11a + 12 + 3a + 2$
58. $13a + 15 + 2a + 11$
59. $5(3x + 2) - 4$
60. $2(5x + 4) - 3$
61. $12 + 5(3x - 2)$
62. $14 + 2(5x - 1)$
63. $7(3a + 2b) + 5(4a + 2b)$
64. $11(6a + 3b) + 4(12a + 5b)$

Practice PLUS

In Exercises 65–66, name the property used to go from step to step each time that "(why?)" occurs.

65. $7 + 2(x + 9)$

$= 7 + (2x + 18)$ (why?)

$= 7 + (18 + 2x)$ (why?)

$= (7 + 18) + 2x$ (why?)

$= 25 + 2x$

$= 2x + 25$ (why?)

66. $5(x + 4) + 3x$

$= (5x + 20) + 3x$ (why?)

$= (20 + 5x) + 3x$ (why?)

$= 20 + (5x + 3x)$ (why?)

$= 20 + (5 + 3)x$ (why?)

$= 20 + 8x$

$= 8x + 20$ (why?)

In Exercises 67–76, write each English phrase as an algebraic expression. Then simplify the expression. Let x represent the number.

67. the sum of 7 times a number and twice the number

68. the sum of 8 times a number and twice the number

69. the product of 3 and a number, which is then subtracted from the product of 12 and a number

70. the product of 5 and a number, which is then subtracted from the product of 11 and a number

71. six times the product of 4 and a number

72. nine times the product of 3 and a number

73. six times the sum of 4 and a number

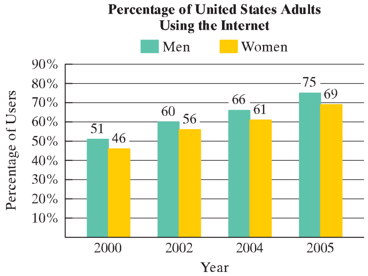
74. nine times the sum of 3 and a number

75. eight increased by the product of 5 and one less than a number

76. nine increased by the product of 3 and 2 less than a number

Application Exercises

The graph shows the percentage of U.S. men and women who used the Internet for four selected years. Exercises 77–78 involve mathematical models for the data.



Percentage of United States Adults Using the Internet

Source: Pew Internet and American Life Project

77. The percentage of U.S. men, M, who used the Internet n years after 2000 can be modeled by the formula

$$M = 2(2n + 25) + 0.5(n + 2).$$

a. Simplify the formula.

b. Use the simplified form of the mathematical model to find the percentage of U.S. men who used the Internet in 2005. Does the model underestimate or overestimate the actual percent shown by the bar graph for 2005? By how much?

78. The percentage of U.S. women, W, who used the Internet n years after 2000 can be modeled by the formula

$$W = 2(2n + 20) + 0.3(n + 20).$$

a. Simplify the formula.

b. Use the simplified form of the mathematical model to find the percentage of U.S. women who used the Internet in 2005. Does the model underestimate or overestimate the actual percent shown by the bar graph for 2005? By how much?

Writing in Mathematics

79. What is a term? Provide an example with your description.

80. What are like terms? Provide an example with your description.

81. What are equivalent algebraic expressions?

82. State a commutative property and give an example.

83. State an associative property and give an example.

84. State a distributive property and give an example.

85. Explain how to add like terms. Give an example.

86. What does it mean to simplify an algebraic expression?

87. An algebra student incorrectly used the distributive property and wrote $3(5x + 7) = 15x + 7$. If you were that student's teacher, what would you say to help the student avoid this kind of error?

88. You can transpose the letters in the word "conversation" to form the phrase "voices rant on." From "total abstainers" we can form "sit not at ale bars." What two algebraic properties do each of these transpositions (called *anagrams*) remind you of? Explain your answer.

Critical Thinking Exercises

Make Sense? *In Exercises 89–92, determine whether each statement "makes sense" or does not make sense" and explain your reasoning.*

89. I applied the commutative property and rewrote $x - 4$ as $4 - x$.

90. Just as the commutative properties change groupings, the associative properties change order.

91. I did not use the distributive property to simplify $3(2x + 5x)$.

92. The commutative, associative, and distributive properties remind me of the rules of a game.

In Exercises 93–96, determine whether each statement is true or false. If the statement is false, make the necessary change(s) to produce a true statement.

93. $(24 \div 6) \div 2 = 24 \div (6 \div 2)$

94. $2x + 5 = 5x + 2$

95. $a + (bc) = (a + b)(a + c)$; In words, addition can be distributed over multiplication.

96. Like terms contain the same coefficients.

Preview Exercises

Exercises 97–99 will help you prepare for the material covered in the next section. In each exercise, write an integer that is the result of the given situation.

97. You earn \$150, but then you misplace \$90.

98. You lose \$30 and then you misplace \$10.

99. The temperature is 30 degrees, and then it drops by 35 degrees.

MID-CHAPTER CHECK POINT Section 1.1–Section 1.4

What You Know: Algebra uses variables, or letters that represent a variety of different numbers. These variables appear in algebraic expressions, equations, and formulas. Mathematical models use variables to describe real-world phenomena. We reviewed operations with fractions and saw how fractions are used in algebra. We defined the real numbers and represented them as points on a number line. Finally, we introduced some basic rules of algebra and used the commutative, associative, and distributive properties to simplify algebraic expressions.

1. Evaluate for $x = 6$: $2 + 10x$.

2. Evaluate for $x = \frac{3}{5}$: $10x - 4$.

3. Evaluate for $x = 3$ and $y = 10$:

$$\frac{xy}{2} + 4(y - x).$$

In Exercises 4–5, translate from English to an algebraic expression or equation, whichever is appropriate. Let the variable x represent the number.

4. Two less than $\frac{1}{4}$ of a number

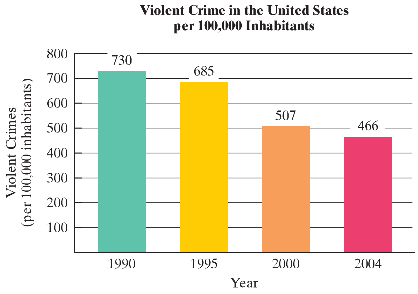
5. Five more than the quotient of a number and 6 gives 19.

In Exercises 6–7, determine whether the given number is a solution of the equation.

6. $3(x + 2) = 4x - 1$; 6

7. $8y = 12\left(y - \frac{1}{2}\right)$; $\frac{3}{4}$

8. Because the population of the United States continues to grow, criminologists use the crime rate per 100,000 population to compare crime statistics from different years. The bar graph shows the number of violent crimes in the United States per 100,000 inhabitants for four selected years.



Source: Federal Bureau of Investigation

Here is a mathematical model that approximates the data displayed by the bar graph:

$$V = 747 - 21n.$$

Number of years after 1990

Number of violent crimes per 100,000 inhabitants

a. Use the formula to find the number of violent crimes per 100,000 inhabitants in 2000. Does the mathematical model underestimate or overestimate the actual number shown by the bar graph for 2000? By how much?

b. If trends from 1990 through 2004 continue, use the formula to project the number of violent crimes per 100,000 inhabitants in 2010.

In Exercises 9–15, perform the indicated operation. Where possible, reduce answers to lowest terms.

9. $\frac{7}{10} - \frac{8}{15}$

10. $\frac{2}{3} \cdot \frac{3}{4}$

11. $\frac{5}{22} + \frac{5}{33}$

12. $\frac{3}{5} \div \frac{9}{10}$

13. $\frac{23}{105} - \frac{2}{105}$

14. $2\frac{7}{9} \div 3$

15. $5\frac{2}{9} - 3\frac{1}{6}$

16. The formula $C = \frac{5}{9}(F - 32)$ expresses the relationship between Fahrenheit temperature, F, and Celsius temperature, C. If the temperature is 50°F, use the formula to find the equivalent temperature on the Celsius scale.

17. Insert either $<$ or $>$ in the shaded area to make a true statement:

-8000 ▨ $-8\frac{1}{4}$.

18. Express $\frac{1}{11}$ as a decimal.

19. Find the absolute value: $|-19.3|$.

20. List all the rational numbers in this set:

$$\left\{-11, -\frac{3}{7}, 0, 0.45, \sqrt{23}, \sqrt{25}\right\}.$$

In Exercises 21–23, rewrite $5(x + 3)$ as an equivalent expression using the given property.

21. the commutative property of multiplication

22. the commutative property of addition

23. the distributive property

In Exercises 24–25, simplify each algebraic expression.

24. $7(9x + 3) + \frac{1}{3}(6x)$

25. $2(3x + 5y) + 4(x + 6y)$

SECTION 1.5

Objectives

- **1** Add numbers with a number line.
- **2** Find sums using identity and inverse properties.
- **3** Add numbers without a number line.
- **4** Use addition rules to simplify algebraic expressions.
- **5** Solve applied problems using a series of additions.

Addition of Real Numbers

It has not been a good day! First, you lost your wallet with \$30 in it. Then, to get through the day, you borrowed \$10, which you somehow misplaced. Your loss of \$30 followed by a loss of \$10 is an overall loss of \$40. This can be written

$$-30 + (-10) = -40.$$

The result of adding two or more numbers is called the **sum** of the numbers. The sum of -30 and -10 is -40. You can think of gains and losses of money to find sums. For example, to find $17 + (-13)$, think of a gain of \$17 followed by a loss of \$13. There is an overall gain of \$4. Thus, $17 + (-13) = 4$. In the same way, to find $-17 + 13$, think of a loss of \$17 followed by a gain of \$13. There is an overall loss of \$4, so $-17 + 13 = -4$.

1 Add numbers with a number line.

Adding with a Number Line

We use the number line to help picture the addition of real numbers. On the next page is the procedure for finding $a + b$, the sum of a and b, using the number line.

ANSWERS TO SELECTED EXERCISES

CHAPTER 1

Section 1.1

Check Point Exercises

1. a. 26 **b.** 32 **2. a.** 37 **b.** 2 **3. a.** $6x$ **b.** $4 + x$ **c.** $3x + 5$ **d.** $12 - 2x$ **e.** $\frac{15}{x}$ **4. a.** not a solution **b.** solution
5. a. $\frac{x}{6} = 5$ **b.** $7 - 2x = 1$ **6. a.** 65% **b.** 60% **c.** overestimates by 5%

Exercise Set 1.1

1. 12 **3.** 8 **5.** 20 **7.** 7 **9.** 17 **11.** 18 **13.** 5 **15.** 19 **17.** 24 **19.** 13 **21.** 10 **23.** 5 **25.** $x + 4$ **27.** $x - 4$
29. $x + 4$ **31.** $x - 9$ **33.** $9 - x$ **35.** $3x - 5$ **37.** $12x - 1$ **39.** $\frac{10}{x} + \frac{x}{10}$ **41.** $\frac{x}{30} + 6$ **43.** solution **45.** solution
47. not a solution **49.** solution **51.** not a solution **53.** solution **55.** solution **57.** not a solution **59.** $4x = 28$ **61.** $\frac{14}{x} = \frac{1}{2}$
63. $20 - x = 5$ **65.** $2x + 6 = 16$ **67.** $3x - 5 = 7$ **69.** $4x + 5 = 33$ **71.** $4(x + 5) = 33$ **73.** $5x = 24 - x$ **75.** 8 **77.** 59
79. a. 14 **b.** yes **81. a.** 6 **b.** no **83. a.** 11%; overestimates by 1% **b.** 9%; It gives the percent displayed for 2004.
85. a. 35.6%; overestimates by 0.6% **b.** 29.8%; overestimates by 3.8% **87. a.** 44 **b.** 164 **99.** makes sense
101. makes sense **103.** true **105.** true **107.** Choices of variables may vary. Samples are given: $h =$ hours worked,
$s =$ salary; $s = 20h$. **109. a.** O'Donnell: \$0.65; Vieira: \$1.79; Couric: \$2.05 **b.** O'Donnell **110.** $\frac{6}{35}$ **111.** $\frac{10}{21}$ **112.** $\frac{4}{17}$

Section 1.2

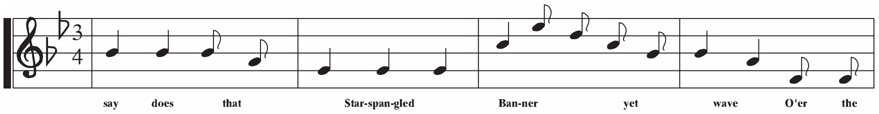
Check Point Exercises

1. $\frac{21}{8}$ **2.** $1\frac{2}{3}$ **3.** $2 \cdot 2 \cdot 3 \cdot 3$ **4. a.** $\frac{2}{3}$ **b.** $\frac{7}{4}$ **c.** $\frac{13}{15}$ **d.** $\frac{1}{5}$ **5. a.** $\frac{8}{33}$ **b.** $\frac{18}{5}$ or $3\frac{3}{5}$ **c.** $\frac{2}{7}$ **d.** $\frac{51}{10}$ or $5\frac{1}{10}$ **6. a.** $\frac{10}{3}$ or $3\frac{1}{3}$
b. $\frac{2}{9}$ **c.** $\frac{3}{2}$ or $1\frac{1}{2}$ **7. a.** $\frac{5}{11}$ **b.** $\frac{2}{3}$ **c.** $\frac{9}{4}$ or $2\frac{1}{4}$ **8.** $\frac{14}{21}$ **9. a.** $\frac{11}{10}$ or $1\frac{1}{10}$ **b.** $\frac{7}{12}$ **c.** $\frac{5}{4}$ or $1\frac{1}{4}$ **10.** $\frac{53}{60}$
11. a. solution **b.** solution **12. a.** $\frac{2}{3}(x - 6)$ **b.** $\frac{3}{4}x - 2 = \frac{1}{5}x$ **13.** 25°C

Exercise Set 1.2

1. $\frac{19}{8}$ **3.** $\frac{38}{5}$ **5.** $\frac{135}{16}$ **7.** $4\frac{3}{5}$ **9.** $8\frac{4}{9}$ **11.** $35\frac{11}{20}$ **13.** $2 \cdot 11$ **15.** $2 \cdot 2 \cdot 5$ **17.** prime **19.** $2 \cdot 2 \cdot 3 \cdot 3$ **21.** $2 \cdot 2 \cdot 5 \cdot 7$
23. prime **25.** $3 \cdot 3 \cdot 3 \cdot 3$ **27.** $2 \cdot 2 \cdot 2 \cdot 2 \cdot 3 \cdot 5$ **29.** $\frac{5}{8}$ **31.** $\frac{5}{6}$ **33.** $\frac{7}{10}$ **35.** $\frac{2}{5}$ **37.** $\frac{22}{25}$ **39.** $\frac{60}{43}$ **41.** $\frac{2}{15}$ **43.** $\frac{21}{88}$
45. $\frac{36}{7}$ **47.** $\frac{1}{12}$ **49.** $\frac{15}{14}$ **51.** 6 **53.** $\frac{15}{16}$ **55.** $\frac{9}{5}$ **57.** $\frac{5}{9}$ **59.** 3 **61.** $\frac{7}{10}$ **63.** $\frac{1}{2}$ **65.** 6 **67.** $\frac{6}{11}$ **69.** $\frac{2}{3}$ **71.** $\frac{5}{4}$
73. $\frac{1}{6}$ **75.** 2 **77.** $\frac{7}{10}$ **79.** $\frac{9}{10}$ **81.** $\frac{19}{24}$ **83.** $\frac{7}{18}$ **85.** $\frac{7}{12}$ **87.** $\frac{41}{80}$ **89.** $1\frac{5}{12}$ or $\frac{17}{12}$ **91.** solution **93.** solution
95. not a solution **97.** solution **99.** not a solution **101.** solution **103.** $\frac{1}{5}x$ **105.** $x - \frac{1}{4}x$ **107.** $x - \frac{1}{4} = \frac{1}{2}x$ **109.** $\frac{1}{7}x + \frac{1}{8}x = 12$
111. $\frac{2}{3}(x + 6)$ **113.** $\frac{2}{3}x + 6 = x - 3$ **115.** $\frac{3a}{20}$ **117.** $\frac{20}{x}$ **119.** $\frac{4}{15}$ **121.** not a solution **123.** 20°C **125. a.** 140 beats per minute
b. 160 beats per minute **127. a.** $H = \frac{9}{10}(220 - a)$ **b.** 162 beats per minute **129.** 474 billion; underestimates by 2 billion
141. makes sense **143.** makes sense **145.** false **147.** true

149.

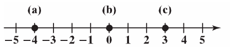


150. 5 **151.** $\frac{5}{2}$ **152.** -4

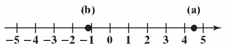
Section 1.3

Check Point Exercises

1. a. -500 **b.** -282 **2.**



3.

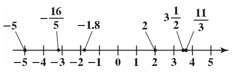


4. a. 0.375 **b.** $0.\overline{45}$ **5. a.** $\sqrt{9}$ **b.** $0, \sqrt{9}$ **c.** $-9, 0, \sqrt{9}$ **d.** $-9, -1.3, 0, 0.\overline{3}, \sqrt{9}$ **e.** $\frac{\pi}{2}, \sqrt{10}$ **f.** $-9, -1.3, 0, 0.\overline{3}, \frac{\pi}{2}, \sqrt{9}, \sqrt{10}$

6. a. $>$ **b.** $<$ **c.** $<$ **d.** $<$ **7. a.** true **b.** true **c.** false **8. a.** 4 **b.** 6 **c.** $\sqrt{2}$

Exercise Set 1.3

1. -20 **3.** 8 **5.** -3000 **7.** -4 billion **9–19.**



21. 0.75 **23.** 0.35 **25.** 0.875 **27.** $0.\overline{81}$

29. -0.5 **31.** $-0.8\overline{3}$ **33. a.** $\sqrt{100}$ **b.** $0, \sqrt{100}$ **c.** $-9, 0, \sqrt{100}$ **d.** $-9, -\frac{4}{5}, 0, 0.25, 9.2, \sqrt{100}$ **e.** $\sqrt{3}$

f. $-9, -\frac{4}{5}, 0, 0.25, \sqrt{3}, 9.2, \sqrt{100}$ **35. a.** $\sqrt{64}$ **b.** $0, \sqrt{64}$ **c.** $-11, 0, \sqrt{64}$ **d.** $-11, -\frac{5}{6}, 0, 0.75, \sqrt{64}$ **e.** $\sqrt{5}, \pi$

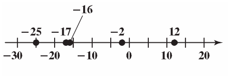
f. $-11, -\frac{5}{6}, 0, 0.75, \sqrt{5}, \pi, \sqrt{64}$ **37.** 0 **39.** Answers will vary; $\frac{1}{2}$ is an example. **41.** Answers will vary; 6 is an example.

43. Answers will vary; π is an example. **45.** $<$ **47.** $>$ **49.** $>$ **51.** $<$ **53.** $>$ **55.** $<$ **57.** $<$ **59.** $>$

61. $>$ **63.** true **65.** true **67.** true **69.** false **71.** 6 **73.** 7 **75.** $\frac{5}{6}$ **77.** $\sqrt{11}$ **79.** $>$ **81.** $=$ **83.** $<$ **85.** $=$

87. rational numbers **89.** integers **91.** all real numbers **93.** whole numbers

95. a.



b. Rhode Island, Georgia, Louisiana, Florida, Hawaii

109. does not make sense **111.** makes sense **113.** false **115.** false **117.** false **119.** $-\frac{1}{2}d$ **121.** -3.464; -4 and -3

123. -0.236; -1 and 0 **124.** 27; 27; Both expressions have the same value. **125.** 32; 32; Both expressions have the same value.

126. 28; 28; Both expressions have the same value.

Section 1.4

Check Point Exercises

1. a. 3 terms **b.** 6 **c.** 11 **d.** $6x$ and $2x$ **2. a.** $14 + x$ **b.** $y7$ **3. a.** $17 + 5x$ **b.** $x5 + 17$ **4. a.** $20 + x$ or $x + 20$ **b.** $30x$

5. $12 + x$ or $x + 12$ **6.** $5x + 15$ **7.** $24y + 42$ **8. a.** $10x$ **b.** $5a$ **9. a.** $18x + 10$ **b.** $14x + 8y$ **10.** $25x + 21$ **11.** $38x + 23y$

Exercise Set 1.4

1. a. 2 **b.** 3 **c.** 5 **d.** no **3. a.** 3 **b.** 1 **c.** 2 **d.** yes; x and $5x$ **5. a.** 3 **b.** 4 **c.** 1 **d.** no **7.** $4 + y$ **9.** $3x + 5$

11. $5y + 4x$ **13.** $5(3 + x)$ **15.** $x9$ **17.** $x + 6y$ **19.** $x7 + 23$ **21.** $(x + 3)5$ **23.** $(7 + 5) + x = 12 + x$ **25.** $(7 \cdot 4)x = 28x$

27. $3x + 15$ **29.** $16x + 24$ **31.** $4 + 2r$ **33.** $5x + 5y$ **35.** $3x - 6$ **37.** $8x - 10$ **39.** $\frac{5}{2}x - 6$ **41.** $8x + 28$ **43.** $6x + 18 + 12y$

45. $15x - 10 + 20y$ **47.** $17x$ **49.** $8a$ **51.** $14 + x$ **53.** $11y + 3$ **55.** $9x + 1$ **57.** $14a + 14$ **59.** $15x + 6$ **61.** $15x + 2$

63. $41a + 24b$ **65.** Distributive property; Commutative property of addition; Associative property of addition; Commutative property of addition

67. $7x + 2x$; $9x$ **69.** $12x - 3x$; $9x$ **71.** $6(4x)$; $24x$ **73.** $6(4 + x)$; $24 + 6x$ **75.** $8 + 5(x - 1)$; $5x + 3$ **77. a.** $M = 4.5n + 51$

b. 73.5%; underestimates by 1.5% **89.** does not make sense **91.** makes sense **93.** false **95.** false **97.** 60 **98.** -40 **99.** -5

Mid-Chapter Check Point Exercises

1. 62 **2.** 2 **3.** 43 **4.** $\frac{1}{4}x - 2$ **5.** $\frac{x}{6} + 5 = 19$ **6.** not a solution **7.** not a solution **8. a.** 537; overestimates by 30 per 100,000 inhabitants **b.** 327 per 100,000 inhabitants **9.** $\frac{1}{6}$ **10.** $\frac{1}{2}$ **11.** $\frac{25}{66}$ **12.** $\frac{2}{3}$ **13.** $\frac{1}{5}$ **14.** $\frac{25}{27}$ **15.** $\frac{37}{18}$ or $2\frac{1}{18}$ **16.** 10°C

17. $<$ **18.** $0.\overline{09}$ **19.** 19.3 **20.** $-11, -\frac{3}{7}, 0, 0.45, \sqrt{25}$ **21.** $(x + 3)5$ **22.** $5(3 + x)$ **23.** $5x + 15$ **24.** $65x + 21$

25. $10x + 34y$