

SECTION

5.1

## Adding and Subtracting Polynomials

### Objectives

- 1 Understand the vocabulary used to describe polynomials.
- 2 Add polynomials.
- 3 Subtract polynomials.
- 4 Graph equations defined by polynomials of degree 2.

More education results in a higher income. The mathematical models

$$M = -18x^3 + 923x^2 - 9603x + 48,446$$

$$\text{and } W = 17x^3 - 450x^2 + 6392x - 14,764$$

describe the median, or middlemost, annual income for men,  $M$ , and women,  $W$ , who have completed  $x$  years of education. We'll be working with these models and the data upon which they are based in the exercise set.



The algebraic expressions that appear on the right side of the models are examples of *polynomials*. A **polynomial** is a single term or the sum of two or more terms containing variables with whole-number exponents. These particular polynomials each contain four terms. Equations containing polynomials are used in such diverse areas as science, business, medicine, psychology, and sociology. In this section, we present basic ideas about polynomials. We then use our knowledge of combining like terms to find sums and differences of polynomials.

- 1 Understand the vocabulary used to describe polynomials.

### Describing Polynomials

Consider the polynomial

$$7x^3 - 9x^2 + 13x - 6.$$

We can express this polynomial as

$$7x^3 + (-9x^2) + 13x + (-6).$$

The polynomial contains four terms. It is customary to write the terms in the order of descending powers of the variables. This is the **standard form** of a polynomial.

We begin this chapter by limiting our discussion to polynomials containing only one variable. Each term of such a polynomial in  $x$  is of the form  $ax^n$ . The **degree** of  $ax^n$  is  $n$ . For example, the degree of the term  $7x^3$  is 3.

### Great Question!

**If the degree of a nonzero constant is 0, why doesn't the constant 0 also have degree 0?**

We can express 0 in many ways, including  $0x$ ,  $0x^2$ , and  $0x^3$ . It is impossible to assign a unique exponent to the variable. This is why 0 has no defined degree.

### The Degree of $ax^n$

If  $a \neq 0$  and  $n$  is a whole number, the degree of  $ax^n$  is  $n$ . The degree of a nonzero constant term is 0. The constant 0 has no defined degree.

Here is an example of a polynomial and the degree of each of its four terms:

$$6x^4 - 3x^3 - 2x - 5.$$

degree 4      degree 3      degree 1      degree of nonzero constant: 0

Notice that the exponent on  $x$  for the term  $-2x$ , meaning  $-2x^1$ , is understood to be 1. For this reason, the degree of  $-2x$  is 1.

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A polynomial is simplified when it contains no grouping symbols and no like terms. A simplified polynomial that has exactly one term is called a **monomial**. A **binomial** is a simplified polynomial that has two terms. A **trinomial** is a simplified polynomial with three terms. Simplified polynomials with four or more terms have no special names.

The **degree of a polynomial** is the greatest degree of all the terms of the polynomial. For example,  $4x^2 + 3x$  is a binomial of degree 2 because the degree of the first term is 2, and the degree of the other term is less than 2. Also,  $7x^5 - 2x^2 + 4$  is a trinomial of degree 5 because the degree of the first term is 5, and the degrees of the other terms are less than 5.

Up to now, we have used  $x$  to represent the variable in a polynomial. However, any letter can be used. For example,

- $7x^5 - 3x^3 + 8$  is a polynomial (in  $x$ ) of degree 5. Because there are three terms, the polynomial is a trinomial.
- $6y^3 + 4y^2 - y + 3$  is a polynomial (in  $y$ ) of degree 3. Because there are four terms, the polynomial has no special name.
- $z^7 + \sqrt{2}$  is a polynomial (in  $z$ ) of degree 7. Because there are two terms, the polynomial is a binomial.

**2** Add polynomials.

**Adding Polynomials**

Recall that *like terms* are terms containing exactly the same variables to the same powers. Polynomials are added by combining like terms. For example, we can add the monomials  $-9x^3$  and  $13x^3$  as follows:

$$-9x^3 + 13x^3 = (-9 + 13)x^3 = 4x^3.$$

These like terms both contain  $x$  to the third power.

Add coefficients and keep the same variable factor,  $x^3$ .

**EXAMPLE 1** Adding Polynomials

Add:  $(-9x^3 + 7x^2 - 5x + 3) + (13x^3 + 2x^2 - 8x - 6)$ .

**Solution** The like terms are  $-9x^3$  and  $13x^3$ , containing the same variable to the same power ( $x^3$ ), as well as  $7x^2$  and  $2x^2$  (both containing  $x^2$ ),  $-5x$  and  $-8x$  (both containing  $x$ ), and the constant terms 3 and  $-6$ . We begin by grouping these pairs of like terms.

$$\begin{aligned} & (-9x^3 + 7x^2 - 5x + 3) + (13x^3 + 2x^2 - 8x - 6) \\ &= (-9x^3 + 13x^3) + (7x^2 + 2x^2) + (-5x - 8x) + (3 - 6) && \text{Group like terms.} \\ &= 4x^3 + 9x^2 + (-13x) + (-3) && \text{Combine like terms.} \\ &= 4x^3 + 9x^2 - 13x - 3 && \text{Express addition of opposites as subtraction. } \blacksquare \end{aligned}$$

**✓ CHECK POINT 1** Add:  $(-11x^3 + 7x^2 - 11x - 5) + (16x^3 - 3x^2 + 3x - 15)$ .

Polynomials can also be added by arranging like terms in columns. Then combine like terms, column by column.

**EXAMPLE 2** Adding Polynomials Vertically

Add:  $(-9x^3 + 7x^2 - 5x + 3) + (13x^3 + 2x^2 - 8x - 6)$ .

**Solution**

$$\begin{array}{r} -9x^3 + 7x^2 - 5x + 3 \\ 13x^3 + 2x^2 - 8x - 6 \\ \hline 4x^3 + 9x^2 - 13x - 3 \end{array}$$

*Line up like terms vertically.*

*Add the like terms in each column.*

This is the same answer that we found in Example 1. ■

✓ **CHECK POINT 2** Add the polynomials in Check Point 1 using a vertical format. Begin by arranging like terms in columns.

**Great Question!**

**What's the advantage of using a vertical format to add polynomials instead of a horizontal format?**

A vertical format often makes it easier to see the like terms.

**3** Subtract polynomials.

**Subtracting Polynomials**

We subtract real numbers by adding the opposite, or additive inverse, of the number being subtracted. For example,

$$8 - 3 = 8 + (-3) = 5.$$

Subtraction of polynomials also involves opposites. If the sum of two polynomials is 0, the polynomials are **opposites**, or **additive inverses**, of each other. Here is an example:

$$(4x^2 - 6x - 7) + (-4x^2 + 6x + 7) = 0.$$

The opposite of  $4x^2 - 6x - 7$  is  $-4x^2 + 6x + 7$ , and vice versa.

Observe that the opposite of  $4x^2 - 6x - 7$  can be obtained by changing the sign of each of its coefficients:

<b>Polynomial</b>	Change 4 to -4, change -6 to 6, and change -7 to 7.	<b>Opposite</b>
$4x^2 - 6x - 7$		$-4x^2 + 6x + 7$

In general, **the opposite of a polynomial is that polynomial with the sign of every coefficient changed.** Just as we did with real numbers, we subtract one polynomial from another by adding the opposite of the polynomial being subtracted.

**Subtracting Polynomials**

To subtract two polynomials, add the first polynomial and the opposite of the polynomial being subtracted.

**EXAMPLE 3** Subtracting Polynomials

Subtract:  $(7x^2 + 3x - 4) - (4x^2 - 6x - 7)$ .

**Solution**

$$\begin{aligned} &(7x^2 + 3x - 4) - (4x^2 - 6x - 7) \\ &= (7x^2 + 3x - 4) + (-4x^2 + 6x + 7) && \text{Add the opposite of the polynomial being subtracted.} \\ &= (7x^2 - 4x^2) + (3x + 6x) + (-4 + 7) && \text{Group like terms.} \\ &= 3x^2 + 9x + 3 && \text{Combine like terms.} \quad \blacksquare \end{aligned}$$

✓ **CHECK POINT 3** Subtract:  $(9x^2 + 7x - 2) - (2x^2 - 4x - 6)$ .

**Great Question!**

I'm confused by what it means to subtract one polynomial from a second polynomial. Which polynomial should I write first?

Be careful of the order in Example 4. For example, subtracting 2 from 5 means  $5 - 2$ . In general, subtracting  $B$  from  $A$  means  $A - B$ . The order of the resulting algebraic expression is not the same as the order in English. The polynomial following the word *from* is the one to write first.

**EXAMPLE 4** Subtracting Polynomials

Subtract  $2x^3 - 6x^2 - 3x + 9$  from  $7x^3 - 8x^2 + 9x - 6$ .

**Solution**

$$\begin{aligned}
 & (7x^3 - 8x^2 + 9x - 6) - (2x^3 - 6x^2 - 3x + 9) \\
 &= (7x^3 - 8x^2 + 9x - 6) + (-2x^3 + 6x^2 + 3x - 9) \quad \text{Change the sign of each coefficient.} \\
 &= (7x^3 - 2x^3) + (-8x^2 + 6x^2) \quad \text{Add the opposite of the polynomial being subtracted.} \\
 &\quad + (9x + 3x) + (-6 - 9) \quad \text{Group like terms.} \\
 &= 5x^3 + (-2x^2) + 12x + (-15) \quad \text{Combine like terms.} \\
 &= 5x^3 - 2x^2 + 12x - 15 \quad \text{Express addition of opposites as subtraction.}
 \end{aligned}$$

**CHECK POINT 4** Subtract  $3x^3 - 8x^2 - 5x + 6$  from  $10x^3 - 5x^2 + 7x - 2$ .

Subtraction can also be performed in columns.

**EXAMPLE 5** Subtracting Polynomials Vertically

Use the method of subtracting in columns to find

$$(12y^3 - 9y^2 - 11y - 3) - (4y^3 - 5y + 8).$$

**Solution** Arrange like terms in columns.

$$\begin{array}{r}
 12y^3 - 9y^2 - 11y - 3 \\
 -(4y^3 \quad \quad - 5y + 8) \\
 \hline
 8y^3 - 9y^2 - 6y - 11
 \end{array}$$

Annotations: "Add the opposite of the polynomial being subtracted." (pointing to the second polynomial), "Change the sign of each coefficient of  $4y^3 - 5y + 8$ ." (pointing to the signs in the second polynomial), "Leave space for the missing term." (pointing to the space between  $4y^3$  and  $-5y$ ), "Combine like terms." (pointing to the result).

**CHECK POINT 5** Use the method of subtracting in columns to find

$$(8y^3 - 10y^2 - 14y - 2) - (5y^3 - 3y + 6).$$

**Graphing Equations Defined by Polynomials**

Look at the picture of this gymnast. He has created a perfect balance in which the two halves of his body are mirror images of each other. Graphs of equations defined by polynomials of degree 2, such as  $y = x^2 - 4$ , have this mirrorlike quality. We can obtain their graphs, shaped like bowls or inverted bowls, using the point-plotting method for graphing an equation in two variables.





- 4** Graph equations defined by polynomials of degree 2.

**EXAMPLE 6** Graphing an Equation Defined by a Polynomial of Degree 2

Graph the equation:  $y = x^2 - 4$ .

**Solution** The given equation involves two variables,  $x$  and  $y$ . However, because the variable  $x$  is squared, it is not a linear equation in two variables.

$$y = x^2 - 4$$

This is not in the form  $y = mx + b$  because  $x$  is squared.

Although the graph is not a line, it is still a picture of all the ordered-pair solutions of  $y = x^2 - 4$ . Thus, we can use the point-plotting method to obtain the graph.

**Step 1.** Find several ordered pairs that are solutions of the equation. To find some solutions of  $y = x^2 - 4$ , we select integers for  $x$ , starting with  $-3$  and ending with  $3$ .

**Great Question!**

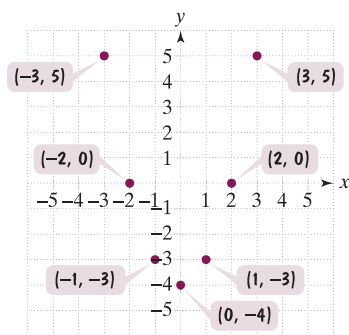
When I graphed lines, I used two, or possibly three, points. Why isn't this enough to use when graphing equations that are not linear?

If the graph of an equation is not a straight line, extra points are needed to get a better general idea of the graph's shape.

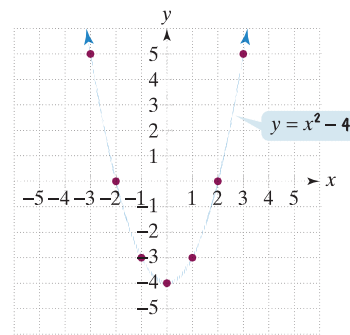
Start with $x$ .	Compute $y$ .	Form the ordered pair $(x, y)$ .
$x$	$y = x^2 - 4$	$(x, y)$
-3	$y = (-3)^2 - 4 = 9 - 4 = 5$	$(-3, 5)$
-2	$y = (-2)^2 - 4 = 4 - 4 = 0$	$(-2, 0)$
-1	$y = (-1)^2 - 4 = 1 - 4 = -3$	$(-1, -3)$
0	$y = 0^2 - 4 = 0 - 4 = -4$	$(0, -4)$
1	$y = 1^2 - 4 = 1 - 4 = -3$	$(1, -3)$
2	$y = 2^2 - 4 = 4 - 4 = 0$	$(2, 0)$
3	$y = 3^2 - 4 = 9 - 4 = 5$	$(3, 5)$

**Step 2.** Plot these ordered pairs as points in the rectangular coordinate system. The seven ordered pairs in the table of values are plotted in **Figure 5.1(a)**.

**Step 3.** Connect the points with a smooth curve. The seven points are joined with a smooth curve in **Figure 5.1(b)**. The graph of  $y = x^2 - 4$  is a curve where the part of the graph to the right of the  $y$ -axis is a reflection of the part to the left of it, and vice versa. The arrows on both ends of the curve indicate that it extends indefinitely in both directions.



**Figure 5.1(a)** Some solutions of  $y = x^2 - 4$  plotted as points



**Figure 5.1(b)** The graph of  $y = x^2 - 4$

**CHECK POINT 6** Graph the equation:  $y = x^2 - 1$ . Select integers for  $x$ , starting with  $-3$  and ending with  $3$ .

### Blitzer Bonus

#### Modeling *American Idol* with a Polynomial of Degree 2

The graph in **Figure 5.2** indicates that the ratings of *American Idol* from season 1 (2002) through season 9 (2010) have a mirrorlike quality. This suggests modeling the show's average number of viewers with a polynomial of degree 2.



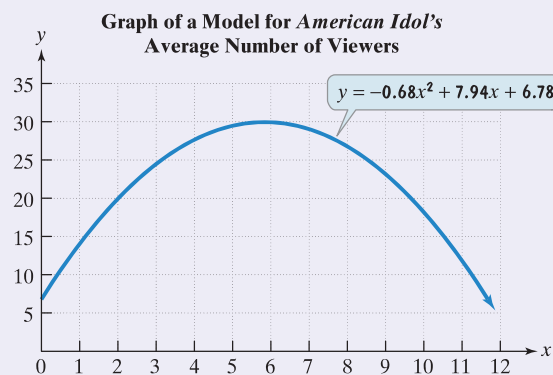
**Figure 5.2**

Source: The Nielsen Company

The equation

$$y = -0.68x^2 + 7.94x + 6.78$$

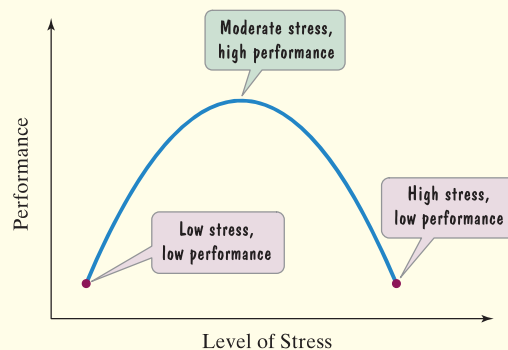
models *American Idol*'s average number of viewers,  $y$ , in millions, where  $x$  is the show's season number. The graph of the model in **Figure 5.3** is shaped like an inverted bowl. Can you see why projections based on this graph have the show's producers looking for a shake-up?



**Figure 5.3**

### Achieving Success

**Address your stress.** Stress levels can help or hinder performance. The graph of the polynomial equation of degree 2 in **Figure 5.4** serves as a model that shows people under both low stress and high stress perform worse than their moderate-stress counterparts.



**Figure 5.4**

Source: Herbert Benson, *Your Maximum Mind*, Random House, 1987.

## CONCEPT AND VOCABULARY CHECK

Fill in each blank so that the resulting statement is true.

1. A polynomial is a single term or the sum of two or more terms containing variables with exponents that are \_\_\_\_\_ numbers.
2. It is customary to write the terms of a polynomial in the order of descending powers of the variable. This is called the \_\_\_\_\_ form of a polynomial.
3. A simplified polynomial that has exactly one term is called a/an \_\_\_\_\_.
4. A simplified polynomial that has two terms is called a/an \_\_\_\_\_.
5. A simplified polynomial that has three terms is called a/an \_\_\_\_\_.
6. The degree of  $ax^n$  is \_\_\_\_\_, provided  $a \neq 0$ .
7. The degree of a polynomial is the \_\_\_\_\_ degree of all the terms of the polynomial.
8. Polynomials are added by combining \_\_\_\_\_ terms.
9. To subtract two polynomials, add the first polynomial and the \_\_\_\_\_ of the polynomial being subtracted.

### 5.1 EXERCISE SET

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#### Practice Exercises

In Exercises 1–16, identify each polynomial as a monomial, a binomial, or a trinomial. Give the degree of the polynomial.

- |                       |                        |
|-----------------------|------------------------|
| 1. $3x + 7$           | 2. $5x - 2$            |
| 3. $x^3 - 2x$         | 4. $x^5 - 7x$          |
| 5. $8x^2$             | 6. $10x^2$             |
| 7. 5                  | 8. 9                   |
| 9. $x^2 - 3x + 4$     | 10. $x^2 - 9x + 2$     |
| 11. $7y^2 - 9y^4 + 5$ | 12. $3y^2 - 14y^5 + 6$ |
| 13. $15x - 7x^3$      | 14. $9x - 5x^3$        |
| 15. $-9y^{23}$        | 16. $-11y^{26}$        |

In Exercises 17–38, add the polynomials.

17.  $(9x + 8) + (-17x + 5)$
18.  $(8x - 5) + (-13x + 9)$
19.  $(4x^2 + 6x - 7) + (8x^2 + 9x - 2)$
20.  $(11x^2 + 7x - 4) + (27x^2 + 10x - 20)$
21.  $(7x^2 - 11x) + (3x^2 - x)$
22.  $(-3x^2 + x) + (4x^2 + 8x)$
23.  $(4x^2 - 6x + 12) + (x^2 + 3x + 1)$
24.  $(-7x^2 + 8x + 3) + (2x^2 + x + 8)$
25.  $(4y^3 + 7y - 5) + (10y^2 - 6y + 3)$
26.  $(2y^3 + 3y + 10) + (3y^2 + 5y - 22)$
27.  $(2x^2 - 6x + 7) + (3x^3 - 3x)$
28.  $(4x^3 + 5x + 13) + (-4x^2 + 22)$
29.  $(4y^2 + 8y + 11) + (-2y^3 + 5y + 2)$
30.  $(7y^3 + 5y - 1) + (2y^2 - 6y + 3)$
31.  $(-2y^6 + 3y^4 - y^2) + (-y^6 + 5y^4 + 2y^2)$
32.  $(7r^4 + 5r^2 + 2r) + (-18r^4 - 5r^2 - r)$

$$33. \left(9x^3 - x^2 - x - \frac{1}{3}\right) + \left(x^3 + x^2 + x + \frac{4}{3}\right)$$

$$34. \left(12x^3 - x^2 - x + \frac{4}{3}\right) + \left(x^3 + x^2 + x - \frac{1}{3}\right)$$

$$35. \left(\frac{1}{5}x^4 + \frac{1}{3}x^3 + \frac{3}{8}x^2 + 6\right) + \left(-\frac{3}{5}x^4 + \frac{2}{3}x^3 - \frac{1}{2}x^2 - 6\right)$$

$$36. \left(\frac{2}{5}x^4 + \frac{2}{3}x^3 + \frac{5}{8}x^2 + 7\right) + \left(-\frac{4}{5}x^4 + \frac{1}{3}x^3 - \frac{1}{4}x^2 - 7\right)$$

$$37. (0.03x^5 - 0.1x^3 + x + 0.03) + (-0.02x^5 + x^4 - 0.7x + 0.3)$$

$$38. (0.06x^5 - 0.2x^3 + x + 0.05) + (-0.04x^5 + 2x^4 - 0.8x + 0.5)$$

In Exercises 39–54, use a vertical format to add the polynomials.

$$39. \begin{array}{r} 5y^3 - 7y^2 \\ 6y^3 + 4y^2 \\ \hline \end{array}$$

$$40. \begin{array}{r} 13x^4 - x^2 \\ 7x^4 + 2x^2 \\ \hline \end{array}$$

$$41. \begin{array}{r} 3x^2 - 7x + 4 \\ -5x^2 + 6x - 3 \\ \hline \end{array}$$

$$42. \begin{array}{r} 7x^2 - 5x - 6 \\ -9x^2 + 4x + 6 \\ \hline \end{array}$$

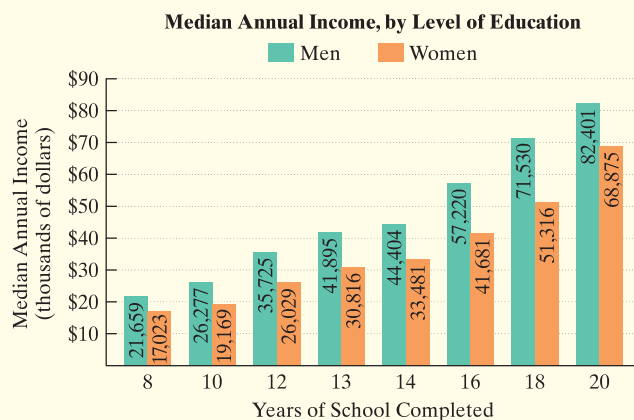
$$43. \begin{array}{r} \frac{1}{4}x^4 - \frac{2}{3}x^3 - 5 \\ -\frac{1}{2}x^4 + \frac{1}{5}x^3 + 4.7 \\ \hline \end{array}$$



100. Subtract  $-3x^3 - 7x + 5$  from the sum of  $2x^2 + 4x - 7$  and  $-5x^3 - 2x - 3$ .
101. Subtract  $-y^2 + 7y^3$  from the difference between  $-5 + y^2 + 4y^3$  and  $-8 - y + 7y^3$ . Express the answer in standard form.
102. Subtract  $-2y^2 + 8y^3$  from the difference between  $-6 + y^2 + 5y^3$  and  $-12 - y + 13y^3$ . Express the answer in standard form.

### Application Exercises

As you complete more years of education, you can count on a greater income. The bar graph shows the median, or middlemost, annual income for Americans, by level of education, for a recent year.



Source: Bureau of the Census

Here are polynomial models that describe the median annual income for men,  $M$ , and for women,  $W$ , who have completed  $x$  years of education:

$$M = 177x^2 + 288x + 7075$$

$$W = 255x^2 - 2956x + 24,336$$

$$M = -18x^3 + 923x^2 - 9603x + 48,446$$

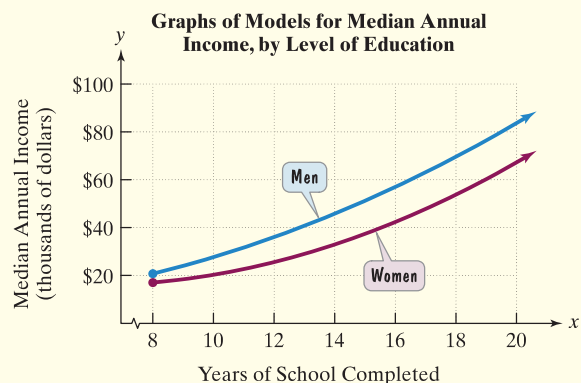
$$W = 17x^3 - 450x^2 + 6392x - 14,764.$$

Exercises 103–106 are based on these models and the data displayed by the graph.

103. a. Use the equations defined by polynomials of degree 3 to find a mathematical model for  $M - W$ .
  - b. According to the model in part (a), what is the difference in the median annual income between men and women with 14 years of education?
  - c. According to the data displayed by the graph, what is the actual difference in the median annual income between men and women with 14 years of education? Did the model in part (b) underestimate or overestimate this difference? By how much?
104. a. Use the equations defined by polynomials of degree 3 to find a mathematical model for  $M - W$ .
  - b. According to the model in part (a), what is the difference in the median annual income between men and women with 16 years of education?

- c. According to the data displayed by the graph, what is the actual difference in the median annual income between men and women with 16 years of education? Did the model in part (b) underestimate or overestimate this difference? By how much?

105. a. Use the equation defined by a polynomial of degree 2 to find the median annual income for a man with 16 years of education. Does this underestimate or overestimate the median income shown by the bar graph in the previous column? By how much?
  - b. Shown in a rectangular coordinate system are the graphs of the polynomial models of degree 2 that describe median annual income, by level of education. Identify your solution from part (a) as a point on the appropriate graph.



- c. Use the appropriate graph in part (b) to estimate, to the nearest thousand dollars, the median annual income for a woman with 16 years of education.
106. a. Use the equation defined by a polynomial of degree 2 to find the median annual income for a woman with 18 years of education. Does this underestimate or overestimate the median income shown by the bar graph in the previous column? By how much?
  - b. Shown in Exercise 105(b) are rectangular coordinate graphs of the polynomial models of degree 2 that describe median annual income, by level of education. Identify your solution from part (a) as a point on the appropriate graph.
  - c. Use the appropriate graph in Exercise 105(b) to estimate, to the nearest thousand dollars, the median annual income for a man with 18 years of education.

### Writing in Mathematics

107. What is a polynomial?
108. What is a monomial? Give an example with your explanation.
109. What is a binomial? Give an example with your explanation.



110. What is a trinomial? Give an example with your explanation.
111. What is the degree of a polynomial? Provide an example with your explanation.
112. Explain how to add polynomials.
113. Explain how to subtract polynomials.

### Critical Thinking Exercises

**Make Sense?** In Exercises 114–117, determine whether each statement “makes sense” or “does not make sense” and explain your reasoning.

114. I add like monomials by adding both their coefficients and the exponents that appear on their common variable factor.
115. By looking at the first term of a polynomial, I can determine its degree.
116. As long as I understand how to add and subtract polynomials, I can select the format, horizontal or vertical, that works best for me.
117. I used two points and a checkpoint to graph  $y = x^2 - 4$ .

In Exercises 118–121, determine whether each statement is true or false. If the statement is false, make the necessary change(s) to produce a true statement.

118. It is not possible to write a binomial with degree 0.
119.  $\frac{1}{5x^2} + \frac{1}{3x}$  is a binomial.
120.  $(2x^2 - 8x + 6) - (x^2 - 3x + 5) = x^2 - 5x + 1$  for any value of  $x$ .

121. In the polynomial  $3x^2 - 5x + 13$ , the coefficient of  $x$  is 5.
122. What polynomial must be subtracted from  $5x^2 - 2x + 1$  so that the difference is  $8x^2 - x + 3$ ?
123. The number of people who catch a cold  $t$  weeks after January 1 is  $5t - 3t^2 + t^3$ . The number of people who recover  $t$  weeks after January 1 is  $t - t^2 + \frac{1}{3}t^3$ . Write a polynomial in standard form for the number of people who are still ill with a cold  $t$  weeks after January 1.
124. Explain why it is not possible to add two polynomials of degree 3 and get a polynomial of degree 4.

### Review Exercises

125. Simplify:  $(-10)(-7) \div (1 - 8)$ .  
(Section 1.8, Example 8)
126. Subtract:  $-4.6 - (-10.2)$ .  
(Section 1.6, Example 2)
127. Solve:  $3(x - 2) = 9(x + 2)$ .  
(Section 2.3, Example 3)

### Preview Exercises

Exercises 128–130 will help you prepare for the material covered in the next section.

128. Find the missing exponent, designated by the question mark, in the final step.  
$$x^3 \cdot x^4 = (x \cdot x \cdot x) \cdot (x \cdot x \cdot x \cdot x) = x^?$$
129. Use the distributive property to multiply:  $3x(x + 5)$ .
130. Simplify:  $x(x + 2) + 3(x + 2)$ .

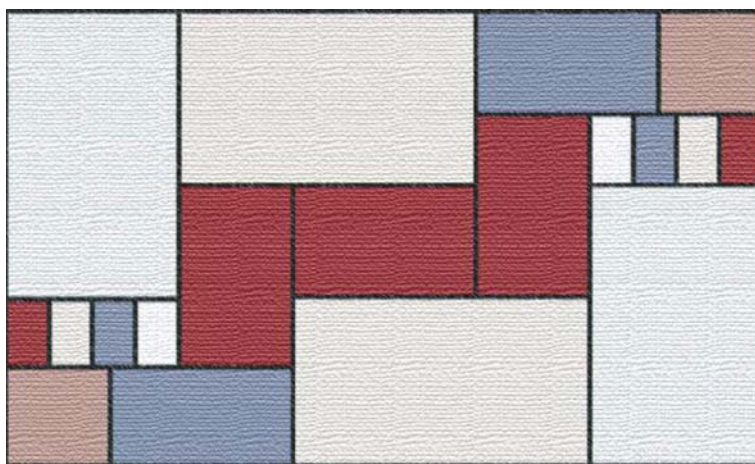
## SECTION

# 5.2

## Multiplying Polynomials

### Objectives

- 1 Use the product rule for exponents.
- 2 Use the power rule for exponents.
- 3 Use the products-to-powers rule.
- 4 Multiply monomials.
- 5 Multiply a monomial and a polynomial.
- 6 Multiply polynomials when neither is a monomial.



The ancient Greeks believed that the most visually pleasing rectangles have a ratio of length to width of approximately 1.618 to 1. With the exception of the squares on the lower left and the upper right, the interior of this geometric figure is filled entirely with these *golden rectangles*. Furthermore, the large rectangle is also a golden rectangle.



The total area of the large rectangle shown on the previous page can be found in many ways. This is because the area of any large rectangular region is related to the areas of the smaller rectangles that make up that region. In this section, we apply areas of rectangles as a way to picture the multiplication of polynomials. Before studying how polynomials are multiplied, we must develop some rules for working with exponents.

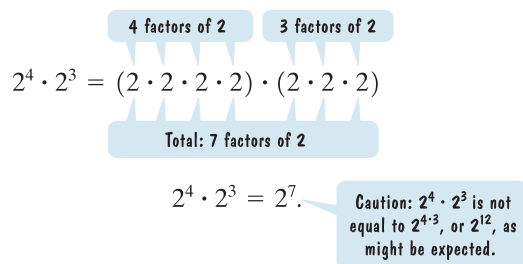
**1** Use the product rule for exponents.

### The Product Rule for Exponents

We have seen that exponents are used to indicate repeated multiplication. For example,  $2^4$ , where 2 is the base and 4 is the exponent, indicates that 2 occurs as a factor four times:

$$2^4 = 2 \cdot 2 \cdot 2 \cdot 2.$$

Now consider the multiplication of two exponential expressions, such as  $2^4 \cdot 2^3$ . We are multiplying 4 factors of 2 and 3 factors of 2. We have a total of 7 factors of 2:



We can quickly find the exponent, 7, of the product by adding 4 and 3, the original exponents:

$$2^4 \cdot 2^3 = 2^{4+3} = 2^7.$$

This suggests the following rule:

#### The Product Rule

$$b^m \cdot b^n = b^{m+n}$$

When multiplying exponential expressions with the same base, add the exponents. Use this sum as the exponent of the common base.

#### EXAMPLE 1 Using the Product Rule

Multiply each expression using the product rule:

- a.  $2^2 \cdot 2^3$       b.  $x^7 \cdot x^9$       c.  $y \cdot y^5$       d.  $y^3 \cdot y^2 \cdot y^5$

#### Solution

- a.  $2^2 \cdot 2^3 = 2^{2+3} = 2^5$  or 32  
 b.  $x^7 \cdot x^9 = x^{7+9} = x^{16}$   
 c.  $y \cdot y^5 = y^1 \cdot y^5 = y^{1+5} = y^6$   
 d.  $y^3 \cdot y^2 \cdot y^5 = y^{3+2+5} = y^{10}$  ■

**CHECK POINT 1** Multiply each expression using the product rule:

- a.  $2^2 \cdot 2^4$       b.  $x^6 \cdot x^4$       c.  $y \cdot y^7$       d.  $y^4 \cdot y^3 \cdot y^2$

#### Great Question!

**Can I use the product rule to multiply an expression such as  $x^7 \cdot y^9$ ?**

The product rule does not apply to exponential expressions with different bases:

$x^7 \cdot y^9$ , or  $x^7 y^9$ , cannot be simplified.

- 2** Use the power rule for exponents.

### The Power Rule for Exponents

The next property of exponents applies when an exponential expression is raised to a power. Here is an example:

$$(3^2)^4.$$

The exponential expression  $3^2$  is raised to the fourth power.

There are 4 factors of  $3^2$ . Thus,

$$(3^2)^4 = 3^2 \cdot 3^2 \cdot 3^2 \cdot 3^2 = 3^{2+2+2+2} = 3^8.$$

Add exponents when multiplying with the same base.

We can obtain the answer,  $3^8$ , by multiplying the exponents:

$$(3^2)^4 = 3^{2 \cdot 4} = 3^8.$$

By generalizing  $(3^2)^4 = 3^{2 \cdot 4} = 3^8$ , we obtain the following rule:

#### The Power Rule (Powers to Powers)

$$(b^m)^n = b^{mn}$$

When an exponential expression is raised to a power, multiply the exponents. Place the product of the exponents on the base and remove the parentheses.

#### EXAMPLE 2 Using the Power Rule

Simplify each expression using the power rule:

- a.  $(2^3)^5$       b.  $(x^6)^4$       c.  $[(-3)^7]^5$ .

#### Solution

- a.  $(2^3)^5 = 2^{3 \cdot 5} = 2^{15}$   
b.  $(x^6)^4 = x^{6 \cdot 4} = x^{24}$   
c.  $[(-3)^7]^5 = (-3)^{7 \cdot 5} = (-3)^{35}$  ■

✓ **CHECK POINT 2** Simplify each expression using the power rule:

- a.  $(3^4)^5$       b.  $(x^9)^{10}$       c.  $[(-5)^7]^3$ .

### Great Question!

Can you show me examples that illustrate the difference between the product rule and the power rule?

Do not confuse the product and power rules. Note the following differences:

- $x^4 \cdot x^7 = x^{4+7} = x^{11}$
- $(x^4)^7 = x^{4 \cdot 7} = x^{28}$ .

- 3** Use the products-to-powers rule.

### The Products-to-Powers Rule for Exponents

The next property of exponents applies when we are raising a product to a power. Here is an example:

$$(2x)^4.$$

The product  $2x$  is raised to the fourth power.

There are four factors of  $2x$ . Thus,

$$(2x)^4 = 2x \cdot 2x \cdot 2x \cdot 2x = 2 \cdot 2 \cdot 2 \cdot 2 \cdot x \cdot x \cdot x \cdot x = 2^4 x^4.$$

We can obtain the answer,  $2^4 x^4$ , by raising each factor within the parentheses to the fourth power:

$$(2x)^4 = 2^4 x^4.$$

Generalizing from  $(2x)^4 = 2^4x^4$  suggests the following rule:

### Products to Powers

$$(ab)^n = a^n b^n$$

When a product is raised to a power, raise each factor to the power.

### EXAMPLE 3 Using the Products-to-Powers Rule

Simplify each expression using the products-to-powers rule:

a.  $(5x)^3$       b.  $(-2y^4)^5$ .

#### Solution

a.  $(5x)^3 = 5^3x^3$   
 $= 125x^3$

Raise each factor to the third power.  
 $5^3 = 5 \cdot 5 \cdot 5 = 125$

b.  $(-2y^4)^5 = (-2)^5(y^4)^5$   
 $= (-2)^5y^{4 \cdot 5}$   
 $= -32y^{20}$

Raise each factor to the fifth power.  
 To raise an exponential expression to a power, multiply exponents:  $(b^m)^n = b^{mn}$ .  
 $(-2)^5 = (-2)(-2)(-2)(-2)(-2) = -32$  ■

✓ **CHECK POINT 3** Simplify each expression using the products-to-powers rule:

a.  $(2x)^4$       b.  $(-4y^2)^3$ .

### Great Question!

**What are some common errors to avoid when simplifying exponential expressions?**

Here's a partial list. The first column shows the correct simplification. The second column illustrates a common error.

Correct	Incorrect	Description of Error
$b^3 \cdot b^4 = b^{3+4} = b^7$	<del><math>b^3 \cdot b^4 = b^{12}</math></del>	Exponents should be added, not multiplied.
$3^2 \cdot 3^4 = 3^{2+4} = 3^6$	<del><math>3^2 \cdot 3^4 = 9^{2+4} = 9^6</math></del>	The common base should be retained, not multiplied.
$(x^5)^3 = x^{5 \cdot 3} = x^{15}$	<del><math>(x^5)^3 = x^{5+3} = x^8</math></del>	Exponents should be multiplied, not added, when raising a power to a power.
$(4x)^3 = 4^3x^3 = 64x^3$	<del><math>(4x)^3 = 4x^3</math></del>	Both factors should be cubed.

## 4 Multiply monomials.

### Multiplying Monomials

Now that we have developed three properties of exponents, we are ready to turn to polynomial multiplication. We begin with the product of two monomials, such as  $-8x^6$  and  $5x^3$ . This product is obtained by multiplying the coefficients,  $-8$  and  $5$ , and then multiplying the variables using the product rule for exponents.

$$(-8x^6)(5x^3) = -8 \cdot 5 \cdot x^6 \cdot x^3 = -8 \cdot 5x^{6+3} = -40x^9$$

Multiply coefficients and add exponents.

### Great Question!

**Because monomials with the same base and different exponents can be multiplied, can they also be added?**

No. Don't confuse adding and multiplying monomials.

**Addition:**

$$5x^4 + 6x^4 = 11x^4$$

**Multiplication:**

$$\begin{aligned}(5x^4)(6x^4) &= (5 \cdot 6)(x^4 \cdot x^4) \\ &= 30x^{4+4} \\ &= 30x^8\end{aligned}$$

Only like terms can be added or subtracted, but unlike terms may be multiplied.

**Addition:**

$5x^4 + 3x^2$  cannot be simplified.

**Multiplication:**

$$\begin{aligned}(5x^4)(3x^2) &= (5 \cdot 3)(x^4 \cdot x^2) \\ &= 15x^{4+2} \\ &= 15x^6\end{aligned}$$

- 5** Multiply a monomial and a polynomial.

### Multiplying Monomials

To multiply monomials with the same variable base, multiply the coefficients and then multiply the variables. Use the product rule for exponents to multiply the variables: Keep the variable and add the exponents.

#### EXAMPLE 4 Multiplying Monomials

Multiply: a.  $(2x)(4x^2)$       b.  $(-10x^6)(6x^{10})$ .

**Solution**

$$\begin{aligned}\text{a. } (2x)(4x^2) &= (2 \cdot 4)(x \cdot x^2) \\ &= 8x^{1+2} \\ &= 8x^3\end{aligned}$$

Multiply the coefficients and multiply the variables.

Add exponents:  $b^m \cdot b^n = b^{m+n}$ .

Simplify.

$$\begin{aligned}\text{b. } (-10x^6)(6x^{10}) &= (-10 \cdot 6)(x^6 \cdot x^{10}) \\ &= -60x^{6+10} \\ &= -60x^{16}\end{aligned}$$

Multiply the coefficients and multiply the variables.

Add exponents:  $b^m \cdot b^n = b^{m+n}$ .

Simplify. ■

**CHECK POINT 4** Multiply: a.  $(7x^2)(10x)$       b.  $(-5x^4)(4x^5)$ .

### Multiplying a Monomial and a Polynomial That Is Not a Monomial

We use the distributive property to multiply a monomial and a polynomial that is not a monomial. For example,

$$3x^2(2x^3 + 5x) = 3x^2 \cdot 2x^3 + 3x^2 \cdot 5x = 3 \cdot 2x^{2+3} + 3 \cdot 5x^{2+1} = 6x^5 + 15x^3.$$

Monomial

Binomial

Multiply coefficients and add exponents.

#### Multiplying a Monomial and a Polynomial That Is Not a Monomial

To multiply a monomial and a polynomial, use the distributive property to multiply each term of the polynomial by the monomial.

#### EXAMPLE 5 Multiplying a Monomial and a Polynomial

Multiply: a.  $2x(x + 4)$       b.  $3x^2(4x^3 - 5x + 2)$ .

**Solution**

$$\begin{aligned}\text{a. } 2x(x + 4) &= 2x \cdot x + 2x \cdot 4 \\ &= 2 \cdot 1x^{1+1} + 2 \cdot 4x \\ &= 2x^2 + 8x\end{aligned}$$

Use the distributive property.

To multiply the monomials, multiply coefficients and add exponents.

Simplify.

$$\begin{aligned}\text{b. } 3x^2(4x^3 - 5x + 2) &= 3x^2 \cdot 4x^3 - 3x^2 \cdot 5x + 3x^2 \cdot 2 \\ &= 3 \cdot 4x^{2+3} - 3 \cdot 5x^{2+1} + 3 \cdot 2x^2 \\ &= 12x^5 - 15x^3 + 6x^2\end{aligned}$$

Use the distributive property.

To multiply the monomials, multiply coefficients and add exponents.

Simplify. ■

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Rectangles often make it possible to visualize polynomial multiplication. For example, **Figure 5.5** shows a rectangle with length  $2x$  and width  $x + 4$ . The area of the large rectangle is

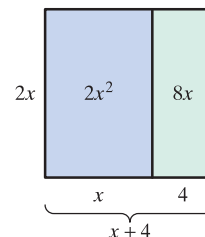
$$2x(x + 4).$$

The sum of the areas of the two smaller rectangles is

$$2x^2 + 8x.$$

Conclusion:

$$2x(x + 4) = 2x^2 + 8x$$



**Figure 5.5**

**✓ CHECK POINT 5** Multiply:

a.  $3x(x + 5)$

b.  $6x^2(5x^3 - 2x + 3)$ .

**6** Multiply polynomials when neither is a monomial.

### Multiplying Polynomials when Neither Is a Monomial

How do we multiply two polynomials if neither is a monomial? For example, consider

$$(2x + 3)(x^2 + 4x + 5).$$

Binomial

Trinomial

One way to perform this multiplication is to distribute  $2x$  throughout the trinomial

$$2x(x^2 + 4x + 5)$$

and  $3$  throughout the trinomial

$$3(x^2 + 4x + 5).$$

Then combine the like terms that result. In general, the product of two polynomials is the polynomial obtained by multiplying each term of one polynomial by each term of the other polynomial and then combining like terms.

### Using Technology

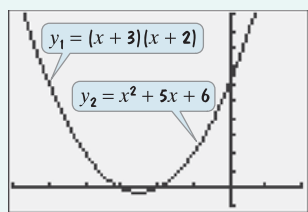
#### Graphic Connections

The graphs of

$$y_1 = (x + 3)(x + 2)$$

and  $y_2 = x^2 + 5x + 6$

are the same.



$[-6, 2, 1]$  by  $[-1, 10, 1]$

This verifies that

$$(x + 3)(x + 2) = x^2 + 5x + 6.$$

### Multiplying Polynomials when Neither Is a Monomial

Multiply each term of one polynomial by each term of the other polynomial. Then combine like terms.

#### EXAMPLE 6 Multiplying Binomials

Multiply: a.  $(x + 3)(x + 2)$       b.  $(3x + 7)(2x - 4)$ .

**Solution** We begin by multiplying each term of the second binomial by each term of the first binomial.

$$\begin{aligned} \text{a. } & (x + 3)(x + 2) \\ & = x(x + 2) + 3(x + 2) \end{aligned}$$

Multiply the second binomial by each term of the first binomial.

$$= x \cdot x + x \cdot 2 + 3 \cdot x + 3 \cdot 2$$

Use the distributive property.

$$= x^2 + 2x + 3x + 6$$

Multiply. Note that  $x \cdot x = x^1 \cdot x^1 = x^{1+1} = x^2$ .

$$= x^2 + 5x + 6$$

Combine like terms.

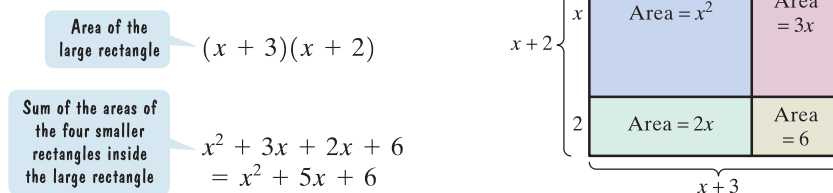
**SECTION 5.2** Multiplying Polynomials **355**

b.  $(3x + 7)(2x - 4)$   
 $= 3x(2x - 4) + 7(2x - 4)$  *Multiply the second binomial by each term of the first binomial.*  
 $= 3x \cdot 2x - 3x \cdot 4 + 7 \cdot 2x - 7 \cdot 4$  *Use the distributive property.*  
 $= 6x^2 - 12x + 14x - 28$  *Multiply.*  
 $= 6x^2 + 2x - 28$  *Combine like terms. ■*

**✓ CHECK POINT 6** Multiply:

a.  $(x + 4)(x + 5)$                       b.  $(5x + 3)(2x - 7)$ .

You can visualize the polynomial multiplication in Example 6(a),  $(x + 3)(x + 2) = x^2 + 5x + 6$ , by analyzing the areas in **Figure 5.6**.



**Figure 5.6**

Conclusion:

$$(x + 3)(x + 2) = x^2 + 5x + 6$$

**EXAMPLE 7** Multiplying a Binomial and a Trinomial

Multiply:  $(2x + 3)(x^2 + 4x + 5)$ .

**Solution**

$(2x + 3)(x^2 + 4x + 5)$   
 $= 2x(x^2 + 4x + 5) + 3(x^2 + 4x + 5)$  *Multiply the trinomial by each term of the binomial.*  
 $= 2x \cdot x^2 + 2x \cdot 4x + 2x \cdot 5 + 3x^2 + 3 \cdot 4x + 3 \cdot 5$  *Use the distributive property.*  
 $= 2x^3 + 8x^2 + 10x + 3x^2 + 12x + 15$  *Multiply monomials: Multiply coefficients and add exponents.*  
 $= 2x^3 + 11x^2 + 22x + 15$  *Combine like terms:*  
 $8x^2 + 3x^2 = 11x^2$  and  $10x + 12x = 22x$ . ■

**✓ CHECK POINT 7** Multiply:  $(5x + 2)(x^2 - 4x + 3)$ .

Another method for solving Example 7 is to use a vertical format similar to that used for multiplying whole numbers.

$$\begin{array}{r}
 x^2 + 4x + 5 \\
 2x + 3 \\
 \hline
 3x^2 + 12x + 15 \quad \leftarrow 3(x^2 + 4x + 5) \\
 2x^3 + 8x^2 + 10x \quad \leftarrow 2x(x^2 + 4x + 5) \\
 \hline
 2x^3 + 11x^2 + 22x + 15
 \end{array}$$

*Write like terms in the same column.*                      *Combine like terms.*

**EXAMPLE 8** Multiplying Polynomials Using a Vertical Format

Multiply:  $(2x^2 - 3x)(5x^3 - 4x^2 + 7x)$ .



**Solution** To use the vertical format to find  $(2x^2 - 3x)(5x^3 - 4x^2 + 7x)$ , it is most convenient to write the polynomial with the greater number of terms in the top row.

$$\begin{array}{r} 5x^3 - 4x^2 + 7x \\ \underline{2x^2 - 3x} \end{array}$$

We now multiply each term in the top polynomial by the last term in the bottom polynomial.

$$\begin{array}{r} 5x^3 - 4x^2 + 7x \\ \underline{2x^2 - 3x} \\ -15x^4 + 12x^3 - 21x^2 \end{array} \quad -3x(5x^3 - 4x^2 + 7x)$$

Then we multiply each term in the top polynomial by  $2x^2$ , the first term in the bottom polynomial. Like terms are placed in columns because the final step involves combining them.

$$\begin{array}{r} 5x^3 - 4x^2 + 7x \\ \underline{2x^2 - 3x} \\ -15x^4 + 12x^3 - 21x^2 \\ 10x^5 - 8x^4 + 14x^3 \\ \hline 10x^5 - 23x^4 + 26x^3 - 21x^2 \end{array}$$

Write like terms in the same column.

$-3x(5x^3 - 4x^2 + 7x)$

$2x^2(5x^3 - 4x^2 + 7x)$

Combine like terms, which are lined up in columns.

**CHECK POINT 8** Multiply using a vertical format:  $(3x^2 - 2x)(2x^3 - 5x^2 + 4x)$ .

## CONCEPT AND VOCABULARY CHECK

Fill in each blank so that the resulting statement is true.

- The product rule for exponents states that  $b^m \cdot b^n =$  \_\_\_\_\_. When multiplying exponential expressions with the same base, \_\_\_\_\_ the exponents.
- The power rule for exponents states that  $(b^m)^n =$  \_\_\_\_\_. When an exponential expression is raised to a power, \_\_\_\_\_ the exponents.
- The products-to-powers rule for exponents states that  $(ab)^n =$  \_\_\_\_\_. When a product is raised to a power, raise each \_\_\_\_\_ to the power.
- To multiply  $2x^2(x^2 + 5x + 7)$ , use the \_\_\_\_\_ property to multiply each term of the polynomial \_\_\_\_\_ by the monomial \_\_\_\_\_.
- To multiply  $(4x + 7)(x^2 + 8x + 3)$ , begin by multiplying each term of  $x^2 + 8x + 3$  by \_\_\_\_\_. Then multiply each term of  $x^2 + 8x + 3$  by \_\_\_\_\_. Then combine \_\_\_\_\_ terms.

## 5.2 EXERCISE SET

MyMathLab®



### Practice Exercises

In Exercises 1–8, multiply each expression using the product rule.

- |                              |                              |
|------------------------------|------------------------------|
| 1. $x^{15} \cdot x^3$        | 2. $x^{12} \cdot x^4$        |
| 3. $y \cdot y^{11}$          | 4. $y \cdot y^{19}$          |
| 5. $x^2 \cdot x^6 \cdot x^3$ | 6. $x^4 \cdot x^3 \cdot x^5$ |
| 7. $7^9 \cdot 7^{10}$        | 8. $8^7 \cdot 8^{10}$        |

In Exercises 9–14, simplify each expression using the power rule.

- |                   |                   |
|-------------------|-------------------|
| 9. $(6^9)^{10}$   | 10. $(6^7)^{10}$  |
| 11. $(x^{15})^3$  | 12. $(x^{12})^4$  |
| 13. $[(-20)^3]^3$ | 14. $[(-50)^4]^4$ |

In Exercises 15–24, simplify each expression using the product-to-powers rule.

- |                 |                    |
|-----------------|--------------------|
| 15. $(2x)^3$    | 16. $(4x)^3$       |
| 17. $(-5x)^2$   | 18. $(-6x)^2$      |
| 19. $(4x^3)^2$  | 20. $(6x^3)^2$     |
| 21. $(-2y^6)^4$ | 22. $(-2y^5)^4$    |
| 23. $(-2x^7)^5$ | 24. $(-2x^{11})^5$ |

In Exercises 25–34, multiply the monomials.

- |  |  |
|--|--|
| 25. $(7x)(2x)$   | 26. $(8x)(3x)$   |
| 27. $(6x)(4x^2)$   | 28. $(10x)(3x^2)$  |
| 29. $(-5y^4)(3y^3)$  | 30. $(-6y^4)(2y^3)$  |
| 31. $\left(-\frac{1}{2}a^3\right)\left(-\frac{1}{4}a^2\right)$ | 32. $\left(-\frac{1}{3}a^4\right)\left(-\frac{1}{2}a^2\right)$ |
| 33. $(2x^2)(-3x)(8x^4)$  | 34. $(3x^3)(-2x)(5x^6)$  |

In Exercises 35–54, find each product of the monomial and the polynomial.

- |                            |                 |
|----------------------------|-----------------|
| 35. $4x(x + 3)$            | 36. $6x(x + 5)$ |
| 37. $x(x - 3)$             | 38. $x(x - 7)$  |
| 39. $2x(x - 6)$            | 40. $3x(x - 5)$ |
| 41. $-4y(3y + 5)$          |                 |
| 42. $-5y(6y + 7)$          |                 |
| 43. $4x^2(x + 2)$          |                 |
| 44. $5x^2(x + 6)$          |                 |
| 45. $2y^2(y^2 + 3y)$       |                 |
| 46. $4y^2(y^2 + 2y)$       |                 |
| 47. $2y^2(3y^2 - 4y + 7)$  |                 |
| 48. $4y^2(5y^2 - 6y + 3)$  |                 |
| 49. $(3x^3 + 4x^2)(2x)$    |                 |
| 50. $(4x^3 + 5x^2)(2x)$    |                 |
| 51. $(x^2 + 5x - 3)(-2x)$  |                 |
| 52. $(x^3 - 2x + 2)(-4x)$  |                 |
| 53. $-3x^2(-4x^2 + x - 5)$ |                 |
| 54. $-6x^2(3x^2 - 2x - 7)$ |                 |

In Exercises 55–78, find each product. In each case, neither factor is a monomial.

- |                       |
|-----------------------|
| 55. $(x + 3)(x + 5)$  |
| 56. $(x + 4)(x + 6)$  |
| 57. $(2x + 1)(x + 4)$ |
| 58. $(2x + 5)(x + 3)$ |
| 59. $(x + 3)(x - 5)$  |
| 60. $(x + 4)(x - 6)$  |
| 61. $(x - 11)(x + 9)$ |
| 62. $(x - 12)(x + 8)$ |
| 63. $(2x - 5)(x + 4)$ |
| 64. $(3x - 4)(x + 5)$ |

- |  |
|--|
| 65. $\left(\frac{1}{4}x + 4\right)\left(\frac{3}{4}x - 1\right)$ |
| 66. $\left(\frac{1}{5}x + 5\right)\left(\frac{3}{5}x - 1\right)$ |
| 67. $(x + 1)(x^2 + 2x + 3)$                                      |
| 68. $(x + 2)(x^2 + x + 5)$                                       |
| 69. $(y - 3)(y^2 - 3y + 4)$                                      |
| 70. $(y - 2)(y^2 - 4y + 3)$                                      |
| 71. $(2a - 3)(a^2 - 3a + 5)$                                     |
| 72. $(2a - 1)(a^2 - 4a + 3)$                                     |
| 73. $(x + 1)(x^3 + 2x^2 + 3x + 4)$                               |
| 74. $(x + 1)(x^3 + 4x^2 + 7x + 3)$                               |
| 75. $\left(x - \frac{1}{2}\right)(4x^3 - 2x^2 + 5x - 6)$         |
| 76. $\left(x - \frac{1}{3}\right)(3x^3 - 6x^2 + 5x - 9)$         |
| 77. $(x^2 + 2x + 1)(x^2 - x + 2)$                                |
| 78. $(x^2 + 3x + 1)(x^2 - 2x - 1)$                               |

In Exercises 79–92, use a vertical format to find each product.

- |                                  |               |
|----------------------------------|---------------|
| 79. $x^2 - 5x + 3$               | $x + 8$       |
| 80. $x^2 - 7x + 9$               | $x + 4$       |
| 81. $x^2 - 3x + 9$               | $2x - 3$      |
| 82. $y^2 - 5y + 3$               | $4y - 5$      |
| 83. $2x^3 + x^2 + 2x + 3$        | $x + 4$       |
| 84. $3y^3 + 2y^2 + y + 4$        | $y + 3$       |
| 85. $4z^3 - 2z^2 + 5z - 4$       | $3z - 2$      |
| 86. $5z^3 - 3z^2 + 7z - 4$       | $2z - 4$      |
| 87. $7x^3 - 5x^2 + 6x$           | $3x^2 - 4x$   |
| 88. $9y^3 - 7y^2 + 5y$           | $3y^2 + 5y$   |
| 89. $2y^5 - 3y^3 + y^2 - 2y + 3$ | $2y - 1$      |
| 90. $n^4 - n^3 + n^2 - n + 1$    | $2n + 3$      |
| 91. $x^2 + 7x - 3$               | $x^2 - x - 1$ |
| 92. $x^2 + 6x - 4$               | $x^2 - x - 2$ |

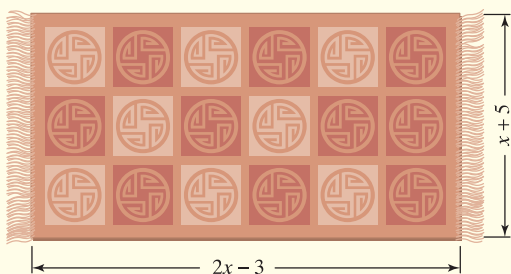
**Practice PLUS**

In Exercises 93–100, perform the indicated operations.

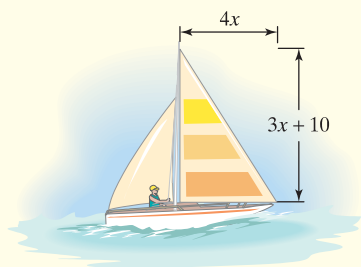
- 93.  $(x + 4)(x - 5) - (x + 3)(x - 6)$
- 94.  $(x + 5)(x - 6) - (x + 2)(x - 9)$
- 95.  $4x^2(5x^3 + 3x - 2) - 5x^3(x^2 - 6)$
- 96.  $3x^2(6x^3 + 2x - 3) - 4x^3(x^2 - 5)$
- 97.  $(y + 1)(y^2 - y + 1) + (y - 1)(y^2 + y + 1)$
- 98.  $(y + 1)(y^2 - y + 1) - (y - 1)(y^2 + y + 1)$
- 99.  $(y + 6)^2 - (y - 2)^2$
- 100.  $(y + 5)^2 - (y - 4)^2$

**Application Exercises**

- 101. Find a trinomial for the area of the rectangular rug shown below whose sides are  $x + 5$  feet and  $2x - 3$  feet.

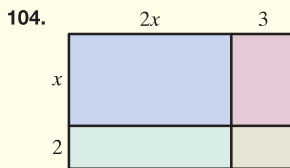
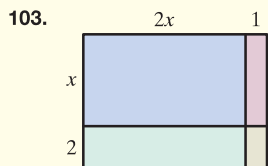


- 102. The base of a triangular sail is  $4x$  feet and its height is  $3x + 10$  feet. Write a binomial in terms of  $x$  for the area of the sail.



In Exercises 103–104,

- a. Express the area of the large rectangle as the product of two binomials.
- b. Find the sum of the areas of the four smaller rectangles.
- c. Use polynomial multiplication to show that your expressions for area in parts (a) and (b) are equal.



**Writing in Mathematics**

- 105. Explain the product rule for exponents. Use  $2^3 \cdot 2^5$  in your explanation.
- 106. Explain the power rule for exponents. Use  $(3^2)^4$  in your explanation.
- 107. Explain how to simplify an expression that involves a product raised to a power. Provide an example with your explanation.
- 108. Explain how to multiply monomials. Give an example.
- 109. Explain how to multiply a monomial and a polynomial that is not a monomial. Give an example.
- 110. Explain how to multiply polynomials when neither is a monomial. Give an example.
- 111. Explain the difference between performing these two operations:  
 $2x^2 + 3x^2$  and  $(2x^2)(3x^2)$ .
- 112. Discuss situations in which a vertical format, rather than a horizontal format, is useful for multiplying polynomials.

**Critical Thinking Exercises**

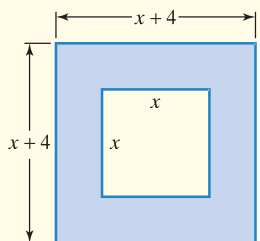
**Make Sense?** In Exercises 113–116, determine whether each statement “makes sense” or “does not make sense” and explain your reasoning.

- 113. I’m working with two monomials that I cannot add, although I can multiply them.
- 114. I’m working with two monomials that I can add, although I cannot multiply them.
- 115. Other than multiplying monomials, the distributive property is used to multiply other kinds of polynomials.
- 116. I used the product rule for exponents to multiply  $x^7$  and  $y^9$ .

In Exercises 117–120, determine whether each statement is true or false. If the statement is false, make the necessary change(s) to produce a true statement.

- 117.  $4x^3 \cdot 3x^4 = 12x^{12}$
- 118.  $5x^2 \cdot 4x^6 = 9x^8$
- 119.  $(y - 1)(y^2 + y + 1) = y^3 - 1$
- 120. Some polynomial multiplications can only be performed by using a vertical format.

121. Find a polynomial in descending powers of  $x$  representing the area of the shaded region.



122. Find each of the products in parts (a)–(c).
- $(x - 1)(x + 1)$
  - $(x - 1)(x^2 + x + 1)$
  - $(x - 1)(x^3 + x^2 + x + 1)$
- d. Using the pattern found in parts (a)–(c), find  $(x - 1)(x^4 + x^3 + x^2 + x + 1)$  without actually multiplying.
123. Find the missing factor.

$$\left(\quad\right)\left(-\frac{1}{4}xy^3\right) = 2x^5y^3$$

### Review Exercises

124. Solve:  $4x - 7 > 9x - 2$ . (Section 2.7, Example 7)
125. Graph  $3x - 2y = 6$  using intercepts. (Section 3.2, Example 4)
126. Find the slope of the line passing through the points  $(-2, 8)$  and  $(1, 6)$ . (Section 3.3, Example 1)

### Preview Exercises

Exercises 127–129 will help you prepare for the material covered in the next section. In each exercise, find the indicated products. Then, if possible, state a fast method for finding these products. (You may already be familiar with some of these methods from a high school algebra course.)

127. a.  $(x + 3)(x + 4)$   
b.  $(x + 5)(x + 20)$
128. a.  $(x + 3)(x - 3)$   
b.  $(x + 5)(x - 5)$
129. a.  $(x + 3)^2$   
b.  $(x + 5)^2$

## SECTION

# 5.3

## Special Products

### Objectives

- Use FOIL in polynomial multiplication.
  - Multiply the sum and difference of two terms.
  - Find the square of a binomial sum.
  - Find the square of a binomial difference.
- 1 Use FOIL in polynomial multiplication.



Let's cut to the chase. Are there fast methods for finding products of polynomials? The answer is beepingly "yes." (Or should that be (BEEP)<sup>2</sup> yes?) In this section, we'll cut to the chase by using the distributive property to develop patterns that will let you multiply certain binomials quite rapidly.

### The Product of Two Binomials: FOIL

Frequently, we need to find the product of two binomials. One way to perform this multiplication is to distribute each term in the first binomial through the second binomial. For example, we can find the product of the binomials  $3x + 2$  and  $4x + 5$  as follows:

$$\begin{aligned} (3x + 2)(4x + 5) &= 3x(4x + 5) + 2(4x + 5) \\ &= 3x(4x) + 3x(5) + 2(4x) + 2(5) \\ &= 12x^2 + 15x + 8x + 10. \end{aligned}$$

Distribute  $3x$  over  $4x + 5$ .

Distribute  $2$  over  $4x + 5$ .

We'll combine these like terms later. For now, our interest is in how to obtain each of these four terms.

We can also find the product of  $3x + 2$  and  $4x + 5$  using a method called FOIL, which is based on our work shown on the previous page. Any two binomials can be quickly multiplied by using the FOIL method, in which **F** represents the product of the **first** terms in each binomial, **O** represents the product of the **outside** terms, **I** represents the product of the **inside** terms, and **L** represents the product of the **last**, or second, terms in each binomial. For example, we can use the FOIL method to find the product of the binomials  $3x + 2$  and  $4x + 5$  as follows:

$$(3x + 2)(4x + 5) = 12x^2 + 15x + 8x + 10$$

Product of First terms
Product of Outside terms
Product of Inside terms
Product of Last terms

$$= 12x^2 + 23x + 10 \quad \text{Combine like terms.}$$

In general, here's how to use the FOIL method to find the product of  $ax + b$  and  $cx + d$ :

### Using the FOIL Method to Multiply Binomials

$$(ax + b)(cx + d) = ax \cdot cx + ax \cdot d + b \cdot cx + b \cdot d$$

Product of First terms
Product of Outside terms
Product of Inside terms
Product of Last terms

### EXAMPLE 1 Using the FOIL Method

Multiply:  $(x + 3)(x + 4)$ .

#### Solution

F: First terms  $= x \cdot x = x^2$   $(x + 3)(x + 4)$   
 O: Outside terms  $= x \cdot 4 = 4x$   $(x + 3)(x + 4)$   
 I: Inside terms  $= 3 \cdot x = 3x$   $(x + 3)(x + 4)$   
 L: Last terms  $= 3 \cdot 4 = 12$   $(x + 3)(x + 4)$

$$(x + 3)(x + 4) = x \cdot x + x \cdot 4 + 3 \cdot x + 3 \cdot 4$$

$$= x^2 + 4x + 3x + 12$$

$$= x^2 + 7x + 12 \quad \text{Combine like terms.} \quad \blacksquare$$

**CHECK POINT 1** Multiply:  $(x + 5)(x + 6)$ .

**EXAMPLE 2** Using the FOIL Method

Multiply:  $(3x + 4)(5x - 3)$ .

**Solution**

$$\begin{aligned}
 (3x + 4)(5x - 3) &= 3x \cdot 5x + 3x(-3) + 4 \cdot 5x + 4(-3) \\
 &= 15x^2 - 9x + 20x - 12 \\
 &= 15x^2 + 11x - 12
 \end{aligned}$$

Combine like terms. ■

✓ **CHECK POINT 2** Multiply:  $(7x + 5)(4x - 3)$ .

**EXAMPLE 3** Using the FOIL Method

Multiply:  $(2 - 5x)(3 - 4x)$ .

**Solution**

$$\begin{aligned}
 (2 - 5x)(3 - 4x) &= 2 \cdot 3 + 2(-4x) + (-5x)(3) + (-5x)(-4x) \\
 &= 6 - 8x - 15x + 20x^2 \\
 &= 6 - 23x + 20x^2
 \end{aligned}$$

Combine like terms.

The product can also be expressed in standard form as  $20x^2 - 23x + 6$ . ■

✓ **CHECK POINT 3** Multiply:  $(4 - 2x)(5 - 3x)$ .

**2** Multiply the sum and difference of two terms.

**Multiplying the Sum and Difference of Two Terms**

We can use the FOIL method to multiply  $A + B$  and  $A - B$  as follows:

$$(A + B)(A - B) = A^2 - AB + AB - B^2 = A^2 - B^2.$$

Notice that the outside and inside products have a sum of 0 and the terms cancel. The FOIL multiplication provides us with a quick rule for multiplying the sum and difference of two terms, referred to as a *special-product formula*.

**The Product of the Sum and Difference of Two Terms**

$$(A + B)(A - B) = A^2 - B^2$$

The product of the sum and the difference of the same two terms is the square of the first term minus the square of the second term.

**EXAMPLE 4** Finding the Product of the Sum and Difference of Two Terms

Multiply: **a.**  $(4y + 3)(4y - 3)$     **b.**  $(3x - 7)(3x + 7)$     **c.**  $(5a^4 + 6)(5a^4 - 6)$ .



**Solution** Use the special-product formula shown.

$$(A + B)(A - B) = A^2 - B^2$$

First term squared - Second term squared = Product

- a.  $(4y + 3)(4y - 3) = (4y)^2 - 3^2 = 16y^2 - 9$
- b.  $(3x - 7)(3x + 7) = (3x)^2 - 7^2 = 9x^2 - 49$
- c.  $(5a^4 + 6)(5a^4 - 6) = (5a^4)^2 - 6^2 = 25a^8 - 36$  ■

- ✓ **CHECK POINT 4** Multiply: a.  $(7y + 8)(7y - 8)$   
 b.  $(4x - 5)(4x + 5)$  c.  $(2a^3 + 3)(2a^3 - 3)$ .

**3** Find the square of a binomial sum.

### The Square of a Binomial

Let's now find  $(A + B)^2$ , the square of a binomial sum. To do so, we begin with the FOIL method and look for a general rule.

$$(A + B)^2 = (A + B)(A + B) = A \cdot A + A \cdot B + A \cdot B + B \cdot B = A^2 + 2AB + B^2$$

F   O   I   L

This result implies the following rule, which is another example of a special-product formula:

#### The Square of a Binomial Sum

$$(A + B)^2 = A^2 + 2AB + B^2$$

The square of a binomial sum is first term squared plus 2 times the product of the terms plus last term squared.

### Great Question!

When finding  $(x + 3)^2$ , why can't I just write  $x^2 + 3^2$ , or  $x^2 + 9$ ?

Caution! The square of a sum is *not* the sum of the squares.

~~$(A + B)^2 \neq A^2 + B^2$~~

The middle term  $2AB$  is missing.

~~$(x + 3)^2 \neq x^2 + 9$~~

Incorrect!

Show that  $(x + 3)^2$  and  $x^2 + 9$  are not equal by substituting 5 for  $x$  in each expression and simplifying.

#### EXAMPLE 5 Finding the Square of a Binomial Sum

Multiply:

- a.  $(x + 3)^2$       b.  $(3x + 7)^2$ .

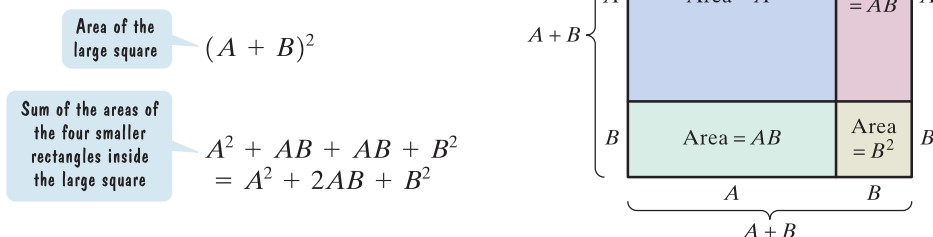
**Solution** Use the special-product formula shown.

$$(A + B)^2 = A^2 + 2AB + B^2$$

	(First Term) <sup>2</sup>	+	2 · Product of the Terms	+	(Last Term) <sup>2</sup>	= Product
a. $(x + 3)^2 =$	$x^2$	+	$2 \cdot x \cdot 3$	+	$3^2$	$= x^2 + 6x + 9$
b. $(3x + 7)^2 =$	$(3x)^2$	+	$2(3x)(7)$	+	$7^2$	$= 9x^2 + 42x + 49$

- ✓ **CHECK POINT 5** Multiply: a.  $(x + 10)^2$       b.  $(5x + 4)^2$ .

The formula for the square of a binomial sum can be interpreted geometrically by analyzing the areas in **Figure 5.7**.



**Figure 5.7**

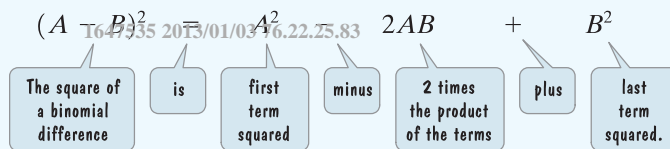
Conclusion:

$$(A + B)^2 = A^2 + 2AB + B^2$$

- 4** Find the square of a binomial difference.

A similar pattern occurs for  $(A - B)^2$ , the square of a binomial difference. Using the FOIL method on  $(A - B)^2$ , we obtain the following rule:

### The Square of a Binomial Difference



### EXAMPLE 6 Finding the Square of a Binomial Difference

Multiply:

- a.  $(x - 4)^2$       b.  $(5y - 6)^2$

#### Solution

Use the special-product formula shown.

$(A - B)^2 =$	$A^2$	$-$	$2AB$	$+$	$B^2$	
	(First Term) <sup>2</sup>	$-$	2 · Product of the Terms	$+$	(Last Term) <sup>2</sup>	= Product
a. $(x - 4)^2 =$	$x^2$	$-$	$2 \cdot x \cdot 4$	$+$	$4^2$	$= x^2 - 8x + 16$
b. $(5y - 6)^2 =$	$(5y)^2$	$-$	$2(5y)(6)$	$+$	$6^2$	$= 25y^2 - 60y + 36$

#### ✓ CHECK POINT 6 Multiply:

- a.  $(x - 9)^2$       b.  $(7x - 3)^2$

The table at the top of the next page summarizes the FOIL method and the three special products. The special products occur so frequently in algebra that it is convenient to memorize the form or pattern of these formulas.

**FOIL and Special Products**

Let  $A, B, C,$  and  $D$  be real numbers, variables, or algebraic expressions.

<p><b>FOIL</b></p> <div style="text-align: center;"> <span style="border: 1px solid black; border-radius: 50%; padding: 2px 5px;">F</span> <span style="border: 1px solid black; border-radius: 50%; padding: 2px 5px;">O</span> <span style="border: 1px solid black; border-radius: 50%; padding: 2px 5px;">I</span> <span style="border: 1px solid black; border-radius: 50%; padding: 2px 5px;">L</span> </div> $(A + B)(C + D) = AC + AD + BC + BD$	<p><b>Example</b></p> <div style="text-align: center;"> <span style="border: 1px solid black; border-radius: 50%; padding: 2px 5px;">F</span> <span style="border: 1px solid black; border-radius: 50%; padding: 2px 5px;">O</span> <span style="border: 1px solid black; border-radius: 50%; padding: 2px 5px;">I</span> <span style="border: 1px solid black; border-radius: 50%; padding: 2px 5px;">L</span> </div> $(2x + 3)(4x + 5) = (2x)(4x) + (2x)(5) + (3)(4x) + (3)(5)$ $= 8x^2 + 10x + 12x + 15$ $= 8x^2 + 22x + 15$
<p><b>Sum and Difference of Two Terms</b></p> $(A + B)(A - B) = A^2 - B^2$	<p><b>Example</b></p> $(2x + 3)(2x - 3) = (2x)^2 - 3^2$ $= 4x^2 - 9$
<p><b>Square of a Binomial</b></p> $(A + B)^2 = A^2 + 2AB + B^2$ $(A - B)^2 = A^2 - 2AB + B^2$	<p><b>Example</b></p> $(2x + 3)^2 = (2x)^2 + 2(2x)(3) + 3^2$ $= 4x^2 + 12x + 9$ $(2x - 3)^2 = (2x)^2 - 2(2x)(3) + 3^2$ $= 4x^2 - 12x + 9$

**Achieving Success**

**Manage your time.** Use a day planner such as the one shown below. (Go online and search “day planner” or “day scheduler” to find a schedule grid that you can print and use.)

**Sample Day Planner**

	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday	Sunday
5:00 A.M.							
6:00 A.M.							
7:00 A.M.							
8:00 A.M.							
9:00 A.M.							
10:00 A.M.							
11:00 A.M.							
12:00 P.M.							
1:00 P.M.							
2:00 P.M.							
3:00 P.M.							
4:00 P.M.							
5:00 P.M.							
6:00 P.M.							
7:00 P.M.							
8:00 P.M.							
9:00 P.M.							
10:00 P.M.							
11:00 P.M.							
Midnight							

- On the Sunday before the week begins, fill in the time slots with fixed items such as school, extracurricular activities, work, etc.
- Because your education should be a top priority, decide what times you would like to study and do homework. Fill in these activities on the day planner.
- Plan other flexible activities such as exercise, socializing, etc. around the times already established.
- Be flexible. Things may come up that are unavoidable or some items may take longer than planned. However, be honest with yourself: Playing video games with friends is not “unavoidable.”
- Stick to your schedule. At the end of the week, you will look back and be impressed at all the things you have accomplished.

**CONCEPT AND VOCABULARY CHECK**

Fill in each blank so that the resulting statement is true.

- For  $(x + 5)(2x + 3)$ , the product of the first terms is \_\_\_\_\_, the product of the outside terms is \_\_\_\_\_, the product of the inside terms is \_\_\_\_\_, and the product of the last terms is \_\_\_\_\_.
- $(A + B)(A - B) =$  \_\_\_\_\_. The product of the sum and the difference of the same two terms is the square of the first term \_\_\_\_\_ the square of the second term.
- $(A + B)^2 =$  \_\_\_\_\_. The square of a binomial sum is the first term \_\_\_\_\_ plus 2 times the \_\_\_\_\_ plus the last term \_\_\_\_\_.
- $(A - B)^2 =$  \_\_\_\_\_. The square of a binomial difference is the first term squared \_\_\_\_\_ 2 times the \_\_\_\_\_ plus the last term \_\_\_\_\_.
- True or false:  $(x + 5)(x - 5) = x^2 - 25$ . \_\_\_\_\_
- True or false:  $(x + 5)^2 = x^2 + 25$ . \_\_\_\_\_

5.3 EXERCISE SET

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**Practice Exercises**

In Exercises 1–24, use the FOIL method to find each product. Express the product in descending powers of the variable.

- $(x + 4)(x + 6)$
- $(x + 8)(x + 2)$
- $(y - 7)(y + 3)$
- $(y - 3)(y + 4)$
- $(2x - 3)(x + 5)$
- $(3x - 5)(x + 7)$
- $(4y + 3)(y - 1)$
- $(5y + 4)(y - 2)$
- $(2x - 3)(5x + 3)$
- $(2x - 5)(7x + 2)$
- $(3y - 7)(4y - 5)$
- $(4y - 5)(7y - 4)$
- $(7 + 3x)(1 - 5x)$
- $(2 + 5x)(1 - 4x)$
- $(5 - 3y)(6 - 2y)$
- $(7 - 2y)(10 - 3y)$
- $(5x^2 - 4)(3x^2 - 7)$
- $(7x^2 - 2)(3x^2 - 5)$
- $(6x - 5)(2 - x)$
- $(4x - 3)(2 - x)$
- $(x + 5)(x^2 + 3)$
- $(x + 4)(x^2 + 5)$
- $(8x^3 + 3)(x^2 + 5)$
- $(7x^3 + 5)(x^2 + 2)$

In Exercises 25–44, multiply using the rule for finding the product of the sum and difference of two terms.

- $(x + 3)(x - 3)$
- $(y + 5)(y - 5)$
- $(3x + 2)(3x - 2)$
- $(2x + 5)(2x - 5)$
- $(3r - 4)(3r + 4)$
- $(5z - 2)(5z + 2)$
- $(3 + r)(3 - r)$
- $(4 + s)(4 - s)$
- $(5 - 7x)(5 + 7x)$
- $(4 - 3y)(4 + 3y)$
- $\left(2x + \frac{1}{2}\right)\left(2x - \frac{1}{2}\right)$
- $\left(3y + \frac{1}{3}\right)\left(3y - \frac{1}{3}\right)$
- $(y^2 + 1)(y^2 - 1)$
- $(y^2 + 2)(y^2 - 2)$
- $(r^3 + 2)(r^3 - 2)$
- $(m^3 + 4)(m^3 - 4)$
- $(1 - y^4)(1 + y^4)$
- $(2 - s^5)(2 + s^5)$
- $(x^{10} + 5)(x^{10} - 5)$
- $(x^{12} + 3)(x^{12} - 3)$

In Exercises 45–62, multiply using the rules for the square of a binomial.

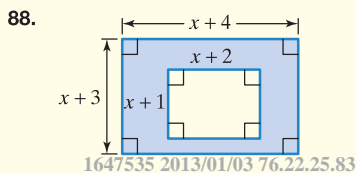
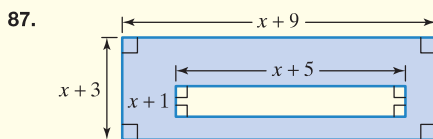
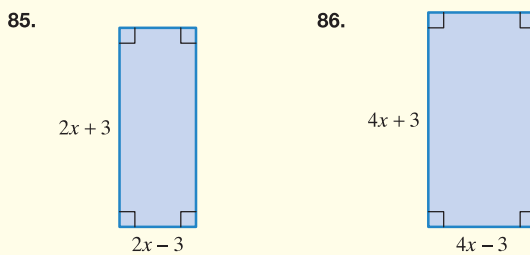
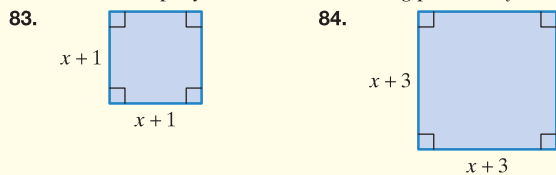
- |                  |                  |
|------------------|------------------|
| 45. $(x + 2)^2$  | 46. $(x + 5)^2$  |
| 47. $(2x + 5)^2$ | 48. $(5x + 2)^2$ |

49.  $(x - 3)^2$                       50.  $(x - 6)^2$   
 51.  $(3y - 4)^2$   
 52.  $(4y - 3)^2$   
 53.  $(4x^2 - 1)^2$   
 54.  $(5x^2 - 3)^2$   
 55.  $(7 - 2x)^2$   
 56.  $(9 - 5x)^2$   
 57.  $\left(2x + \frac{1}{2}\right)^2$   
 58.  $\left(3x + \frac{1}{3}\right)^2$   
 59.  $\left(4y - \frac{1}{4}\right)^2$   
 60.  $\left(2y - \frac{1}{2}\right)^2$   
 61.  $(x^8 + 3)^2$   
 62.  $(x^8 + 5)^2$

In Exercises 63–82, multiply using the method of your choice.

63.  $(x - 1)(x^2 + x + 1)$   
 64.  $(x + 1)(x^2 - x + 1)$   
 65.  $(x - 1)^2$   
 66.  $(x + 1)^2$   
 67.  $(3y + 7)(3y - 7)$   
 68.  $(4y + 9)(4y - 9)$   
 69.  $3x^2(4x^2 + x + 9)$   
 70.  $5x^2(7x^2 + x + 6)$   
 71.  $(7y + 3)(10y - 4)$   
 72.  $(8y + 3)(10y - 5)$   
 73.  $(x^2 + 1)^2$   
 74.  $(x^2 + 2)^2$   
 75.  $(x^2 + 1)(x^2 + 2)$   
 76.  $(x^2 + 2)(x^2 + 3)$   
 77.  $(x^2 + 4)(x^2 - 4)$   
 78.  $(x^2 + 5)(x^2 - 5)$   
 79.  $(2 - 3x^5)^2$                       80.  $(2 - 3x^6)^2$   
 81.  $\left(\frac{1}{4}x^2 + 12\right)\left(\frac{3}{4}x^2 - 8\right)$   
 82.  $\left(\frac{1}{4}x^2 + 16\right)\left(\frac{3}{4}x^2 - 4\right)$

In Exercises 83–88, find the area of each shaded region. Write the answer as a polynomial in descending powers of  $x$ .



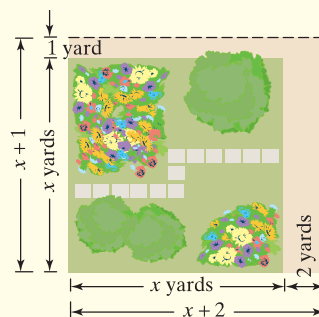
### Practice PLUS

In Exercises 89–96, multiply by the method of your choice.

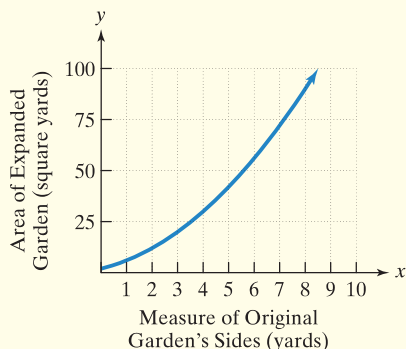
89.  $[(2x + 3)(2x - 3)]^2$   
 90.  $[(3x + 2)(3x - 2)]^2$   
 91.  $(4x^2 + 1)[(2x + 1)(2x - 1)]$   
 92.  $(9x^2 + 1)[(3x + 1)(3x - 1)]$   
 93.  $(x + 2)^3$   
 94.  $(x + 4)^3$   
 95.  $[(x + 3) - y][(x + 3) + y]$   
 96.  $[(x + 5) - y][(x + 5) + y]$

### Application Exercises

The square garden shown in the figure measures  $x$  yards on each side. The garden is to be expanded so that one side is increased by 2 yards and an adjacent side is increased by 1 yard.

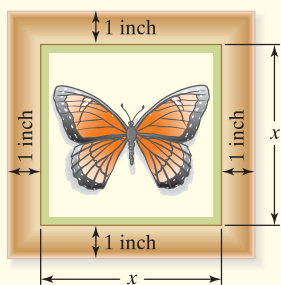


The graph shows the area of the expanded garden,  $y$ , in terms of the length of one of its original sides,  $x$ . Use this information to solve Exercises 97–100.



97. Use the gardens pictured at the bottom of the previous page to write a product of two binomials that expresses the area of the larger garden.
98. Use the gardens pictured at the bottom of the previous page to write a polynomial in descending powers of  $x$  that expresses the area of the larger garden.
99. If the original garden measures 6 yards on a side, use your expression from Exercise 97 to find the area of the larger garden. Then identify your solution as a point on the graph shown above.
100. If the original garden measures 8 yards on a side, use your polynomial from Exercise 98 to find the area of the larger garden. Then identify your solution as a point on the graph shown above.

The square painting in the figure measures  $x$  inches on each side. The painting is uniformly surrounded by a frame that measures 1 inch wide. Use this information to solve Exercises 101–102.



101. Write a polynomial in descending powers of  $x$  that expresses the area of the square that includes the painting and the frame.
102. Write an algebraic expression that describes the area of the frame. (*Hint:* The area of the frame is the area of the square that includes the painting and the frame minus the area of the painting.)

### Writing in Mathematics

103. Explain how to multiply two binomials using the FOIL method. Give an example with your explanation.

104. Explain how to find the product of the sum and difference of two terms. Give an example with your explanation.
105. Explain how to square a binomial sum. Give an example with your explanation.
106. Explain how to square a binomial difference. Give an example with your explanation.
107. Explain why the graph for Exercises 97–100 is shown only in quadrant I.

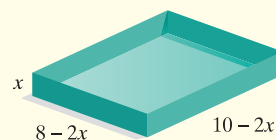
### Critical Thinking Exercises

**Make Sense?** In Exercises 108–111, determine whether each statement “makes sense” or “does not make sense” and explain your reasoning.

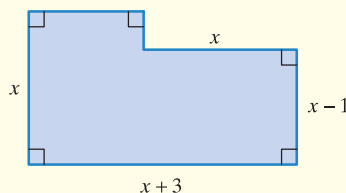
108. Squaring a binomial sum is as simple as squaring each of the two terms and then writing their sum.
109. I can distribute the exponent 2 on each factor of  $(5x)^2$ , but I cannot do the same thing on each term of  $(x + 5)^2$ .
110. Instead of using the formula for the square of a binomial sum, I prefer to write the binomial sum twice and then apply the FOIL method.
111. Special-product formulas for  $(A + B)(A - B)$ ,  $(A + B)^2$ , and  $(A - B)^2$  have patterns that make their multiplications quicker than using the FOIL method.

In Exercises 112–115, determine whether each statement is true or false. If the statement is false, make the necessary change(s) to produce a true statement.

112.  $(3 + 4)^2 = 3^2 + 4^2$
113.  $(2y + 7)^2 = 4y^2 + 28y + 49$
114.  $(3x^2 + 2)(3x^2 - 2) = 9x^2 - 4$
115.  $(x - 5)^2 = x^2 - 5x + 25$
116. What two binomials must be multiplied using the FOIL method to give a product of  $x^2 - 8x - 20$ ?
117. Express the volume of the box as a polynomial in standard form.



118. Express the area of the plane figure shown as a polynomial in standard form.





### Technology Exercises

In Exercises 119–122, use a graphing utility to graph each side of the equation in the same viewing rectangle. (Call the left side  $y_1$  and the right side  $y_2$ .) If the graphs coincide, verify that the multiplication has been performed correctly. If the graphs do not appear to coincide, this indicates that the multiplication is incorrect. In these exercises, correct the right side of the equation. Then graph the left side and the corrected right side to verify that the graphs coincide.

119.  $(x + 1)^2 = x^2 + 1$ ; Use a  $[-5, 5, 1]$  by  $[0, 20, 1]$  viewing rectangle.
120.  $(x + 2)^2 = x^2 + 2x + 4$ ; Use a  $[-6, 5, 1]$  by  $[0, 20, 1]$  viewing rectangle.
121.  $(x + 1)(x - 1) = x^2 - 1$ ; Use a  $[-6, 5, 1]$  by  $[-2, 18, 1]$  viewing rectangle.
122.  $(x - 2)(x + 2) + 4 = x^2$ ; Use a  $[-6, 5, 1]$  by  $[-2, 18, 1]$  viewing rectangle.

### Review Exercises

In Exercises 123–124, solve each system by the method of your choice.

123. 
$$\begin{cases} 2x + 3y = 1 \\ y = 3x - 7 \end{cases}$$
  
(Section 4.2, Example 1)

124. 
$$\begin{cases} 3x + 4y = 7 \\ 2x + 7y = 9 \end{cases}$$
  
(Section 4.3, Example 3)

125. Graph:  $y \leq \frac{1}{3}x$ .  
(Section 3.6, Example 3)

### Preview Exercises

Exercises 126–128 will help you prepare for the material covered in the next section.

126. Use the order of operations to evaluate  
$$x^3y + 2xy^2 + 5x - 2$$
  
for  $x = -2$  and  $y = 3$ .
127. Use the second step to combine the like terms.  
$$5xy + 6xy = (5 + 6)xy = ?$$
128. Multiply using FOIL:  $(x + 2y)(3x + 5y)$ .

## SECTION

# 5.4

## Polynomials in Several Variables

### Objectives

- 1 Evaluate polynomials in several variables.
- 2 Understand the vocabulary of polynomials in two variables.
- 3 Add and subtract polynomials in several variables.
- 4 Multiply polynomials in several variables.



The next time you visit a lumberyard and go rummaging through piles of wood, think *polynomials*, although polynomials a bit different from those we have encountered so far. The construction industry uses a polynomial in two variables to determine the number of board feet that can be manufactured from a tree with a diameter of  $x$  inches and a length of  $y$  feet. This polynomial is

$$\frac{1}{4}x^2y - 2xy + 4y.$$

We call a polynomial containing two or more variables a **polynomial in several variables**. These polynomials can be evaluated, added, subtracted, and multiplied just like polynomials that contain only one variable.

- 1 Evaluate polynomials in several variables.

### Evaluating a Polynomial in Several Variables

Two steps can be used to evaluate a polynomial in several variables.

#### Evaluating a Polynomial in Several Variables

1. Substitute the given value for each variable.
2. Perform the resulting computation using the order of operations.

#### EXAMPLE 1 Evaluating a Polynomial in Two Variables

Evaluate  $2x^3y + xy^2 + 7x - 3$  for  $x = -2$  and  $y = 3$ .

**Solution** We begin by substituting  $-2$  for  $x$  and  $3$  for  $y$  in the polynomial.

$$\begin{aligned} & 2x^3y + xy^2 + 7x - 3 && \text{This is the given polynomial.} \\ & = 2(-2)^3 \cdot 3 + (-2) \cdot 3^2 + 7(-2) - 3 && \text{Replace } x \text{ with } -2 \text{ and } y \text{ with } 3. \\ & = 2(-8) \cdot 3 + (-2) \cdot 9 + 7(-2) - 3 && \text{Evaluate exponential expressions:} \\ & && (-2)^3 = (-2)(-2)(-2) = -8 \text{ and} \\ & && 3^2 = 3 \cdot 3 = 9. \\ & = -48 + (-18) + (-14) - 3 && \text{Perform the indicated multiplications.} \\ & = -83 && \text{Add from left to right. } \blacksquare \end{aligned}$$

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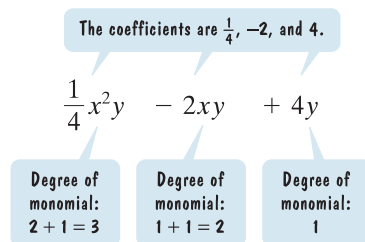
**CHECK POINT 1** Evaluate  $3x^3y + xy^2 + 5y + 6$  for  $x = -1$  and  $y = 5$ .

- 2 Understand the vocabulary of polynomials in two variables.

### Describing Polynomials in Two Variables

In this section, we will limit our discussion of polynomials in several variables to two variables.

In general, a **polynomial in two variables**,  $x$  and  $y$ , contains the sum of one or more monomials in the form  $ax^ny^m$ . The constant,  $a$ , is the **coefficient**. The exponents,  $n$  and  $m$ , represent whole numbers. The **degree** of the monomial  $ax^ny^m$  is  $n + m$ . We'll use the polynomial from the construction industry to illustrate these ideas.



The **degree of a polynomial in two variables** is the highest degree of all its terms. For the preceding polynomial, the degree is 3.

**EXAMPLE 2** Using the Vocabulary of Polynomials

Determine the coefficient of each term, the degree of each term, and the degree of the polynomial:

$$7x^2y^3 - 17x^4y^2 + xy - 6y^2 + 9.$$

**Solution**

Term	Coefficient	Degree (Sum of Exponents on the Variables)
$7x^2y^3$	7	$2 + 3 = 5$
$-17x^4y^2$	-17	$4 + 2 = 6$
$xy$	1	$1 + 1 = 2$
$-6y^2$	-6	2
9	9	0

Think of  $xy$  as  $1x^1y^1$ .

The degree of the polynomial is the highest degree of all its terms, which is 6. ■

**✓ CHECK POINT 2** Determine the coefficient of each term, the degree of each term, and the degree of the polynomial:

$$8x^4y^5 - 7x^3y^2 - x^2y - 5x + 11.$$

**3** Add and subtract polynomials in several variables.

**Adding and Subtracting Polynomials in Several Variables**

Polynomials in several variables are added by combining like terms. For example, we can add the monomials  $-7xy^2$  and  $13xy^2$  as follows:

$$-7xy^2 + 13xy^2 = (-7 + 13)xy^2 = 6xy^2.$$

These like terms both contain the variable factors  $x$  and  $y^2$ .

Add coefficients and keep the same variable factors,  $xy^2$ .

**EXAMPLE 3** Adding Polynomials in Two Variables

Add:  $(6xy^2 - 5xy + 7) + (9xy^2 + 2xy - 6)$ .

**Solution**

$$\begin{aligned} & (6xy^2 - 5xy + 7) + (9xy^2 + 2xy - 6) \\ &= (6xy^2 + 9xy^2) + (-5xy + 2xy) + (7 - 6) && \text{Group like terms.} \\ &= 15xy^2 - 3xy + 1 && \text{Combine like terms by adding coefficients and keeping the same variable factors.} \quad \blacksquare \end{aligned}$$

**✓ CHECK POINT 3** Add:  $(-8x^2y - 3xy + 6) + (10x^2y + 5xy - 10)$ .

We subtract polynomials in two variables just as we did when subtracting polynomials in one variable. Add the first polynomial and the opposite of the polynomial being subtracted.

**EXAMPLE 4** Subtracting Polynomials in Two Variables

Subtract:

$$(5x^3 - 9x^2y + 3xy^2 - 4) - (3x^3 - 6x^2y - 2xy^2 + 3).$$

**Solution**

$$\begin{aligned}
 & (5x^3 - 9x^2y + 3xy^2 - 4) - (3x^3 - 6x^2y - 2xy^2 + 3) \\
 &= (5x^3 - 9x^2y + 3xy^2 - 4) + (-3x^3 + 6x^2y + 2xy^2 - 3) \\
 &= (5x^3 - 3x^3) + (-9x^2y + 6x^2y) + (3xy^2 + 2xy^2) + (-4 - 3) \\
 &= 2x^3 - 3x^2y + 5xy^2 - 7
 \end{aligned}$$

Change the sign of each coefficient.

Add the opposite of the polynomial being subtracted.

Group like terms.

Combine like terms by adding coefficients and keeping the same variable factors. ■

✓ **CHECK POINT 4** Subtract:

$$(7x^3 - 10x^2y + 2xy^2 - 5) - (4x^3 - 12x^2y - 3xy^2 + 5).$$

**4** Multiply polynomials in several variables.

**Multiplying Polynomials in Several Variables**

The product of monomials forms the basis of polynomial multiplication. As with monomials in one variable, multiplication can be done mentally by multiplying coefficients and adding exponents on variables with the same base.

**EXAMPLE 5** Multiplying Monomials

Multiply:  $(7x^2y)(5x^3y^2)$ .

**Solution**

$$\begin{aligned}
 & (7x^2y)(5x^3y^2) \\
 &= (7 \cdot 5)(x^2 \cdot x^3)(y \cdot y^2) \\
 &= 35x^{2+3}y^{1+2} \\
 &= 35x^5y^3
 \end{aligned}$$

This regrouping can be worked mentally. Multiply coefficients and add exponents on variables with the same base. Simplify. ■

✓ **CHECK POINT 5** Multiply:  $(6xy^3)(10x^4y^2)$ .

How do we multiply a monomial and a polynomial that is not a monomial? As we did with polynomials in one variable, multiply each term of the polynomial by the monomial.

**EXAMPLE 6** Multiplying a Monomial and a Polynomial

Multiply:  $3x^2y(4x^3y^2 - 6x^2y + 2)$ .

**Solution**

$$\begin{aligned}
 & 3x^2y(4x^3y^2 - 6x^2y + 2) \\
 &= 3x^2y \cdot 4x^3y^2 - 3x^2y \cdot 6x^2y + 3x^2y \cdot 2 \\
 &= 12x^{2+3}y^{1+2} - 18x^{2+2}y^{1+1} + 6x^2y \\
 &= 12x^5y^3 - 18x^4y^2 + 6x^2y
 \end{aligned}$$

Use the distributive property. Multiply coefficients and add exponents on variables with the same base. Simplify. ■

✓ **CHECK POINT 6** Multiply:  $6xy^2(10x^4y^5 - 2x^2y + 3)$ .

FOIL and the special-products formulas can be used to multiply polynomials in several variables.

**EXAMPLE 7** Multiplying Polynomials in Two Variables

Multiply: **a.**  $(x + 4y)(3x - 5y)$  **b.**  $(5x + 3y)^2$ .

**Solution** We will perform the multiplication in part (a) using the FOIL method. We will multiply in part (b) using the formula for the square of a binomial,  $(A + B)^2$ .

**a.**  $(x + 4y)(3x - 5y)$

$$\begin{aligned}
 & \begin{array}{cccc} \text{F} & & \text{O} & & \text{I} & & \text{L} \end{array} \\
 & = (x)(3x) + (x)(-5y) + (4y)(3x) + (4y)(-5y) \\
 & = 3x^2 - 5xy + 12xy - 20y^2 \\
 & = 3x^2 + 7xy - 20y^2
 \end{aligned}$$

Multiply these binomials using the FOIL method.

Combine like terms.

$$(A + B)^2 = A^2 + 2 \cdot A \cdot B + B^2$$

**b.**  $(5x + 3y)^2 = (5x)^2 + 2(5x)(3y) + (3y)^2$   
 $= 25x^2 + 30xy + 9y^2$  ■

✓ **CHECK POINT 7** Multiply:  
**a.**  $(7x - 6y)(3x - y)$  **b.**  $(2x + 4y)^2$ .

**EXAMPLE 8** Multiplying Polynomials in Two Variables

Multiply: **a.**  $(4x^2y + 3y)(4x^2y - 3y)$  **b.**  $(x + y)(x^2 - xy + y^2)$ .

**Solution** We perform the multiplication in part (a) using the formula for the product of the sum and difference of two terms. We perform the multiplication in part (b) by multiplying each term of the trinomial,  $x^2 - xy + y^2$ , by  $x$  and  $y$ , respectively, and then adding like terms.

$$(A + B) \cdot (A - B) = A^2 - B^2$$

**a.**  $(4x^2y + 3y)(4x^2y - 3y) = (4x^2y)^2 - (3y)^2$   
 $= 16x^4y^2 - 9y^2$

**b.**  $(x + y)(x^2 - xy + y^2)$   
 $= x(x^2 - xy + y^2) + y(x^2 - xy + y^2)$   
 $= x \cdot x^2 - x \cdot xy + x \cdot y^2 + y \cdot x^2 - y \cdot xy + y \cdot y^2$   
 $= x^3 - x^2y + xy^2 + x^2y - xy^2 + y^3$   
 $= x^3 + y^3$

Multiply the trinomial by each term of the binomial.

Use the distributive property.

Add exponents on variables with the same base.

Combine like terms:  
 $-x^2y + x^2y = 0$  and  
 $xy^2 - xy^2 = 0$ . ■

✓ **CHECK POINT 8** Multiply:  
**a.**  $(6xy^2 + 5x)(6xy^2 - 5x)$  **b.**  $(x - y)(x^2 + xy + y^2)$ .

**CONCEPT AND VOCABULARY CHECK**

Fill in each blank so that the resulting statement is true.

- The coefficient of the monomial  $-18x^4y^2$  is \_\_\_\_\_.
- The degree of the monomial  $-18x^4y^2$  is \_\_\_\_\_.
- The coefficient of the monomial  $ax^ny^m$  is \_\_\_\_\_ and the degree is \_\_\_\_\_.
- The degree of  $x^3y^2$  is \_\_\_\_\_ and the degree of  $x^2y^7$  is \_\_\_\_\_, so the degree of  $x^3y^2 - 8x^2y^7$  is \_\_\_\_\_.
- True or false: The monomials  $7xy^2$  and  $2x^2y$  can be added. \_\_\_\_\_
- True or false: The monomials  $7xy^2$  and  $2x^2y$  can be multiplied. \_\_\_\_\_

**5.4 EXERCISE SET**

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**Practice Exercises**

In Exercises 1–6, evaluate each polynomial for  $x = 2$  and  $y = -3$ .

- |                      |                      |
|----------------------|----------------------|
| 1. $x^2 + 2xy + y^2$ | 2. $x^2 + 3xy + y^2$ |
| 3. $xy^3 - xy + 1$   | 4. $x^3y - xy + 2$   |
| 5. $2x^2y - 5y + 3$  | 6. $3x^2y - 4y + 5$  |

In Exercises 7–8, determine the coefficient of each term, the degree of each term, and the degree of the polynomial.

- $x^3y^2 - 5x^2y^7 + 6y^2 - 3$
- $12x^4y - 5x^3y^7 - x^2 + 4$

In Exercises 9–20, add or subtract as indicated.

- $(5x^2y - 3xy) + (2x^2y - xy)$
- $(-2x^2y + xy) + (4x^2y + 7xy)$
- $(4x^2y + 8xy + 11) + (-2x^2y + 5xy + 2)$
- $(7x^2y + 5xy + 13) + (-3x^2y + 6xy + 4)$
- $(7x^4y^2 - 5x^2y^2 + 3xy) + (-18x^4y^2 - 6x^2y^2 - xy)$
- $(6x^4y^2 - 10x^2y^2 + 7xy) + (-12x^4y^2 - 3x^2y^2 - xy)$
- $(x^3 + 7xy - 5y^2) - (6x^3 - xy + 4y^2)$
- $(x^4 - 7xy - 5y^3) - (6x^4 - 3xy + 4y^3)$
- $(3x^4y^2 + 5x^3y - 3y) - (2x^4y^2 - 3x^3y - 4y + 6x)$
- $(5x^4y^2 + 6x^3y - 7y) - (3x^4y^2 - 5x^3y - 6y + 8x)$
- $(x^3 - y^3) - (-4x^3 - x^2y + xy^2 + 3y^3)$
- $(x^3 - y^3) - (-6x^3 + x^2y - xy^2 + 2y^3)$
- Add: 
$$\begin{array}{r} 5x^2y^2 - 4xy^2 + 6y^2 \\ -8x^2y^2 + 5xy^2 - y^2 \end{array}$$
- Add: 
$$\begin{array}{r} 7a^2b^2 - 5ab^2 + 6b^2 \\ -10a^2b^2 + 6ab^2 + 6b^2 \end{array}$$

23. Subtract: 
$$\begin{array}{r} 3a^2b^4 - 5ab^2 + 7ab \\ -(-5a^2b^4 - 8ab^2 - ab) \end{array}$$

24. Subtract: 
$$\begin{array}{r} 13x^2y^4 - 17xy^2 + xy \\ -(-7x^2y^4 - 8xy^2 - xy) \end{array}$$

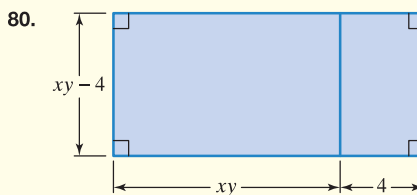
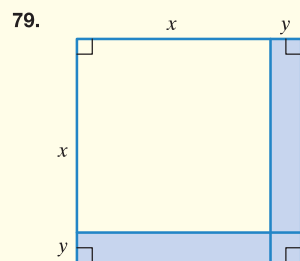
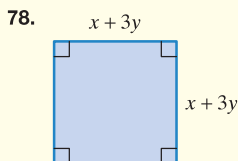
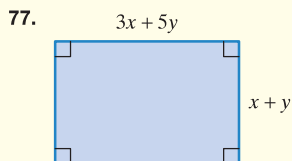
- Subtract  $11x - 5y$  from the sum of  $7x + 13y$  and  $-26x + 19y$ .
- Subtract  $23x - 5y$  from the sum of  $6x + 15y$  and  $x - 19y$ .

In Exercises 27–76, find each product.

- $(5x^2y)(8xy)$
- $(10x^2y)(5xy)$
- $(-8x^3y^4)(3x^2y^5)$
- $(7x^4y^5)(-10x^7y^{11})$
- $9xy(5x + 2y)$
- $7xy(8x + 3y)$
- $5xy^2(10x^2 - 3y)$
- $6x^2y(5x^2 - 9y)$
- $4ab^2(7a^2b^3 + 2ab)$
- $2ab^2(20a^2b^3 + 11ab)$
- $-b(a^2 - ab + b^2)$
- $-b(a^3 - ab + b^3)$
- $(x + 5y)(7x + 3y)$
- $(x + 9y)(6x + 7y)$
- $(x - 3y)(2x + 7y)$
- $(3x - y)(2x + 5y)$
- $(3xy - 1)(5xy + 2)$
- $(7xy + 1)(2xy - 3)$
- $(2x + 3y)^2$
- $(2x + 5y)^2$
- $(xy - 3)^2$

48.  $(xy - 5)^2$
49.  $(x^2 + y^2)^2$
50.  $(2x^2 + y^2)^2$
51.  $(x^2 - 2y^2)^2$
52.  $(x^2 - y^2)^2$
53.  $(3x + y)(3x - y)$
54.  $(x + 5y)(x - 5y)$
55.  $(ab + 1)(ab - 1)$
56.  $(ab + 2)(ab - 2)$
57.  $(x + y^2)(x - y^2)$
58.  $(x^2 + y)(x^2 - y)$
59.  $(3a^2b + a)(3a^2b - a)$
60.  $(5a^2b + a)(5a^2b - a)$
61.  $(3xy^2 - 4y)(3xy^2 + 4y)$
62.  $(7xy^2 - 10y)(7xy^2 + 10y)$
63.  $(a + b)(a^2 - b^2)$
64.  $(a - b)(a^2 + b^2)$
65.  $(x + y)(x^2 + 3xy + y^2)$
66.  $(x + y)(x^2 + 5xy + y^2)$
67.  $(x - y)(x^2 - 3xy + y^2)$
68.  $(x - y)(x^2 - 4xy + y^2)$
69.  $(xy + ab)(xy - ab)$
70.  $(xy + ab^2)(xy - ab^2)$
71.  $(x^2 + 1)(x^4y + x^2 + 1)$
72.  $(x^2 + 1)(xy^4 + y^2 + 1)$
73.  $(x^2y^2 - 3)^2$
74.  $(x^2y^2 - 5)^2$
75.  $(x + y + 1)(x + y - 1)$
76.  $(x + y + 1)(x - y + 1)$

In Exercises 77–80, write a polynomial in two variables that describes the total area of each region shaded in blue. Express each polynomial as the sum or difference of terms.



### Practice PLUS

In Exercises 81–86, find each product. As we said in the Section 5.3 opener, cut to the chase in each part of the polynomial multiplication: Use only the special-product formula for the sum and difference of two terms or the formulas for the square of a binomial.

81.  $[(x^3y^3 + 1)(x^3y^3 - 1)]^2$
82.  $[(1 - a^3b^3)(1 + a^3b^3)]^2$
83.  $(xy - 3)^2(xy + 3)^2$  (Do not begin by squaring a binomial.)
84.  $(ab - 4)^2(ab + 4)^2$  (Do not begin by squaring a binomial.)
85.  $[x + y + z][x - (y + z)]$
86.  $(a - b - c)(a + b + c)$

### Application Exercises

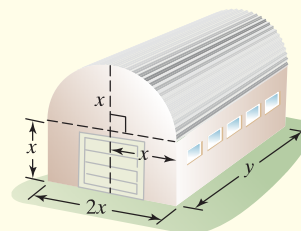
87. The number of board feet,  $N$ , that can be manufactured from a tree with a diameter of  $x$  inches and a length of  $y$  feet is modeled by the formula

$$N = \frac{1}{4}x^2y - 2xy + 4y.$$

A building contractor estimates that 3000 board feet of lumber is needed for a job. The lumber company has just milled a fresh load of timber from 20 trees that averaged 10 inches in diameter and 16 feet in length. Is this enough to complete the job? If not, how many additional board feet of lumber is needed?

88. The storage shed shown in the figure has a volume given by the polynomial

$$2x^2y + \frac{1}{2}\pi x^2y.$$



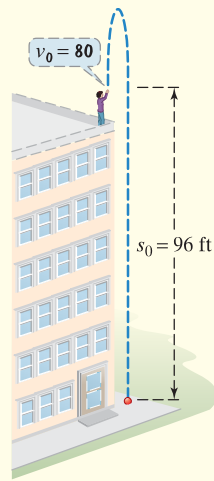
- a. A small business is considering having a shed installed like the one shown in the figure. The shed's height,  $2x$ , is 26 feet and its length,  $y$ , is 27 feet. Using  $x = 13$  and  $y = 27$ , find the volume of the storage shed.
- b. The business requires at least 18,000 cubic feet of storage space. Should they construct the storage shed described in part (a)?



An object that is falling or vertically projected into the air has its height, in feet, above the ground given by

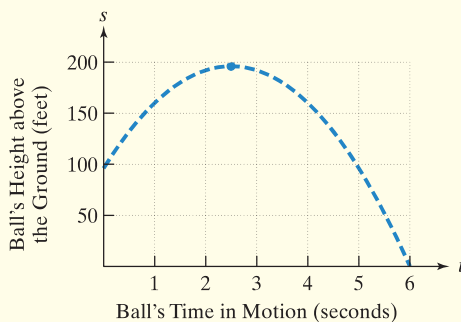
$$s = -16t^2 + v_0t + s_0,$$

where  $s$  is the height, in feet,  $v_0$  is the original velocity of the object, in feet per second,  $t$  is the time the object is in motion, in seconds, and  $s_0$  is the height, in feet, from which the object is dropped or projected. The figure shows that a ball is thrown straight up from a rooftop at an original velocity of 80 feet per second from a height of 96 feet. The ball misses the rooftop on its way down and eventually strikes the ground. Use the formula and this information to solve Exercises 89–91.



89. How high above the ground will the ball be 2 seconds after being thrown?
90. How high above the ground will the ball be 4 seconds after being thrown?
91. How high above the ground will the ball be 6 seconds after being thrown? Describe what this means in practical terms.

The graph visually displays the information about the thrown ball described in Exercises 89–91. The horizontal axis represents the ball's time in motion, in seconds. The vertical axis represents the ball's height above the ground, in feet. Use the graph to solve Exercises 92–97.



92. During which time period is the ball rising?
93. During which time period is the ball falling?
94. Identify your answer from Exercise 90 as a point on the graph.
95. Identify your answer from Exercise 89 as a point on the graph.
96. After how many seconds does the ball strike the ground?
97. After how many seconds does the ball reach its maximum height above the ground? What is a reasonable estimate of this maximum height?

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### Writing in Mathematics

98. What is a polynomial in two variables? Provide an example with your description.
99. Explain how to find the degree of a polynomial in two variables.
100. Suppose that you take up sky diving. Explain how to use the formula for Exercises 89–91 to determine your height above the ground at every instant of your fall.

### Critical Thinking Exercises

**Make Sense?** In Exercises 101–104, determine whether each statement “makes sense” or “does not make sense” and explain your reasoning.

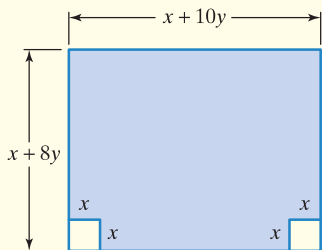
101. I use the same procedures for operations with polynomials in two variables as I did when performing these operations with polynomials in one variable.
102. Adding polynomials in several variables is the same as adding like terms.
103. I used FOIL to find the product of  $x + y$  and  $x^2 - xy + y^2$ .
104. I used FOIL to multiply  $5xy$  and  $3xy + 4$ .

In Exercises 105–108, determine whether each statement is true or false. If the statement is false, make the necessary change(s) to produce a true statement.

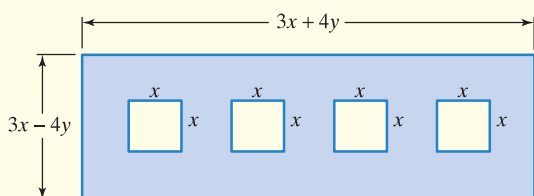
105. The degree of  $5x^{24} - 3x^{16}y^9 - 7xy^2 + 6$  is 24.
106. In the polynomial  $4x^2y + x^3y^2 + 3x^2y^3 + 7y$ , the term  $x^3y^2$  has degree 5 and no numerical coefficient.
107.  $(2x + 3 - 5y)(2x + 3 + 5y) = 4x^2 + 12x + 9 - 25y^2$
108.  $(6x^2y - 7xy - 4) - (6x^2y + 7xy - 4) = 0$

In Exercises 109–110, find a polynomial in two variables that describes the area of the region of each figure shaded in blue. Write the polynomial as the sum or difference of terms.

109.



110.



111. Use the formulas for the volume of a rectangular solid and a cylinder to derive the polynomial in Exercise 88 that describes the volume of the storage building.

### Review Exercises

112. Solve for  $W$ :  $R = \frac{L + 3W}{2}$ . (Section 2.4, Example 4)

113. Subtract:  $-6.4 - (-10.2)$ . (Section 1.6, Example 2)

114. Solve:  $0.02(x - 5) = 0.03 - 0.03(x + 7)$ . (Section 2.3, Example 5)

### Preview Exercises

Exercises 115–117 will help you prepare for the material covered in the next section.

115. Find the missing exponent, designated by the question mark, in the final step.

$$\frac{x^7}{x^3} = \frac{\overset{?}{x} \cdot \overset{?}{x} \cdot \overset{?}{x} \cdot x \cdot x \cdot x \cdot x}{\underset{?}{x} \cdot \underset{?}{x} \cdot \underset{?}{x}} = x^?$$

116. Simplify:  $\frac{(x^2)^3}{5^3}$ .

117. Simplify:  $\frac{(2a^3)^5}{(b^4)^5}$ .

## MID-CHAPTER CHECK POINT

## Section 5.1–Section 5.4



**What You Know:** We learned to add, subtract, and multiply polynomials. We used a number of fast methods for finding products of polynomials, including the FOIL method for multiplying binomials, a special-product formula for the product of the sum and difference of two terms  $[(A + B)(A - B) = A^2 - B^2]$ , and special-product formulas for squaring binomials  $[(A + B)^2 = A^2 + 2AB + B^2; (A - B)^2 = A^2 - 2AB + B^2]$ . Finally, we applied all of these operations to polynomials in several variables.

In Exercises 1–21, perform the indicated operations.

1.  $(11x^2y^3)(-5x^2y^3)$

2.  $11x^2y^3 - 5x^2y^3$

3.  $(3x + 5)(4x - 7)$

4.  $(3x + 5) - (4x - 7)$

5.  $(2x - 5)(x^2 - 3x + 1)$

6.  $(2x - 5) + (x^2 - 3x + 1)$

7.  $(8x - 3)^2$

8.  $(-10x^4)(-7x^5)$

9.  $(x^2 + 2)(x^2 - 2)$

10.  $(x^2 + 2)^2$

11.  $(9a - 10b)(2a + b)$

12.  $7x^2(10x^3 - 2x + 3)$

13.  $(3a^2b^3 - ab + 4b^2) - (-2a^2b^3 - 3ab + 5b^2)$

14.  $2(3y - 5)(3y + 5)$

15.  $(-9x^3 + 5x^2 - 2x + 7) + (11x^3 - 6x^2 + 3x - 7)$

16.  $10x^2 - 8xy - 3(y^2 - xy)$

17.  $(-2x^5 + x^4 - 3x + 10) - (2x^5 - 6x^4 + 7x - 13)$

18.  $(x + 3y)(x^2 - 3xy + 9y^2)$

19.  $(5x^4 + 4)(2x^3 - 1)$

20.  $(y - 6z)^2$

21.  $(2x + 3)(2x - 3) - (5x + 4)(5x - 4)$

22. Graph:  $y = 1 - x^2$ .

SECTION

5.5

## Dividing Polynomials

### Objectives

- 1 Use the quotient rule for exponents.
  - 2 Use the zero-exponent rule.
  - 3 Use the quotients-to-powers rule.
  - 4 Divide monomials.
  - 5 Check polynomial division.
  - 6 Divide a polynomial by a monomial.
- 
- 1 Use the quotient rule for exponents.

In the dramatic arts, ours is the era of the movies. As individuals and as a nation, we've grown up with them.

Our images of love, war, family, country—even of things that terrify us—owe much to what we've seen on screen. In this section's exercise set, we'll model our love for movies with polynomials and polynomial division. Before discussing polynomial division, we must develop some additional rules for working with exponents.



### The Quotient Rule for Exponents

Consider the quotient of two exponential expressions, such as the quotient of  $2^7$  and  $2^3$ . We are dividing 7 factors of 2 by 3 factors of 2. We are left with 4 factors of 2:

$$\frac{2^7}{2^3} = \frac{\overbrace{2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2}^{7 \text{ factors of } 2}}{\underbrace{2 \cdot 2 \cdot 2}_{3 \text{ factors of } 2}} = \frac{\cancel{2} \cdot \cancel{2} \cdot \cancel{2} \cdot 2 \cdot 2 \cdot 2 \cdot 2}{\cancel{2} \cdot \cancel{2} \cdot \cancel{2}} = 2 \cdot 2 \cdot 2 \cdot 2 = 2^4$$

Divide out pairs of factors:  $\frac{2}{2} = 1$ .

Thus,

$$\frac{2^7}{2^3} = 2^4.$$

We can quickly find the exponent, 4, on the quotient by subtracting the original exponents:

$$\frac{2^7}{2^3} = 2^{7-3}.$$

This suggests the following rule:

#### The Quotient Rule

$$\frac{b^m}{b^n} = b^{m-n}, \quad b \neq 0$$

When dividing exponential expressions with the same nonzero base, subtract the exponent in the denominator from the exponent in the numerator. Use this difference as the exponent of the common base.

**EXAMPLE 1** Using the Quotient Rule

Divide each expression using the quotient rule:

a.  $\frac{2^8}{2^4}$       b.  $\frac{x^{13}}{x^3}$       c.  $\frac{y^{15}}{y}$ .

**Solution**

a.  $\frac{2^8}{2^4} = 2^{8-4} = 2^4$  or 16  
b.  $\frac{x^{13}}{x^3} = x^{13-3} = x^{10}$   
c.  $\frac{y^{15}}{y} = \frac{y^{15}}{y^1} = y^{15-1} = y^{14}$  ■

**CHECK POINT 1** Divide each expression using the quotient rule:

a.  $\frac{5^{12}}{5^4}$       b.  $\frac{x^9}{x^2}$       c.  $\frac{y^{20}}{y}$ .

**2** Use the zero-exponent rule.

**Zero as an Exponent**

A nonzero base can be raised to the 0 power. The quotient rule can be used to help determine what zero as an exponent should mean. Consider the quotient of  $b^4$  and  $b^4$ , where  $b$  is not zero. We can determine this quotient in two ways.

$\frac{b^4}{b^4} = 1$        $\frac{b^4}{b^4} = b^{4-4} = b^0$

Any nonzero expression divided by itself is 1.      Use the quotient rule and subtract exponents.

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This means that  $b^0$  must equal 1.

**The Zero-Exponent Rule**

If  $b$  is any real number other than 0,

$$b^0 = 1.$$

**EXAMPLE 2** Using the Zero-Exponent Rule

Use the zero-exponent rule to simplify each expression:

a.  $7^0$       b.  $(-5)^0$       c.  $-5^0$       d.  $10x^0$       e.  $(10x)^0$ .

**Solution**

a.  $7^0 = 1$       Any nonzero number raised to the 0 power is 1.  
b.  $(-5)^0 = 1$       Any nonzero number raised to the 0 power is 1.

c.  $-5^0 = -1$

$-5^0 = -(5^0) = -1$

Only 5 is raised to the 0 power.

d.  $10x^0 = 10 \cdot 1 = 10$

Only  $x$  is raised to the 0 power.

e.  $(10x)^0 = 1$

The entire expression,  $10x$ , is raised to the 0 power. ■

**CHECK POINT 2** Use the zero-exponent rule to simplify each expression:

- a.  $14^0$       b.  $(-10)^0$       c.  $-10^0$       d.  $20x^0$       e.  $(20x)^0$ .

**3** Use the quotients-to-powers rule.

### The Quotients-to-Powers Rule for Exponents

We have seen that when a product is raised to a power, we raise every factor in the product to the power:

$$(ab)^n = a^n b^n.$$

There is a similar property for raising a quotient to a power.

#### Quotients to Powers

If  $a$  and  $b$  are real numbers and  $b$  is nonzero, then

$$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}.$$

When a quotient is raised to a power, raise the numerator to the power and divide by the denominator raised to the power.

#### EXAMPLE 3 Using the Quotients-to-Powers Rule

Simplify each expression using the quotients-to-powers rule:

- a.  $\left(\frac{x}{4}\right)^2$       b.  $\left(\frac{x^2}{5}\right)^3$       c.  $\left(\frac{2a^3}{b^4}\right)^5$ .

#### Solution

a.  $\left(\frac{x}{4}\right)^2 = \frac{x^2}{4^2} = \frac{x^2}{16}$

Square the numerator and the denominator.

b.  $\left(\frac{x^2}{5}\right)^3 = \frac{(x^2)^3}{5^3} = \frac{x^{2 \cdot 3}}{5 \cdot 5 \cdot 5} = \frac{x^6}{125}$

Cube the numerator and the denominator.

c.  $\left(\frac{2a^3}{b^4}\right)^5 = \frac{(2a^3)^5}{(b^4)^5}$

Raise the numerator and the denominator to the fifth power.

$$= \frac{2^5(a^3)^5}{(b^4)^5}$$

Raise each factor in the numerator to the fifth power.

$$= \frac{2^5 a^{3 \cdot 5}}{b^{4 \cdot 5}}$$

To raise exponential expressions to powers, multiply exponents:  $(b^m)^n = b^{mn}$ .

$$= \frac{32a^{15}}{b^{20}}$$

Simplify. ■

**✓ CHECK POINT 3** Simplify each expression using the quotients-to-powers rule:

a.  $\left(\frac{x}{5}\right)^2$       b.  $\left(\frac{x^4}{2}\right)^3$       c.  $\left(\frac{2a^{10}}{b^3}\right)^4$ .

### Great Question!

**What are some common errors to avoid when using the quotient rule or the zero-exponent rule?**

Here's a partial list. The first column shows the correct simplification. The second column illustrates a common error.

Correct	Incorrect	Description of Error
$\frac{2^{20}}{2^4} = 2^{20-4} = 2^{16}$	<del><math>\frac{2^{20}}{2^4} = 2^5</math></del>	Exponents should be subtracted, not divided.
$-8^0 = -1$	<del><math>-8^0 = 1</math></del>	Only 8 is raised to the 0 power.
$\left(\frac{x}{5}\right)^2 = \frac{x^2}{5^2} = \frac{x^2}{25}$	<del><math>\left(\frac{x}{5}\right)^2 = \frac{x^2}{5}</math></del>	The numerator and denominator must both be squared.

### 4 Divide monomials.

### Dividing Monomials

Now that we have developed three additional properties of exponents, we are ready to turn to polynomial division. We begin with the quotient of two monomials, such as  $16x^{14}$  and  $8x^2$ . This quotient is obtained by dividing the coefficients, 16 and 8, and then dividing the variables using the quotient rule for exponents.

$$\frac{16x^{14}}{8x^2} = \frac{16}{8}x^{14-2} = 2x^{12}$$

Divide coefficients and subtract exponents.

### Dividing Monomials

To divide monomials, divide the coefficients and then divide the variables. Use the quotient rule for exponents to divide the variables: Keep the variable and subtract the exponents.

#### EXAMPLE 4 Dividing Monomials

Divide:    a.  $\frac{-12x^8}{4x^2}$       b.  $\frac{2x^3}{8x^3}$       c.  $\frac{15x^5y^4}{3x^2y}$ .

#### Solution

a.  $\frac{-12x^8}{4x^2} = \frac{-12}{4}x^{8-2} = -3x^6$   
 b.  $\frac{2x^3}{8x^3} = \frac{2}{8}x^{3-3} = \frac{1}{4}x^0 = \frac{1}{4} \cdot 1 = \frac{1}{4}$   
 c.  $\frac{15x^5y^4}{3x^2y} = \frac{15}{3}x^{5-2}y^{4-1} = 5x^3y^3$  ■

**✓ CHECK POINT 4** Divide:

a.  $\frac{-20x^{12}}{10x^4}$       b.  $\frac{3x^4}{15x^4}$       c.  $\frac{9x^6y^5}{3xy^2}$ .

### Great Question!

I notice that the exponents on  $x$  are the same in Example 4(b). Do I have to subtract exponents since I know the quotient of  $x^3$  and  $x^3$  is 1?

No. Rather than subtracting exponents for division that results in a 0 exponent, you might prefer to divide out  $x^3$ .

$$\frac{2x^3}{8x^3} = \frac{2}{8} = \frac{1}{4}$$

- 5** Check polynomial division.

### Checking Division of Polynomial Problems

The answer to a division problem can be checked. For example, consider the following problem:

$$\frac{15x^5y^4}{3x^2y} = 5x^3y^3.$$

Dividend: the polynomial you are dividing into

Divisor: the polynomial you are dividing by

Quotient: the answer to your division problem

The quotient is correct if the product of the divisor and the quotient is the dividend. Is the quotient shown in the preceding equation correct?

$$(3x^2y)(5x^3y^3) = 3 \cdot 5x^{2+3}y^{1+3} = 15x^5y^4$$

Divisor      Quotient      This is the dividend.

Because the product of the divisor and the quotient is the dividend, the answer to the division problem is correct.

### Checking Division of Polynomials

To check a quotient in a division problem, multiply the divisor and the quotient. If this product is the dividend, the quotient is correct.

- 6** Divide a polynomial by a monomial.

### Dividing a Polynomial That Is Not a Monomial by a Monomial

To divide a polynomial by a monomial, we divide each term of the polynomial by the monomial. For example,

$$\frac{10x^8 + 15x^6}{5x^3} = \frac{10x^8}{5x^3} + \frac{15x^6}{5x^3} = \frac{10}{5}x^{8-3} + \frac{15}{5}x^{6-3} = 2x^5 + 3x^3.$$

polynomial dividend

monomial divisor

Divide the first term by  $5x^3$ .

Divide the second term by  $5x^3$ .

Is the quotient correct? Multiply the divisor and the quotient.

$$5x^3(2x^5 + 3x^3) = 5x^3 \cdot 2x^5 + 5x^3 \cdot 3x^3 = 5 \cdot 2x^{3+5} + 5 \cdot 3x^{3+3} = 10x^8 + 15x^6$$

Because this product gives the dividend, the quotient is correct.

### Dividing a Polynomial That Is Not a Monomial by a Monomial

To divide a polynomial by a monomial, divide each term of the polynomial by the monomial.

### Great Question!

**Can I cancel identical terms in the dividend and the divisor?**

No. Try to avoid this common error:

**Incorrect:**

~~$$\frac{x^4 - \cancel{x}}{x} = \frac{x^4 - 1}{1} = x^4 - 1$$~~

**Correct:**

$$\begin{aligned} \frac{x^4 - x}{x} &= \frac{x^4}{x} - \frac{x}{x} \\ &= x^{4-1} - x^{1-1} \\ &= x^3 - x^0 \\ &= x^3 - 1 \end{aligned}$$

Don't leave out the 1.



**EXAMPLE 5** Dividing a Polynomial by a Monomial

Find the quotient:  $(-12x^8 + 4x^6 - 8x^3) \div 4x^2$ .

**Solution**

$$\begin{aligned} & \frac{-12x^8 + 4x^6 - 8x^3}{4x^2} && \text{Rewrite the division in a vertical format.} \\ &= \frac{-12x^8}{4x^2} + \frac{4x^6}{4x^2} - \frac{8x^3}{4x^2} && \text{Divide each term of the polynomial by the monomial.} \\ &= \frac{-12}{4}x^{8-2} + \frac{4}{4}x^{6-2} - \frac{8}{4}x^{3-2} && \text{Divide coefficients and subtract exponents.} \\ &= -3x^6 + x^4 - 2x && \text{Simplify.} \end{aligned}$$

To check the answer, multiply the divisor and the quotient.

$$\begin{aligned} & \begin{array}{c} \text{Divisor} \quad \text{Quotient} \\ \downarrow \quad \downarrow \\ 4x^2(-3x^6 + x^4 - 2x) \end{array} = 4x^2(-3x^6) + 4x^2 \cdot x^4 - 4x^2(2x) \\ &= 4(-3)x^{2+6} + 4x^{2+4} - 4 \cdot 2x^{2+1} \\ &= -12x^8 + 4x^6 - 8x^3 \end{aligned}$$

This is the dividend.

Because the product of the divisor and the quotient is the dividend, the answer—that is, the quotient—is correct. ■

✓ **CHECK POINT 5** Find the quotient:  $(-15x^9 + 6x^5 - 9x^3) \div 3x^2$ .

**EXAMPLE 6** Dividing a Polynomial by a Monomial

Divide:  $\frac{16x^5 - 9x^4 + 8x^3}{2x^3}$ .

**Solution**

$$\begin{aligned} & \frac{16x^5 - 9x^4 + 8x^3}{2x^3} && \text{This is the given polynomial division.} \\ &= \frac{16x^5}{2x^3} - \frac{9x^4}{2x^3} + \frac{8x^3}{2x^3} && \text{Divide each term by } 2x^3. \\ &= \frac{16}{2}x^{5-3} - \frac{9}{2}x^{4-3} + \frac{8}{2}x^{3-3} && \text{Divide coefficients and subtract exponents.} \\ & && \text{Did you immediately write the last term as 4?} \\ &= 8x^2 - \frac{9}{2}x + 4x^0 && \text{Simplify.} \\ &= 8x^2 - \frac{9}{2}x + 4 && x^0 = 1, \text{ so } 4x^0 = 4 \cdot 1 = 4. \end{aligned}$$

Check the answer by showing that the product of the divisor and the quotient is the dividend. ■

✓ **CHECK POINT 6** Divide:  $\frac{25x^9 - 7x^4 + 10x^3}{5x^3}$ .

**EXAMPLE 7** Dividing Polynomials in Two Variables

Divide:  $(15x^5y^4 - 3x^3y^2 + 9x^2y) \div 3x^2y$ .

**Solution**

$$\frac{15x^5y^4 - 3x^3y^2 + 9x^2y}{3x^2y}$$

Rewrite the division in a vertical format.

$$= \frac{15x^5y^4}{3x^2y} - \frac{3x^3y^2}{3x^2y} + \frac{9x^2y}{3x^2y}$$

Divide each term of the polynomial by the monomial.

$$= \frac{15}{3}x^{5-2}y^{4-1} - \frac{3}{3}x^{3-2}y^{2-1} + \frac{9}{3}x^{2-2}y^{1-1}$$

Divide coefficients and subtract exponents.

$$= 5x^3y^3 - xy + 3$$

Simplify.

Check the answer by showing that the product of the divisor and the quotient is the dividend. ■

**CHECK POINT 7** Divide:  $(18x^7y^6 - 6x^2y^3 + 60xy^2) \div 6xy^2$ .

**CONCEPT AND VOCABULARY CHECK**

Fill in each blank so that the resulting statement is true.

- The quotient rule for exponents states that  $\frac{b^m}{b^n} = \underline{\hspace{2cm}}$ ,  $b \neq 0$ . When dividing exponential expressions with the same nonzero base,                      the exponents.
- If  $b \neq 0$ ,  $b^0 = \underline{\hspace{2cm}}$ .
- The quotients-to-powers rule for exponents states that  $\left(\frac{a}{b}\right)^n = \underline{\hspace{2cm}}$ ,  $b \neq 0$ . When a quotient is raised to a power, raise both the                      and the                      to the power.
- To divide monomials,                      the coefficients and                      the exponents.
- Consider the following division problem:

$$\frac{20x^6y^4}{10x^2y} = 2x^4y^3.$$

The polynomial in the numerator,  $20x^6y^4$ , is called the                     . The polynomial in the denominator,  $10x^2y$ , is called the                     . The answer to the problem,  $2x^4y^3$ , is called the                     .

- To check the answer to a division problem, multiply the                      and the                     . If this product is the                     , the answer is correct.
- To perform the division  $\frac{20x^8 - 10x^4 + 6x^3}{2x^3}$ , divide each term of                      by                     .

**5.5 EXERCISE SET**

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**Practice Exercises**

In Exercises 1–10, divide each expression using the quotient rule. Express any numerical answers in exponential form.

- |  |  |   |
|--|--|---|
| 1. $\frac{3^{20}}{3^5}$                  | 2. $\frac{3^{30}}{3^{10}}$               | 3. $\frac{x^6}{x^2}$                    |
| 4. $\frac{x^8}{x^4}$                     | 5. $\frac{y^{13}}{y^5}$                  | 6. $\frac{y^{19}}{y^6}$                 |
| 7. $\frac{5^6 \cdot 2^8}{5^3 \cdot 2^4}$ | 8. $\frac{3^6 \cdot 2^8}{3^3 \cdot 2^4}$ | 9. $\frac{x^{100}y^{50}}{x^{25}y^{10}}$ |
| 10. $\frac{x^{200}y^{40}}{x^{25}y^{10}}$ |  |   |

In Exercises 11–24, use the zero-exponent rule to simplify each expression.

- |                         |                                   |                     |
|-------------------------|-----------------------------------|---------------------|
| 11. $2^0$               | 12. $4^0$                         | 13. $(-2)^0$        |
| 14. $(-4)^0$            | 15. $-2^0$                        | 16. $-4^0$          |
| 17. $100y^0$            | 18. $200y^0$                      | 19. $(100y)^0$      |
| 20. $(200y)^0$          | 21. $-5^0 + (-5)^0$               | 22. $-6^0 + (-6)^0$ |
| 23. $-\pi^0 - (-\pi)^0$ | 24. $-\sqrt{3}^0 - (-\sqrt{3})^0$ |                     |

In Exercises 25–36, simplify each expression using the quotients-to-powers rule. If possible, evaluate exponential expressions.

25.  $\left(\frac{x}{3}\right)^2$

26.  $\left(\frac{x}{5}\right)^2$

27.  $\left(\frac{x^2}{4}\right)^3$

28.  $\left(\frac{x^2}{3}\right)^3$

29.  $\left(\frac{2x^3}{5}\right)^2$

30.  $\left(\frac{3x^4}{7}\right)^2$

31.  $\left(\frac{-4}{3a^3}\right)^3$

32.  $\left(\frac{-5}{2a^3}\right)^3$

33.  $\left(\frac{-2a^7}{b^4}\right)^5$

34.  $\left(\frac{-2a^8}{b^3}\right)^5$

35.  $\left(\frac{x^2y^3}{2z}\right)^4$

36.  $\left(\frac{x^3y^2}{2z}\right)^4$

In Exercises 37–52, divide the monomials. Check each answer by showing that the product of the divisor and the quotient is the dividend.

37.  $\frac{30x^{10}}{10x^5}$

38.  $\frac{45x^{12}}{15x^4}$

39.  $\frac{-8x^{22}}{4x^2}$

40.  $\frac{-15x^{40}}{3x^4}$

41.  $\frac{-9y^8}{18y^5}$

42.  $\frac{-15y^{13}}{45y^9}$

43.  $\frac{7y^{17}}{5y^5}$

44.  $\frac{9y^{19}}{7y^{11}}$

45.  $\frac{30x^7y^5}{5x^2y}$

46.  $\frac{40x^9y^5}{2x^2y}$

47.  $\frac{-18x^{14}y^2}{36x^2y^2}$

48.  $\frac{-15x^{16}y^2}{45x^2y^2}$

49.  $\frac{9x^{20}y^{20}}{7x^{20}y^{20}}$

50.  $\frac{7x^{30}y^{30}}{15x^{30}y^{30}}$

51.  $\frac{-5x^{10}y^{12}z^6}{50x^2y^3z^2}$

52.  $\frac{-8x^{12}y^{10}z^4}{40x^2y^3z^2}$

In Exercises 53–78, divide the polynomial by the monomial. Check each answer by showing that the product of the divisor and the quotient is the dividend.

53.  $\frac{10x^4 + 2x^3}{2}$

54.  $\frac{20x^4 + 5x^3}{5}$

55.  $\frac{14x^4 - 7x^3}{7x}$

56.  $\frac{24x^4 - 8x^3}{8x}$

57.  $\frac{y^7 - 9y^2 + y}{y}$

58.  $\frac{y^8 - 11y^3 + y}{y}$

59.  $\frac{24x^3 - 15x^2}{-3x}$

60.  $\frac{10x^3 - 20x^2}{-5x}$

61.  $\frac{18x^5 + 6x^4 + 9x^3}{3x^2}$

62.  $\frac{18x^5 + 24x^4 + 12x^3}{6x^2}$

63.  $\frac{12x^4 - 8x^3 + 40x^2}{4x}$

64.  $\frac{49x^4 - 14x^3 + 70x^2}{-7x}$

65.  $(4x^2 - 6x) \div x$

66.  $(16y^2 - 8y) \div y$

67.  $\frac{30z^3 + 10z^2}{-5z}$

68.  $\frac{12y^4 - 42y^2}{-4y}$

69.  $\frac{8x^3 + 6x^2 - 2x}{2x}$

70.  $\frac{9x^3 + 12x^2 - 3x}{3x}$

71.  $\frac{25x^7 - 15x^5 - 5x^4}{5x^3}$

72.  $\frac{49x^7 - 28x^5 - 7x^4}{7x^3}$

73.  $\frac{18x^7 - 9x^6 + 20x^5 - 10x^4}{-2x^4}$

74.  $\frac{25x^8 - 50x^7 + 3x^6 - 40x^5}{-5x^5}$

75.  $\frac{12x^2y^2 + 6x^2y - 15xy^2}{3xy}$

76.  $\frac{18a^3b^2 - 9a^2b - 27ab^2}{9ab}$

77.  $\frac{20x^7y^4 - 15x^3y^2 - 10x^2y}{-5x^2y}$

78.  $\frac{8x^6y^3 - 12x^8y^2 - 4x^{14}y^6}{-4x^6y^2}$

**Practice PLUS**

In Exercises 79–82, simplify each expression.

79.  $\frac{2x^3(4x + 2) - 3x^2(2x - 4)}{2x^2}$

80.  $\frac{6x^3(3x - 1) + 5x^2(6x - 3)}{3x^2}$

81.  $\left(\frac{18x^2y^4}{9xy^2}\right) - \left(\frac{15x^5y^6}{5x^4y^4}\right)$

82.  $\left(\frac{9x^3 + 6x^2}{3x}\right) - \left(\frac{12x^2y^2 - 4xy^2}{2xy^2}\right)$

83. Divide the sum of  $(y + 5)^2$  and  $(y + 5)(y - 5)$  by  $2y$ .

84. Divide the sum of  $(y + 4)^2$  and  $(y + 4)(y - 4)$  by  $2y$ .

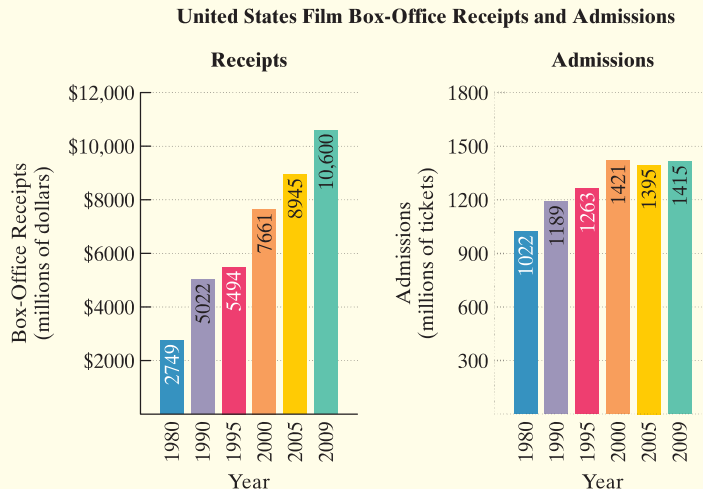
In Exercises 85–86, the variable  $n$  in each exponent represents a natural number. Divide the polynomial by the monomial. Then use polynomial multiplication to check the quotient.

85.  $\frac{12x^{15n} - 24x^{12n} + 8x^{3n}}{4x^{3n}}$

86.  $\frac{35x^{10n} - 15x^{8n} + 25x^{2n}}{5x^{2n}}$

### Application Exercises

The bar graphs show U.S. film box-office receipts, in millions of dollars, and box-office admissions, in millions of tickets sold, for five selected years.



Sources: U.S. Department of Commerce, Motion Picture Association of America, National Association of Theatre Owners

The following polynomial models of degree 2 can be used to describe the data in the bar graphs:

Receipts, in millions of dollars

$$R = 3.6x^2 + 158x + 2790$$

Admissions, in millions of tickets sold

$$A = -0.2x^2 + 21x + 1015.$$

The variable  $x$  represents the number of years after 1980.

Use this information to solve Exercises 87–88.

- 87.**
- Use the data displayed by the bar graphs to find the average admission charge for a film ticket in 2000. Round to two decimal places, or to the nearest cent.
  - Use the models to write an algebraic expression that describes the average admission charge for a film ticket  $x$  years after 1980.
  - Use the model from part (b) to find the average admission charge for a film ticket in 2000. Round to the nearest cent. Does the model underestimate or overestimate the actual average charge that you found in part (a)? By how much?
  - Can the polynomial division for the model in part (b) be performed using the methods that you learned in this section? Explain your answer.
- 88.**
- Use the data displayed by the bar graphs to find the average admission charge for a film ticket in 2005. Round to two decimal places, or to the nearest cent.
  - Use the models to write an algebraic expression that describes the average admission charge for a film ticket  $x$  years after 1980.
  - Use the model from part (b) to find the average admission charge for a film ticket in 2005. Round to the nearest cent. Does the model underestimate or overestimate the actual average charge that you found in part (a)? By how much?
  - Can the polynomial division for the model in part (b) be performed using the methods that you learned in this section? Explain your answer.

### Writing in Mathematics

- Explain the quotient rule for exponents. Use  $\frac{3^6}{3^2}$  in your explanation.
- Explain how to find any nonzero number to the 0 power.
- Explain the difference between  $(-7)^0$  and  $-7^0$ .
- Explain how to simplify an expression that involves a quotient raised to a power. Provide an example with your explanation.
- Explain how to divide monomials. Give an example.
- Explain how to divide a polynomial that is not a monomial by a monomial. Give an example.

95. Are the expressions

$$\frac{12x^2 + 6x}{3x} \quad \text{and} \quad 4x + 2$$

equal for every value of  $x$ ? Explain.

### Critical Thinking Exercises

**Make Sense?** In Exercises 96–99, determine whether each statement “makes sense” or “does not make sense” and explain your reasoning.

96. Because division by 0 is undefined, numbers to 0 powers should not be written in denominators.
97. The quotient rule is applied by dividing the exponent in the numerator by the exponent in the denominator.
98. I divide monomials by dividing coefficients and subtracting exponents.
99. I divide a polynomial by a monomial by dividing each term of the monomial by the polynomial.

In Exercises 100–103, determine whether each statement is true or false. If the statement is false, make the necessary change(s) to produce a true statement.

100.  $x^{10} \div x^2 = x^5$  for all nonzero real numbers  $x$ .

101.  $\frac{12x^3 - 6x}{2x} = 6x^2 - 6x$

102.  $\frac{x^2 + x}{x} = x$

103. If a polynomial in  $x$  of degree 6 is divided by a monomial in  $x$  of degree 2, the degree of the quotient is 4.

104. What polynomial, when divided by  $3x^2$ , yields the trinomial  $6x^6 - 9x^4 + 12x^2$  as a quotient?

In Exercises 105–106, find the missing coefficients and exponents designated by question marks.

105.  $\frac{?x^8 - ?x^6}{3x^2} = 3x^5 - 4x^3$

106.  $\frac{3x^{14} - 6x^{12} - ?x^7}{?x^2} = -x^7 + 2x^5 + 3$

### Review Exercises

107. Find the absolute value:  $|-20.3|$ . (Section 1.3, Example 8)
108. Express  $\frac{7}{8}$  as a decimal. (Section 1.3, Example 4)
109. Graph:  $y = \frac{1}{3}x + 2$ . (Section 3.4, Example 3)

### Preview Exercises

Exercises 110–112 will help you prepare for the material covered in the next section. In each exercise, perform the long division without using a calculator, and then state the quotient and the remainder.

110.  $19 \overline{)494}$

111.  $24 \overline{)2958}$

112.  $98 \overline{)25,187}$

## SECTION

# 5.6

## Dividing Polynomials by Binomials

### Objective

- 1 Divide polynomials by binomials.



“He was an arithmetician rather than a mathematician. None of the humor, the music, or the mysticism of higher mathematics ever entered his head.”

—John Steinbeck

“Arithmetic has a very great and elevating effect. He who can properly divide is to be considered a god.”

—Plato

“You cannot ask us to take sides against arithmetic.”

—Winston Churchill

**SECTION 5.6** Dividing Polynomials by Binomials **387**

So, what's the deal? Will performing the repetitive procedure of long division (don't reach for that calculator!) have an elevating effect? Or will it confine you to a computational box that allows neither humor nor music to enter? Forget the box: Mathematician Wilhelm Leibniz believed that music is nothing but unconscious arithmetic. But do think elevation, if not to the level of an ancient Greek god, then to new, algebraic highs. The bottom line: Understanding long division of whole numbers lays the foundation for performing the division of a polynomial by a binomial, such as

$$x + 3 \overline{)x^2 + 10x + 21.}$$

Divisor has two terms and is a binomial.

The polynomial dividend has three terms and is a trinomial.

In this section, you will learn how to perform such divisions.

**The Steps in Dividing a Polynomial by a Binomial**

Dividing a polynomial by a binomial may remind you of long division. Let's review long division of whole numbers by dividing 3983 by 26.

Divisor  $26 \overline{)3983}$  Dividend

$$\begin{array}{r} 1 \\ 26 \overline{)3983} \\ \underline{26} \phantom{0} \\ 138 \\ \underline{130} \\ 83 \end{array}$$

DIVIDE:  $\frac{39}{26} = 1$  plus a remainder.  
 MULTIPLY:  $1 \cdot 26 = 26$ .  
 SUBTRACT:  $39 - 26 = 13$ . BRING DOWN the next digit in the dividend.

$$\begin{array}{r} 15 \\ 26 \overline{)3983} \\ \underline{26} \phantom{0} \\ 138 \\ \underline{130} \\ 83 \end{array}$$

DIVIDE:  $\frac{138}{26} = 5$  plus a remainder.  
 MULTIPLY:  $5 \cdot 26 = 130$ .  
 SUBTRACT:  $138 - 130 = 8$ . BRING DOWN the next digit in the dividend.

$$\begin{array}{r} 153 \\ 26 \overline{)3983} \\ \underline{26} \phantom{0} \\ 138 \\ \underline{130} \\ 83 \\ \underline{78} \\ 5 \end{array}$$

DIVIDE:  $\frac{83}{26} = 3$  plus a remainder.  
 MULTIPLY:  $3 \cdot 26 = 78$ .  
 SUBTRACT:  $83 - 78 = 5$ . There are no more digits to bring down, so the remainder is 5.

The quotient is 153 and the remainder is 5. This can be written as

$$153 \frac{5}{26}$$

Quotient

Remainder

Divisor

$$153\frac{5}{26}$$

We see that  $26 \overline{)3983} \frac{5}{26}$ .

This answer can be checked. Multiply the divisor and the quotient. Then add the remainder. If the result is the dividend, the answer is correct. In this case, we have

$$\begin{array}{ccccccc}
 26(153) & + & 5 & = & 3978 & + & 5 & = & 3983. \\
 \text{Divisor} & & \text{Quotient} & & \text{Remainder} & & & & \text{This is the dividend}
 \end{array}$$

**1** Divide polynomials by binomials.

Because we obtained the dividend, the answer to the division problem,  $153\frac{5}{26}$ , is correct.

When a divisor is a binomial, the four steps used to divide whole numbers—**divide, multiply, subtract, bring down the next term**—form the repetitive procedure for dividing a polynomial by a binomial.

**EXAMPLE 1** Dividing a Polynomial by a Binomial

Divide  $x^2 + 10x + 21$  by  $x + 3$ .

**Solution** The following steps illustrate how polynomial division is very similar to numerical division.

$$x + 3 \overline{)x^2 + 10x + 21}$$

Arrange the terms of the dividend ( $x^2 + 10x + 21$ ) and the divisor ( $x + 3$ ) in descending powers of  $x$ .

$$x + 3 \overline{)x^2 + 10x + 21}$$

DIVIDE  $x^2$  (the first term in the dividend) by  $x$  (the first term in the divisor):  $\frac{x^2}{x} = x$ . Align like terms.

$$\begin{array}{r}
 x \\
 x + 3 \overline{)x^2 + 10x + 21} \\
 \underline{x^2 + 3x} \phantom{+ 21} \\
 7x + 21
 \end{array}$$

$x(x + 3) = x^2 + 3x$

MULTIPLY each term in the divisor ( $x + 3$ ) by  $x$ , aligning terms of the product under like terms in the dividend.

$$\begin{array}{r}
 x \\
 x + 3 \overline{)x^2 + 10x + 21} \\
 \underline{x^2 + 3x} \phantom{+ 21} \\
 7x + 21
 \end{array}$$

SUBTRACT  $x^2 + 3x$  from  $x^2 + 10x$  by changing the sign of each term in the lower expression and adding.

Change signs of the polynomial being subtracted.

$$\begin{array}{r}
 x \\
 x + 3 \overline{)x^2 + 10x + 21} \\
 \underline{x^2 + 3x} \phantom{+ 21} \\
 7x + 21
 \end{array}$$

BRING DOWN 21 from the original dividend and add algebraically to form a new dividend.

$$\begin{array}{r}
 x + 7 \\
 x + 3 \overline{)x^2 + 10x + 21} \\
 \underline{x^2 + 3x} \phantom{+ 21} \\
 7x + 21
 \end{array}$$

FIND the second term of the quotient. Divide the first term of  $7x + 21$  by  $x$ , the first term of the divisor:  $\frac{7x}{x} = 7$ .

$$\begin{array}{r}
 x + 7 \\
 x + 3 \overline{)x^2 + 10x + 21} \\
 \underline{x^2 + 3x} \phantom{+ 21} \\
 7x + 21 \\
 \underline{7x + 21} \\
 0
 \end{array}$$

$7(x + 3) = 7x + 21$

MULTIPLY the divisor ( $x + 3$ ) by 7, aligning under like terms in the new dividend. Then subtract to obtain the remainder of 0.

Remainder

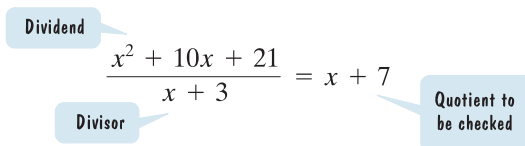


**SECTION 5.6** Dividing Polynomials by Binomials **389**

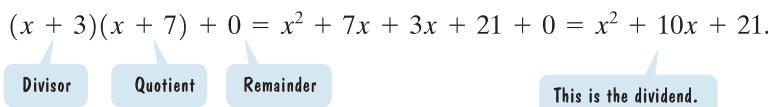
The quotient is  $x + 7$  and the remainder is 0. We will not list a remainder of 0 in the answer. Thus,

$$\frac{x^2 + 10x + 21}{x + 3} = x + 7. \quad \blacksquare$$

When dividing polynomials by binomials, the answer can be checked. Find the product of the divisor and the quotient and add the remainder. If the result is the dividend, the answer to the division problem is correct. For example, let's check our work in Example 1.


$$\frac{x^2 + 10x + 21}{x + 3} = x + 7$$

Multiply the divisor and the quotient and add the remainder, 0:


$$(x + 3)(x + 7) + 0 = x^2 + 7x + 3x + 21 + 0 = x^2 + 10x + 21.$$

Because we obtained the dividend, the quotient is correct.

**CHECK POINT 1** Divide  $x^2 + 14x + 45$  by  $x + 9$ .

Before considering additional examples, let's summarize the general procedure for dividing a polynomial by a binomial.

### Dividing a Polynomial by a Binomial

1. **Arrange the terms** of both the dividend and the divisor in descending powers of the variable.
2. **Divide** the first term in the dividend by the first term in the divisor. The result is the first term of the quotient.
3. **Multiply** every term in the divisor by the first term in the quotient. Write the resulting product beneath the dividend with like terms lined up.
4. **Subtract** the product from the dividend.
5. **Bring down** the next term in the original dividend and write it next to the remainder to form a new dividend.
6. Use this new expression as the dividend and repeat this process until the remainder can no longer be divided. This will occur when the degree of the remainder (the highest exponent on a variable in the remainder) is less than the degree of the divisor.

In our next division, we will obtain a nonzero remainder.

### EXAMPLE 2 Dividing a Polynomial by a Binomial

Divide:  $\frac{7x - 9 - 4x^2 + 4x^3}{2x - 1}$ .

**Solution** We begin by writing the dividend,  $7x - 9 - 4x^2 + 4x^3$ , in descending powers of  $x$ .

$$7x - 9 - 4x^2 + 4x^3 = 4x^3 - 4x^2 + 7x - 9$$

Think of 9 as  $9x^0$ . The powers descend from 3 to 0.

$$2x - 1 \overline{)4x^3 - 4x^2 + 7x - 9}$$

This is the problem with the dividend in descending powers of  $x$ .

$$2x - 1 \overline{)4x^3 - 4x^2 + 7x - 9}$$

DIVIDE:  $\frac{4x^3}{2x} = 2x^2$ .

$$2x^2(2x - 1) = 4x^3 - 2x^2$$

$$2x - 1 \overline{)4x^3 - 4x^2 + 7x - 9}$$

MULTIPLY:  $2x^2(2x - 1) = 4x^3 - 2x^2$ .

$$2x - 1 \overline{)4x^3 - 4x^2 + 7x - 9}$$

SUBTRACT:  $4x^3 - 4x^2 - (4x^3 - 2x^2)$   
 $= 4x^3 - 4x^2 - 4x^3 + 2x^2$   
 $= -2x^2$ .

Change signs of the polynomial being subtracted.

$$2x - 1 \overline{)4x^3 - 4x^2 + 7x - 9}$$

BRING DOWN  $7x$ . The new dividend is  $-2x^2 + 7x$ .

$$2x - 1 \overline{)4x^3 - 4x^2 + 7x - 9}$$

DIVIDE:  $\frac{-2x^2}{2x} = -x$ .

$$-x(2x - 1) = -2x^2 + x$$

$$2x - 1 \overline{)4x^3 - 4x^2 + 7x - 9}$$

MULTIPLY:  $-x(2x - 1) = -2x^2 + x$ .

$$2x - 1 \overline{)4x^3 - 4x^2 + 7x - 9}$$

SUBTRACT:  $-2x^2 + 7x - (-2x^2 + x)$   
 $= -2x^2 + 7x + 2x^2 - x$   
 $= 6x$ .

$$2x - 1 \overline{)4x^3 - 4x^2 + 7x - 9}$$

BRING DOWN  $-9$ . The new dividend is  $6x - 9$ .

**SECTION 5.6** Dividing Polynomials by Binomials **391**

$$\begin{array}{r}
 2x^2 - x + 3 \\
 2x - 1 \overline{) 4x^3 - 4x^2 + 7x - 9} \\
 \underline{4x^3 - 2x^2} \phantom{+ 7x - 9} \\
 -2x^2 + 7x \phantom{- 9} \\
 \underline{-2x^2 + x} \phantom{- 9} \\
 6x - 9
 \end{array}$$

DIVIDE:  $\frac{6x}{2x} = 3$ .

$$\begin{array}{r}
 2x^2 - x + 3 \\
 2x - 1 \overline{) 4x^3 - 4x^2 + 7x - 9} \\
 \underline{4x^3 - 2x^2} \phantom{+ 7x - 9} \\
 -2x^2 + 7x \phantom{- 9} \\
 \underline{-2x^2 + x} \phantom{- 9} \\
 6x - 9 \\
 \underline{6x - 3} \\
 -6
 \end{array}$$

MULTIPLY:  $3(2x - 1) = 6x - 3$ .

$$\begin{array}{r}
 2x^2 - x + 3 \\
 2x - 1 \overline{) 4x^3 - 4x^2 + 7x - 9} \\
 \underline{4x^3 - 2x^2} \phantom{+ 7x - 9} \\
 -2x^2 + 7x \phantom{- 9} \\
 \underline{-2x^2 + x} \phantom{- 9} \\
 6x - 9 \\
 \ominus \underline{6x - 3} \\
 -6
 \end{array}$$

SUBTRACT:  $6x - 9 - (6x - 3)$   
 $= 6x - 9 - 6x + 3$   
 $= -6$ .

Remainder

The quotient is  $2x^2 - x + 3$  and the remainder is  $-6$ . When there is a nonzero remainder, as in this example, list the quotient, plus the remainder above the divisor. Thus,

$$\frac{7x - 9 - 4x^2 + 4x^3}{2x - 1} = \underbrace{2x^2 - x + 3}_{\text{Quotient}} + \frac{-6}{2x - 1}$$

Remainder above divisor

or

$$\frac{7x - 9 - 4x^2 + 4x^3}{2x - 1} = 2x^2 - x + 3 - \frac{6}{2x - 1}$$

Check this result by showing that the product of the divisor and the quotient,

$$(2x - 1)(2x^2 - x + 3),$$

plus the remainder,  $-6$ , is the dividend,  $7x - 9 - 4x^2 + 4x^3$ . ■

**CHECK POINT 2** Divide:  $\frac{6x + 8x^2 - 12}{2x + 3}$ .

If a power of the variable is missing in a dividend, add that power of the variable with a coefficient of 0 and then divide. In this way, like terms will be aligned as you carry out the division.

**EXAMPLE 3** Dividing a Polynomial with Missing Terms

Divide:  $\frac{8x^3 - 1}{2x - 1}$ .

**Solution** We write the dividend,  $8x^3 - 1$ , as

$$8x^3 + 0x^2 + 0x - 1.$$

Use a coefficient of 0 with missing terms.

By doing this, we will keep all like terms aligned.

$$\begin{array}{r} 4x^2 \\ 2x - 1 \overline{) 8x^3 + 0x^2 + 0x - 1} \\ \underline{8x^3 - 4x^2} \phantom{+ 0x - 1} \\ 4x^2 + 0x \phantom{- 1} \end{array}$$

Divide  $\left(\frac{8x^3}{2x} = 4x^2\right)$ , multiply  $[4x^2(2x - 1) = 8x^3 - 4x^2]$ , subtract, and bring down the next term. The new dividend is  $4x^2 + 0x$ .

$$\begin{array}{r} 4x^2 + 2x \\ 2x - 1 \overline{) 8x^3 + 0x^2 + 0x - 1} \\ \underline{8x^3 - 4x^2} \phantom{+ 0x - 1} \\ 4x^2 + 0x \phantom{- 1} \\ \underline{4x^2 - 2x} \phantom{- 1} \\ 2x - 1 \end{array}$$

Divide  $\left(\frac{4x^2}{2x} = 2x\right)$ , multiply  $[2x(2x - 1) = 4x^2 - 2x]$ , subtract, and bring down the next term. The new dividend is  $2x - 1$ .

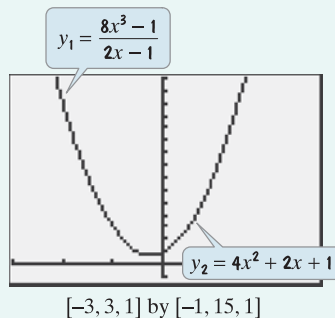
$$\begin{array}{r} 4x^2 + 2x + 1 \\ 2x - 1 \overline{) 8x^3 + 0x^2 + 0x - 1} \\ \underline{8x^3 - 4x^2} \phantom{+ 0x - 1} \\ 4x^2 + 0x \phantom{- 1} \\ \underline{4x^2 - 2x} \phantom{- 1} \\ 2x - 1 \\ \underline{2x - 1} \\ 0 \end{array}$$

Divide  $\left(\frac{2x}{2x} = 1\right)$ , multiply  $[1(2x - 1) = 2x - 1]$ , and subtract. The remainder is 0.

**Using Technology**

**Graphic Connections**

The graphs of  $y_1 = \frac{8x^3 - 1}{2x - 1}$  and  $y_2 = 4x^2 + 2x + 1$  are shown below.



The graphs coincide. Thus,

$$\frac{8x^3 - 1}{2x - 1} = 4x^2 + 2x + 1.$$

Thus,

$$\frac{8x^3 - 1}{2x - 1} = 4x^2 + 2x + 1.$$

Check this result by showing that the product of the divisor and the quotient

$$(2x - 1)(4x^2 + 2x + 1)$$

plus the remainder, 0, is the dividend,  $8x^3 - 1$ . ■

**✓ CHECK POINT 3** Divide:  $\frac{x^3 - 1}{x - 1}$ .

**CONCEPT AND VOCABULARY CHECK**

Fill in each blank so that the resulting statement is true.

1. Consider the following long division problem:

$$2x + 3 \overline{)4x^2 + 9 + 10x^3}$$

We begin the division process by rewriting the dividend as \_\_\_\_\_.

2. Consider the following long division problem:

$$2x + 1 \overline{)8x^2 + 10x - 1}$$

We begin the division process by dividing \_\_\_\_\_ by \_\_\_\_\_. We obtain \_\_\_\_\_. We write this result above \_\_\_\_\_ in the dividend.

3. In the following long division problem, the first step has been completed:

$$\begin{array}{r} 5x \\ 3x - 2 \overline{)15x^2 - 22x + 19} \end{array}$$

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The next step is to multiply \_\_\_\_\_ and \_\_\_\_\_. We obtain \_\_\_\_\_. We write this result below \_\_\_\_\_.

4. In the following long division problem, the first steps have been completed:

$$\begin{array}{r} 2x \\ 3x - 5 \overline{)6x^2 + 8x - 4} \\ \underline{6x^2 - 10x} \end{array}$$

The next step is to subtract \_\_\_\_\_ from \_\_\_\_\_. We obtain \_\_\_\_\_. Then we bring down \_\_\_\_\_ and form the new dividend \_\_\_\_\_.

5. In the following long division problem, most of the steps have been completed:

$$\begin{array}{r} x - 12 \\ x - 5 \overline{)x^2 - 17x + 74} \\ \underline{x^2 - 5x} \\ -12x + 74 \\ \underline{-12x + 60} \\ ? \end{array}$$

Completing the step designated by the question mark, we obtain \_\_\_\_\_. Thus, the quotient is \_\_\_\_\_ and the remainder is \_\_\_\_\_. The answer to this long division problem is \_\_\_\_\_.

**5.6 EXERCISE SET**



**Practice Exercises**

In Exercises 1–36, divide as indicated. Check each answer by showing that the product of the divisor and the quotient, plus the remainder, is the dividend.

1.  $\frac{x^2 + 6x + 8}{x + 2}$

2.  $\frac{x^2 + 7x + 10}{x + 5}$

3.  $\frac{2x^2 + x - 10}{x - 2}$

4.  $\frac{2x^2 + 13x + 15}{x + 5}$

5.  $\frac{x^2 - 5x + 6}{x - 3}$

6.  $\frac{x^2 - 2x - 24}{x + 4}$

7.  $\frac{2y^2 + 5y + 2}{y + 2}$

8.  $\frac{2y^2 - 13y + 21}{y - 3}$

9.  $\frac{x^2 - 3x + 4}{x + 2}$

10.  $\frac{x^2 - 7x + 5}{x + 3}$

11.  $\frac{5y + 10 + y^2}{y + 2}$

12.  $\frac{-8y + y^2 - 9}{y - 3}$

13.  $\frac{x^3 - 6x^2 + 7x - 2}{x - 1}$

14.  $\frac{x^3 + 3x^2 + 5x + 3}{x + 1}$

15.  $\frac{12y^2 - 20y + 3}{2y - 3}$

17.  $\frac{4a^2 + 4a - 3}{2a - 1}$

19.  $\frac{3y - y^2 + 2y^3 + 2}{2y + 1}$

20.  $\frac{9y + 18 - 11y^2 + 12y^3}{4y + 3}$

21.  $\frac{6x^2 - 5x - 30}{2x - 5}$

22.  $\frac{4y^2 + 8y + 3}{2y - 1}$

23.  $\frac{x^3 + 4x - 3}{x - 2}$

24.  $\frac{x^3 + 2x^2 - 3}{x - 2}$

25.  $\frac{4y^3 + 8y^2 + 5y + 9}{2y + 3}$

26.  $\frac{2y^3 - y^2 + 3y + 2}{2y + 1}$

27.  $\frac{6y^3 - 5y^2 + 5}{3y + 2}$

28.  $\frac{4y^3 + 3y + 5}{2y - 3}$

29.  $\frac{27x^3 - 1}{3x - 1}$

30.  $\frac{8x^3 + 27}{2x + 3}$

31.  $\frac{81 - 12y^3 + 54y^2 + y^4 - 108y}{y - 3}$

32.  $\frac{8y^3 + y^4 + 16 + 32y + 24y^2}{y + 2}$

33.  $\frac{4y^2 + 6y}{2y - 1}$

34.  $\frac{10x^2 - 3x}{x + 3}$

35.  $\frac{y^4 - 2y^2 + 5}{y - 1}$

36.  $\frac{y^4 - 6y^2 + 3}{y - 1}$

16.  $\frac{4y^2 - 8y - 5}{2y + 1}$

18.  $\frac{2b^2 - 9b - 5}{2b + 1}$

38.  $\frac{3x^3 + 4x^2 + x + 7}{x^2 + 1}$

39.  $\frac{x^3 - a^3}{x - a}$

40.  $\frac{x^4 - a^4}{x - a}$

41.  $\frac{6x^4 - 5x^3 - 8x^2 + 16x - 8}{3x^2 + 2x - 4}$

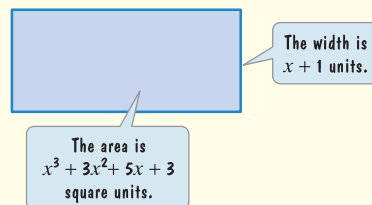
42.  $\frac{2x^4 + 5x^3 - 11x^2 - 20x + 12}{x^2 + x - 6}$

43. Divide the difference between  $4x^3 + x^2 - 2x + 7$  and  $3x^3 - 2x^2 - 7x + 4$  by  $x + 1$ .

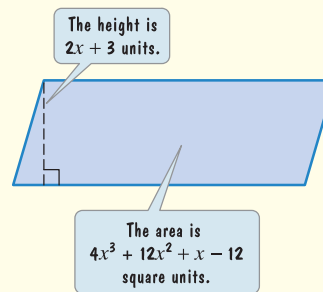
44. Divide the difference between  $4x^3 + 2x^2 - x - 1$  and  $2x^3 - x^2 + 2x - 5$  by  $x + 2$ .

### Application Exercises

45. Write a simplified polynomial that represents the length of the rectangle.



46. Write a simplified polynomial that represents the measure of the base of the parallelogram.



You just signed a contract for a new job. The salary for the first year is \$30,000 and there is to be a percent increase in your salary each year. The algebraic expression

$$\frac{30,000x^n - 30,000}{x - 1}$$

describes your total salary over  $n$  years, where  $x$  is the sum of 1 and the yearly percent increase, expressed as a decimal. Use this information to solve Exercises 47–48.

47. a. Use the given expression and write a quotient of polynomials that describes your total salary over three years.

b. Simplify the expression in part (a) by performing the division.

### Practice PLUS

In Exercises 37–42, divide as indicated.

37.  $\frac{4x^3 - 3x^2 + x + 1}{x^2 + 2}$

- c. Suppose you are to receive an increase of 5% per year. Thus,  $x$  is the sum of 1 and 0.05, or 1.05. Substitute 1.05 for  $x$  in the expression in part (a) as well as in the simplified form of the expression in part (b). Evaluate each expression. What is your total salary over the three-year period?
48. a. Use the expression given on the previous page and write a quotient of polynomials that describes your total salary over four years.
- b. Simplify the expression in part (a) by performing the division.
- c. Suppose you are to receive an increase of 8% per year. Thus,  $x$  is the sum of 1 and 0.08, or 1.08. Substitute 1.08 for  $x$  in the expression in part (a) as well as in the simplified form of the expression in part (b). Evaluate each expression. What is your total salary over the four-year period?

### Writing in Mathematics

49. In your own words, explain how to divide a polynomial by a binomial. Use  $\frac{x^2 + 4}{x + 2}$  in your explanation.
50. When dividing a polynomial by a binomial, explain when to stop dividing.
51. After dividing a polynomial by a binomial, explain how to check the answer.
52. When dividing a binomial into a polynomial with missing terms, explain the advantage of writing the missing terms with zero coefficients.

### Critical Thinking Exercises

**Make Sense?** In Exercises 53–56, determine whether each statement “makes sense” or “does not make sense” and explain your reasoning. Each statement applies to the division problem

$$\frac{x^3 + 1}{x + 1}.$$

53. The purpose of writing  $x^3 + 1$  as  $x^3 + 0x^2 + 0x + 1$  is to keep all like terms aligned.
54. Rewriting  $x^3 + 1$  as  $x^3 + 0x^2 + 0x + 1$  can change the value of the variable expression for certain values of  $x$ .
55. There's no need to apply the long-division process to this problem because I can work the problem in my head and see that the quotient must be  $x^2 + 1$ .
56. The degree of the quotient must be  $3 - 1$ .

In Exercises 57–60, determine whether each statement is true or false. If the statement is false, make the necessary change(s) to produce a true statement.

57. If  $4x^2 + 25x - 3$  is divided by  $4x + 1$ , the remainder is 9.

58. If polynomial division results in a remainder of zero, then the product of the divisor and the quotient is the dividend.
59. A nonzero remainder indicates that the answer to a polynomial long-division problem is not a polynomial.
60. When a polynomial is divided by a binomial, the division process stops when the last term of the dividend is brought down.
61. When a certain polynomial is divided by  $2x + 4$ , the quotient is

$$x - 3 + \frac{17}{2x + 4}.$$

What is the polynomial?

62. Find the number  $k$  such that when  $16x^2 - 2x + k$  is divided by  $2x - 1$ , the remainder is 0.
63. Describe the pattern that you observe in the following quotients and remainders.

$$\frac{x^3 - 1}{x + 1} = x^2 - x + 1 - \frac{2}{x + 1}$$

$$\frac{x^5 - 1}{x + 1} = x^4 - x^3 + x^2 - x + 1 - \frac{2}{x + 1}$$

Use this pattern to find  $\frac{x^7 - 1}{x + 1}$ . Verify your result by dividing.

### Technology Exercises

In Exercises 64–68, use a graphing utility to determine whether the divisions have been performed correctly. Graph each side of the given equation in the same viewing rectangle. The graphs should coincide. If they do not, correct the expression on the right side by using polynomial division. Then use your graphing utility to show that the division has been performed correctly.

64.  $\frac{x^2 - 4}{x - 2} = x + 2$
65.  $\frac{x^2 - 25}{x - 5} = x - 5$
66.  $\frac{2x^2 + 13x + 15}{x - 5} = 2x + 3$
67.  $\frac{6x^2 + 16x + 8}{3x + 2} = 2x - 4$
68.  $\frac{x^3 + 3x^2 + 5x + 3}{x + 1} = x^2 - 2x + 3$

### Review Exercises

69. Graph the solution set of the system:

$$\begin{cases} 2x - y \geq 4 \\ x - y \leq -1. \end{cases}$$

(Section 4.5, Example 2)



70. What is 6% of 20? (Section 2.4, Example 5)
71. Solve:  $\frac{x}{3} + \frac{2}{5} = \frac{x}{5} - \frac{2}{5}$ . (Section 2.3, Example 4)

**Preview Exercises**

Exercises 72–74 will help you prepare for the material covered in the next section.

72. a. Find the missing exponent, designated by the question mark, in each final step.
- $$\frac{7^3}{7^5} = \frac{7 \cdot 7 \cdot 7}{7 \cdot 7 \cdot 7 \cdot 7 \cdot 7} = \frac{1}{7^?}$$
- $$\frac{7^3}{7^5} = 7^{3-5} = 7^?$$
- b. Based on your two results for  $\frac{7^3}{7^5}$ , what can you conclude?
73. Simplify:  $\frac{(2x^3)^4}{x^{10}}$ .
74. Simplify:  $\left(\frac{x^5}{x^2}\right)^3$ .

**SECTION**

**5.7**

**Negative Exponents and Scientific Notation**

*Bigger than the biggest thing ever and then some. Much bigger than that in fact, really amazingly immense, a totally stunning size, real ‘wow, that’s big’, time ... Gigantic multiplied by colossal multiplied by staggeringly huge is the sort of concept we’re trying to get across here.*

Douglas Adams, *The Restaurant at the End of the Universe*

Although Adams’s description may not quite apply to this \$12.4 trillion national debt, exponents can be used to explore the meaning of this “staggeringly huge” number. In this section, you will learn to use exponents to provide a way of putting large and small numbers in perspective.



**Objectives**

- 1 Use the negative exponent rule.
- 2 Simplify exponential expressions.
- 3 Convert from scientific notation to decimal notation.
- 4 Convert from decimal notation to scientific notation.
- 5 Compute with scientific notation.
- 6 Solve applied problems using scientific notation.

**Negative Integers as Exponents**

A nonzero base can be raised to a negative power. The quotient rule can be used to help determine what a negative integer as an exponent should mean. Consider the quotient of  $b^3$  and  $b^5$ , where  $b$  is not zero. We can determine this quotient in two ways.

$$\frac{b^3}{b^5} = \frac{1 \cdot \cancel{b} \cdot \cancel{b} \cdot \cancel{b}}{\cancel{b} \cdot \cancel{b} \cdot \cancel{b} \cdot b \cdot b} = \frac{1}{b^2}$$

After dividing out pairs of factors, we have two factors of  $b$  in the denominator.

$$\frac{b^3}{b^5} = b^{3-5} = b^{-2}$$

Use the quotient rule and subtract exponents.

Notice that  $\frac{b^3}{b^5}$  equals both  $b^{-2}$  and  $\frac{1}{b^2}$ . This means that  $b^{-2}$  must equal  $\frac{1}{b^2}$ . This example is a special case of the **negative exponent rule**.

- 1** Use the negative exponent rule.

**The Negative Exponent Rule**

If  $b$  is any real number other than 0 and  $n$  is a natural number, then

$$b^{-n} = \frac{1}{b^n}.$$

**Great Question!**

**Does a negative exponent make the value of an expression negative?**

No. For example,

$$7^{-2} = \frac{1}{7^2} = \frac{1}{49}$$

is positive. Avoid these common errors:

**Incorrect!**

~~$$7^{-2} = -7^2$$

$$7^{-2} = -\frac{1}{7^2}$$~~

**EXAMPLE 1** Using the Negative Exponent Rule

Use the negative exponent rule to write each expression with a positive exponent. Then simplify the expression.

- a.  $7^{-2}$       b.  $4^{-3}$       c.  $(-2)^{-4}$       d.  $-2^{-4}$       e.  $5^{-1}$

**Solution**

a.  $7^{-2} = \frac{1}{7^2} = \frac{1}{7 \cdot 7} = \frac{1}{49}$

b.  $4^{-3} = \frac{1}{4^3} = \frac{1}{4 \cdot 4 \cdot 4} = \frac{1}{64}$

c.  $(-2)^{-4} = \frac{1}{(-2)^4} = \frac{1}{(-2)(-2)(-2)(-2)} = \frac{1}{16}$

d.  $-2^{-4} = -\frac{1}{2^4} = -\frac{1}{2 \cdot 2 \cdot 2 \cdot 2} = -\frac{1}{16}$

The negative is not inside parentheses and is not taken to the  $-4$  power.

e.  $5^{-1} = \frac{1}{5^1} = \frac{1}{5}$  ■

**✓ CHECK POINT 1** Use the negative exponent rule to write each expression with a positive exponent. Then simplify the expression.

- a.  $6^{-2}$       b.  $5^{-3}$       c.  $(-3)^{-4}$   
 d.  $-3^{-4}$       e.  $8^{-1}$

Negative exponents can also appear in denominators. For example,

$$\frac{1}{2^{-10}} = \frac{1}{\frac{1}{2^{10}}} = 1 \div \frac{1}{2^{10}} = 1 \cdot \frac{2^{10}}{1} = 2^{10}.$$

In general, if a negative exponent appears in a denominator, an expression can be written with a positive exponent using

$$\frac{1}{b^{-n}} = b^n.$$

For example,

$$\frac{1}{2^{-3}} = 2^3 = 8 \quad \text{and} \quad \frac{1}{(-6)^{-2}} = (-6)^2 = 36.$$

Change only the sign of the exponent and not the sign of the base,  $-6$ .

### Negative Exponents in Numerators and Denominators

If  $b$  is any real number other than 0 and  $n$  is a natural number, then

$$b^{-n} = \frac{1}{b^n} \quad \text{and} \quad \frac{1}{b^{-n}} = b^n.$$

When a negative number appears as an exponent, switch the position of the base (from numerator to denominator or from denominator to numerator) and make the exponent positive. The sign of the base does not change.

### EXAMPLE 2 Using Negative Exponents

Write each expression with positive exponents only. Then simplify, if possible.

a.  $\frac{4^{-3}}{5^{-2}}$       b.  $\left(\frac{3}{4}\right)^{-2}$       c.  $\frac{1}{4x^{-3}}$       d.  $\frac{x^{-5}}{y^{-1}}$

#### Solution

a.  $\frac{4^{-3}}{5^{-2}} = \frac{5^2}{4^3} = \frac{5 \cdot 5}{4 \cdot 4 \cdot 4} = \frac{25}{64}$

Switch the position of the bases and make the exponents positive.

b.  $\left(\frac{3}{4}\right)^{-2} = \frac{3^{-2}}{4^{-2}} = \frac{4^2}{3^2} = \frac{4 \cdot 4}{3 \cdot 3} = \frac{16}{9}$

Switch the position of the bases and make the exponents positive.

c.  $\frac{1}{4x^{-3}} = \frac{x^3}{4}$

Switch the position of the base and make the exponent positive. Note that only  $x$  is raised to the  $-3$  power.

d.  $\frac{x^{-5}}{y^{-1}} = \frac{y^1}{x^5} = \frac{y}{x^5}$  ■

**✓ CHECK POINT 2** Write each expression with positive exponents only. Then simplify, if possible.

a.  $\frac{2^{-3}}{7^{-2}}$       b.  $\left(\frac{4}{5}\right)^{-2}$       c.  $\frac{1}{7y^{-2}}$       d.  $\frac{x^{-1}}{y^{-8}}$

## 2 Simplify exponential expressions.

### Simplifying Exponential Expressions

Properties of exponents are used to simplify exponential expressions. An exponential expression is **simplified** when

- Each base occurs only once.
- No parentheses appear.
- No powers are raised to powers.
- No negative or zero exponents appear.

### Simplifying Exponential Expressions

1. If necessary, be sure that each base appears only once, using

$$b^m \cdot b^n = b^{m+n} \quad \text{or} \quad \frac{b^m}{b^n} = b^{m-n}.$$

**Example**

$$x^4 \cdot x^3 = x^{4+3} = x^7$$

2. If necessary, remove parentheses using

$$(ab)^n = a^n b^n \quad \text{or} \quad \left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}.$$

$$(xy)^3 = x^3 y^3$$

3. If necessary, simplify powers to powers using

$$(b^m)^n = b^{mn}.$$

$$(x^4)^3 = x^{4 \cdot 3} = x^{12}$$

4. If necessary, rewrite exponential expressions with zero powers as 1 ( $b^0 = 1$ ). Furthermore, write the answer with positive exponents using

$$b^{-n} = \frac{1}{b^n} \quad \text{or} \quad \frac{1}{b^{-n}} = b^n.$$

$$\frac{x^5}{x^8} = x^{5-8} = x^{-3} = \frac{1}{x^3}$$

The following examples show how to simplify exponential expressions. In each example, assume that any variable in a denominator is not equal to zero.

### Great Question!

Is the procedure in Example 3 the only way to simplify  $x^{-9} \cdot x^4$ ?

There is often more than one way to simplify an exponential expression. For example, you may prefer to simplify Example 3 as follows:

$$x^{-9} \cdot x^4 = \frac{x^4}{x^9} = x^{4-9} = x^{-5} = \frac{1}{x^5}.$$

#### EXAMPLE 3 Simplifying an Exponential Expression

Simplify:  $x^{-9} \cdot x^4$ .

**Solution**

$$\begin{aligned} x^{-9} \cdot x^4 &= x^{-9+4} && b^m \cdot b^n = b^{m+n} \\ &= x^{-5} && \text{The base, } x, \text{ now appears only once.} \\ &= \frac{1}{x^5} && b^{-n} = \frac{1}{b^n} \quad \blacksquare \end{aligned}$$

**CHECK POINT 3** Simplify:  $x^{-12} \cdot x^2$ .

#### EXAMPLE 4 Simplifying Exponential Expressions

Simplify:

a.  $\frac{x^4}{x^{20}}$       b.  $\frac{25x^6}{5x^8}$       c.  $\frac{10y^7}{-2y^{10}}$

**Solution**

a.  $\frac{x^4}{x^{20}} = x^{4-20} = x^{-16} = \frac{1}{x^{16}}$

b.  $\frac{25x^6}{5x^8} = \frac{25}{5} \cdot \frac{x^6}{x^8} = 5x^{6-8} = 5x^{-2} = \frac{5}{x^2}$

c.  $\frac{10y^7}{-2y^{10}} = \frac{10}{-2} \cdot \frac{y^7}{y^{10}} = -5y^{7-10} = -5y^{-3} = -\frac{5}{y^3} \quad \blacksquare$

**CHECK POINT 4** Simplify:

a.  $\frac{x^2}{x^{10}}$       b.  $\frac{75x^3}{5x^9}$       c.  $\frac{50y^8}{-25y^{14}}$

**EXAMPLE 5** Simplifying an Exponential Expression

Simplify:  $\frac{(5x^3)^2}{x^{10}}$ .

**Solution**

$$\begin{aligned} \frac{(5x^3)^2}{x^{10}} &= \frac{5^2(x^3)^2}{x^{10}} && \text{Raise each factor in the product to the second power using } (ab)^n = a^n b^n. \\ &= \frac{5^2 x^{3 \cdot 2}}{x^{10}} && \text{Raise powers to powers using } (b^m)^n = b^{mn}. \\ &= \frac{25x^6}{x^{10}} && \text{Simplify.} \\ &= 25x^{6-10} && \text{When dividing with the same base, subtract exponents: } \frac{b^m}{b^n} = b^{m-n}. \\ &= 25x^{-4} && \text{Simplify. The base, } x, \text{ now appears only once.} \\ &= \frac{25}{x^4} && \text{Rewrite with a positive exponent using } b^{-n} = \frac{1}{b^n}. \quad \blacksquare \end{aligned}$$

**CHECK POINT 5** Simplify:  $\frac{(6x^4)^2}{x^{11}}$ .

**EXAMPLE 6** Simplifying an Exponential Expression

Simplify:  $\left(\frac{x^5}{x^2}\right)^{-3}$ .

**Solution**

**Method 1.** First perform the division within the parentheses.

$$\begin{aligned} \left(\frac{x^5}{x^2}\right)^{-3} &= (x^{5-2})^{-3} && \text{Within parentheses, divide by subtracting exponents: } \frac{b^m}{b^n} = b^{m-n}. \\ &= (x^3)^{-3} && \text{Simplify. The base, } x, \text{ now appears only once.} \\ &= x^{3(-3)} && \text{Raise powers to powers: } (b^m)^n = b^{mn}. \\ &= x^{-9} && \text{Simplify.} \\ &= \frac{1}{x^9} && \text{Rewrite with a positive exponent using } b^{-n} = \frac{1}{b^n}. \end{aligned}$$

**Method 2.** Remove parentheses first by raising the numerator and the denominator to the  $-3$  power.

$$\begin{aligned} \left(\frac{x^5}{x^2}\right)^{-3} &= \frac{(x^5)^{-3}}{(x^2)^{-3}} && \text{Use } \left(\frac{a}{b}\right)^n = \frac{a^n}{b^n} \text{ and raise the numerator and denominator to the } -3 \text{ power.} \\ &= \frac{x^{5(-3)}}{x^{2(-3)}} && \text{Raise powers to powers using } (b^m)^n = b^{mn}. \\ &= \frac{x^{-15}}{x^{-6}} && \text{Simplify.} \\ &= x^{-15-(-6)} && \text{When dividing with the same base, subtract the exponent in the denominator from the exponent in the numerator: } \frac{b^m}{b^n} = b^{m-n}. \\ &= x^{-9} && \text{Subtract: } -15 - (-6) = -15 + 6 = -9. \text{ The base, } x, \text{ now appears only once.} \\ &= \frac{1}{x^9} && \text{Rewrite with a positive exponent using } b^{-n} = \frac{1}{b^n}. \end{aligned}$$

Which method do you prefer? ■

✓ **CHECK POINT 6** Simplify:  $\left(\frac{x^8}{x^4}\right)^{-5}$ .

**Table 5.1** Names of Large Numbers

$10^2$	hundred
$10^3$	thousand
$10^6$	million
$10^9$	billion
$10^{12}$	trillion
$10^{15}$	quadrillion
$10^{18}$	quintillion
$10^{21}$	sextillion
$10^{24}$	septillion
$10^{27}$	octillion
$10^{30}$	nonillion
$10^{100}$	googol
$10^{\text{googol}}$	googolplex

**3** Convert from scientific notation to decimal notation.

## Scientific Notation

In 2009, the United States government spent more than it had collected in taxes, resulting in a budget deficit of \$1.35 trillion. Put into perspective, the \$1.35 trillion could pay for 40,000 players like Alex Rodriguez, whose \$33 million salary in 2009 made him baseball's richest man. Because a trillion is  $10^{12}$  (see **Table 5.1**), the 2009 budget deficit can be expressed as

$$\$1.35 \times 10^{12}.$$

The number  $1.35 \times 10^{12}$  is written in a form called *scientific notation*.

### Scientific Notation

A positive number is written in **scientific notation** when it is expressed in the form

$$a \times 10^n,$$

where  $a$  is a number greater than or equal to 1 and less than 10 ( $1 \leq a < 10$ ) and  $n$  is an integer.

It is customary to use the multiplication symbol,  $\times$ , rather than a dot, when writing a number in scientific notation.

Here are two examples of numbers in scientific notation:

- In 2010, humankind generated  $1.2 \times 10^{21}$  bytes, or 1.2 zettabytes, of digital information. (Put into perspective, if all 6.8 billion ( $6.8 \times 10^9$ ) people on Earth joined Twitter and continuously tweeted for a century, they would crank out one zettabyte of data.) (Source: LiveScience.com)
- The length of the AIDS virus is  $1.1 \times 10^{-4}$  millimeter.

We can use  $n$ , the exponent on the 10 in  $a \times 10^n$ , to change a number in scientific notation to decimal notation. If  $n$  is **positive**, move the decimal point in  $a$  to the **right**  $n$  places. If  $n$  is **negative**, move the decimal point in  $a$  to the **left**  $|n|$  places.

### EXAMPLE 7 Converting from Scientific to Decimal Notation

Write each number in decimal notation:

a.  $2.6 \times 10^7$       b.  $1.1 \times 10^{-4}$ .

**Solution** In each case, we use the exponent on the 10 to move the decimal point. In part (a), the exponent is positive, so we move the decimal point to the right. In part (b), the exponent is negative, so we move the decimal point to the left.

a.  $2.6 \times 10^7 = 26,000,000$

$n = 7$

Move the decimal point 7 places to the right.

b.  $1.1 \times 10^{-4} = 0.00011$

$n = -4$

Move the decimal point  $|-4|$  places, or 4 places, to the left. ■

**CHECK POINT 7** Write each number in decimal notation:

- a.  $7.4 \times 10^9$                       b.  $3.017 \times 10^{-6}$ .

**4** Convert from decimal notation to scientific notation.

To convert a positive number from decimal notation to scientific notation, we reverse the procedure of Example 7.

**Converting from Decimal to Scientific Notation**

Write the number in the form  $a \times 10^n$ .

- Determine  $a$ , the numerical factor. Move the decimal point in the given number to obtain a number greater than or equal to 1 and less than 10.
- Determine  $n$ , the exponent on  $10^n$ . The absolute value of  $n$  is the number of places the decimal point was moved. The exponent  $n$  is positive if the given number is greater than 10 and negative if the given number is between 0 and 1.

**EXAMPLE 8** Converting from Decimal Notation to Scientific Notation

Write each number in scientific notation:

- a. 4,600,000                      b. 0.000023.

**Solution**

a.  $4,600,000 = 4.6 \times 10^6$

This number is greater than 10, so  $n$  is positive in  $a \times 10^n$ .

Move the decimal point in 4,600,000 to get  $1 \leq a < 10$ .

The decimal point moved 6 places from 4,600,000 to 4.6.

b.  $0.000023 = 2.3 \times 10^{-5}$

This number is less than 1, so  $n$  is negative in  $a \times 10^n$ .

Move the decimal point in 0.000023 to get  $1 \leq a < 10$ .

The decimal point moved 5 places from 0.000023 to 2.3. ■

**CHECK POINT 8** Write each number in scientific notation:

- a. 7,410,000,000                      b. 0.000000092.

**5** Compute with scientific notation.

**Computations with Scientific Notation**

Properties of exponents are used to perform computations with numbers that are expressed in scientific notation.

**Using Technology**

You can use your calculator's **EE** (enter exponent) or **EXP** key to convert from decimal to scientific notation. Here is how it's done for 0.000023:

**Many Scientific Calculators**

Keystrokes	Display
.000023 <b>EE</b> <b>=</b>	2.3 - 05

**Many Graphing Calculators**

Use the mode setting for scientific notation.

Keystrokes	Display
.000023 <b>ENTER</b>	2.3E-5



### Computations with Numbers in Scientific Notation

#### Multiplication

$$(a \times 10^n) \times (b \times 10^m) = (a \times b) \times 10^{n+m}$$

Add the exponents on 10 and multiply the other parts of the numbers separately.

#### Division

$$\frac{a \times 10^n}{b \times 10^m} = \left(\frac{a}{b}\right) \times 10^{n-m}$$

Subtract the exponents on 10 and divide the other parts of the numbers separately.

#### Exponentiation

$$(a \times 10^n)^m = a^m \times 10^{nm}$$

Multiply exponents on 10 and raise the other part of the number to the power.

After the computation is completed, the answer may require an adjustment before it is expressed in scientific notation.

### Using Technology

$(4 \times 10^5)(2 \times 10^9)$  On a Calculator:

#### Many Scientific Calculators

4 [EE] 5 [×] 2 [EE] 9 [=]

Display: 8.14

#### Many Graphing Calculators

4 [EE] 5 [×] 2 [EE] 9 [ENTER]

Display: 8E14

### EXAMPLE 9 Computations with Scientific Notation

Perform the indicated computations, writing the answers in scientific notation:

- a.  $(4 \times 10^5)(2 \times 10^9)$       b.  $\frac{1.2 \times 10^6}{4.8 \times 10^{-3}}$       c.  $(5 \times 10^{-4})^3$ .

#### Solution

a.  $(4 \times 10^5)(2 \times 10^9) = (4 \times 2) \times (10^5 \times 10^9)$   
 $= 8 \times 10^{5+9}$   
 $= 8 \times 10^{14}$

Regroup.

Add the exponents on 10 and multiply the other parts.

Simplify.

b.  $\frac{1.2 \times 10^6}{4.8 \times 10^{-3}} = \left(\frac{1.2}{4.8}\right) \times \left(\frac{10^6}{10^{-3}}\right)$   
 $= 0.25 \times 10^{6-(-3)}$

Regroup.

Subtract the exponents on 10 and divide the other parts.

Simplify. Because 0.25 is not between 1 and 10, it must be written in scientific notation.

$$0.25 = 2.5 \times 10^{-1}$$

Add the exponents on 10.

Simplify.

$(ab)^n = a^n b^n$ . Cube each factor in parentheses.

Multiply the exponents and cube the other part of the number.

Simplify. 125 must be written in scientific notation.

$$125 = 1.25 \times 10^2$$

Add the exponents on 10.

Simplify. ■

c.  $(5 \times 10^{-4})^3 = 5^3 \times (10^{-4})^3$   
 $= 125 \times 10^{-4 \cdot 3}$   
 $= 125 \times 10^{-12}$   
 $= 1.25 \times 10^2 \times 10^{-12}$   
 $= 1.25 \times 10^{2+(-12)}$   
 $= 1.25 \times 10^{-10}$

**✓ CHECK POINT 9** Perform the indicated computations, writing the answers in scientific notation:

a.  $(3 \times 10^8)(2 \times 10^2)$

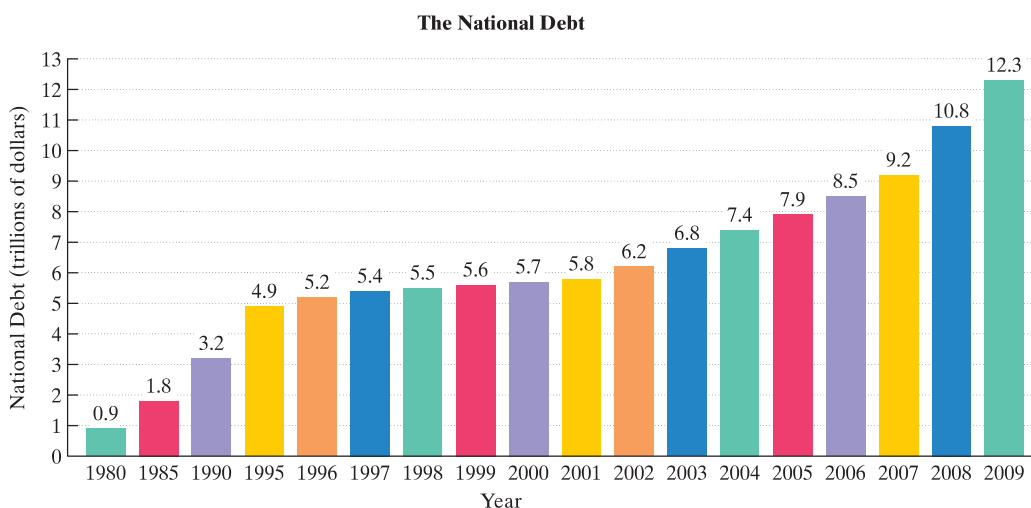
b.  $\frac{8.4 \times 10^7}{4 \times 10^{-4}}$

c.  $(4 \times 10^{-2})^3$ .

**6** Solve applied problems using scientific notation.

### Applications: Putting Numbers in Perspective

Due to tax cuts and spending increases, the United States began accumulating large deficits in the 1980s. To finance the deficit, the government had borrowed \$12.3 trillion as of December 2009. The graph in **Figure 5.8** shows the national debt increasing over time.



**Figure 5.8**  
Source: Office of Management and Budget

Example 10 shows how we can use scientific notation to comprehend the meaning of a number such as 12.3 trillion.

#### EXAMPLE 10 The National Debt

As of December 2009, the national debt was \$12.3 trillion, or  $12.3 \times 10^{12}$  dollars. At that time, the U.S. population was approximately 307,000,000 (307 million), or  $3.07 \times 10^8$ . If the national debt was evenly divided among every individual in the United States, how much would each citizen have to pay?

**Solution** Although it is not necessary to do so, let's express the total debt,  $12.3 \times 10^{12}$  dollars, in scientific notation as  $1.23 \times 10^{13}$ . The amount each citizen must pay is the total debt,  $1.23 \times 10^{13}$  dollars, divided by the number of citizens,  $3.07 \times 10^8$ .

$$\begin{aligned} \frac{1.23 \times 10^{13}}{3.07 \times 10^8} &= \left(\frac{1.23}{3.07}\right) \times \left(\frac{10^{13}}{10^8}\right) \\ &\approx 0.401 \times 10^{13-8} \\ &= 0.401 \times 10^5 \\ &= (4.01 \times 10^{-1}) \times 10^5 \\ &= 4.01 \times 10^4 \\ &= 40,100 \end{aligned}$$

Every U.S. citizen would have to pay approximately \$40,100 to the federal government to pay off the national debt. ■

#### Using Technology

Here is the keystroke sequence for solving Example 10 using a calculator:

1.23 **EE** 13 **÷** 3.07 **EE** 8.

The quotient is displayed by pressing **=** on a scientific calculator or **ENTER** on a graphing calculator. The answer can be displayed in scientific or decimal notation. Consult your manual.

If a number is written in scientific notation,  $a \times 10^n$ , the digits in  $a$  are called **significant digits**.

National Debt:  $1.23 \times 10^{13}$       U.S. Population:  $3.07 \times 10^8$

Three significant digits

Three significant digits

Because these were the given numbers in Example 10, we rounded the answer,  $4.01 \times 10^4$ , to three significant digits. When multiplying or dividing in scientific notation where rounding is necessary and rounding instructions are not given, **round the scientific notation answer to the least number of significant digits found in any of the given numbers**.

**CHECK POINT 10** The cost of President Obama's 2009 economic stimulus package was \$787 billion, or  $7.87 \times 10^{11}$  dollars. If this cost were evenly divided among every individual in the United States (approximately  $3.07 \times 10^8$  people), how much would each citizen have to pay?

## Blitzer Bonus

### Seven Ways to Spend \$1 Trillion

Confronting a national debt of \$12.3 trillion starts with grasping just how colossal \$1 trillion ( $1 \times 10^{12}$ ) actually is. To help you wrap your head around this mind-boggling number, and to put the national debt in further perspective, consider what \$1 trillion will buy:

- 40,816,326 new cars based on an average sticker price of \$24,500 each
- 5,574,136 homes based on the national median price of \$179,400 for existing single-family homes
- one year's salary for 14.7 million teachers based on the average teacher salary of \$68,000 in California
- the annual salaries of all 535 members of Congress for the next 10,742 years based on current salaries of \$174,000 per year
- the salary of basketball superstar LeBron James for 50,000 years based on an annual salary of \$20 million
- annual base pay for 59.5 million U.S. privates (that's 100 times the total number of active-duty soldiers in the Army) based on basic pay of \$16,794 per year
- salaries to hire all 2.8 million residents of the state of Kansas in full-time minimum-wage jobs for the next 23 years based on the federal minimum wage of \$7.25 per hour

Source: Kiplinger.com

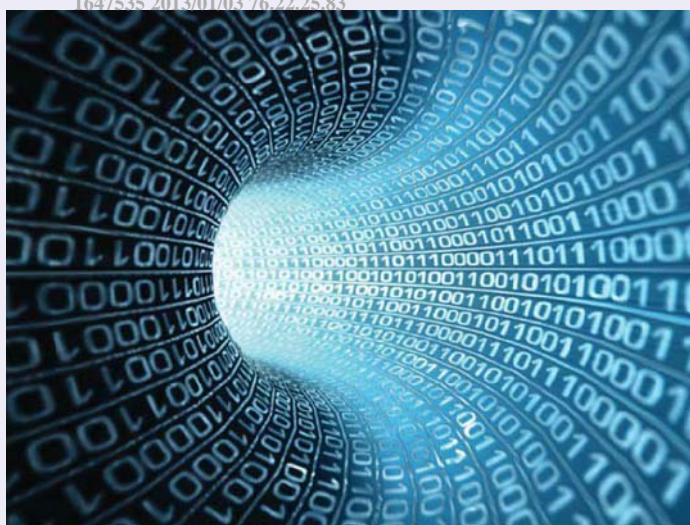


Image © photobank.kiev.ua, 2009

## Achieving Success

**Form a study group with other students in your class.** Working in small groups often serves as an excellent way to learn and reinforce new material. Set up helpful procedures and guidelines for the group. “Talk” math by discussing and explaining the concepts and exercises to one another.



45.  $\frac{x^3}{(x^4)^2}$

47.  $\frac{(y^4)^2}{(4x^3)^2}$

49.  $\frac{x^8}{(6y^4)^3}$

51.  $\frac{(6y^4)^3}{y^{-5}}$

53.  $\left(\frac{x^4}{x^2}\right)^{-3}$

55.  $\left(\frac{4x^5}{2x^2}\right)^{-4}$

57.  $(3x^{-1})^{-2}$

59.  $(-2y^{-1})^{-3}$

61.  $\frac{2x^5 \cdot 3x^7}{15x^6}$

63.  $(x^3)^5 \cdot x^{-7}$

65.  $(2y^3)^4 y^{-6}$

67.  $\frac{(y^3)^4}{(y^2)^7}$

69.  $(y^{10})^{-5}$

71.  $(a^4 b^5)^{-3}$

73.  $(a^{-2} b^6)^{-4}$

75.  $\left(\frac{x^2}{2}\right)^{-2}$

77.  $\left(\frac{x^2}{y^3}\right)^{-3}$

46.  $\frac{x^5}{(x^3)^2}$

48.  $\frac{(y^3)^2}{(5x^3)^2}$

50.  $\frac{x^7}{(4y^5)^3}$

52.  $\frac{(4y^5)^3}{y^{-4}}$

54.  $\left(\frac{x^6}{x^2}\right)^{-3}$

56.  $\left(\frac{6x^7}{2x^2}\right)^{-4}$

58.  $(4x^{-1})^{-2}$

60.  $(-3y^{-1})^{-3}$

62.  $\frac{3x^3 \cdot 5x^{14}}{20x^{14}}$

64.  $(x^4)^3 \cdot x^{-5}$

66.  $(3y^4)^3 y^{-7}$

68.  $\frac{(y^2)^5}{(y^3)^4}$

70.  $(y^{20})^{-5}$

72.  $(a^5 b^3)^{-4}$

74.  $(a^{-7} b^2)^{-5}$

76.  $\left(\frac{x^2}{2}\right)^{-3}$

78.  $\left(\frac{x^3}{y^2}\right)^{-4}$

In Exercises 79–90, write each number in decimal notation without the use of exponents.

79.  $8.7 \times 10^2$

81.  $9.23 \times 10^5$

83.  $3.4 \times 10^0$

85.  $7.9 \times 10^{-1}$

87.  $2.15 \times 10^{-2}$

89.  $7.86 \times 10^{-4}$

80.  $2.75 \times 10^3$

82.  $7.24 \times 10^4$

84.  $9.115 \times 10^0$

86.  $8.6 \times 10^{-1}$

88.  $3.14 \times 10^{-2}$

90.  $4.63 \times 10^{-5}$

In Exercises 91–106, write each number in scientific notation.

91. 32,400

93. 220,000,000

95. 713

97. 6751

99. 0.0027

101. 0.0000202

103. 0.005

105. 3.14159

92. 327,000

94. 370,000,000,000

96. 623

98. 9832

100. 0.00083

102. 0.00000103

104. 0.006

106. 2.71828

In Exercises 107–126, perform the indicated computations. Write the answers in scientific notation.

107.  $(2 \times 10^3)(3 \times 10^2)$

108.  $(3 \times 10^4)(3 \times 10^2)$

109.  $(2 \times 10^5)(8 \times 10^3)$

110.  $(4 \times 10^3)(5 \times 10^4)$

111.  $\frac{12 \times 10^6}{4 \times 10^2}$

113.  $\frac{15 \times 10^4}{5 \times 10^{-2}}$

115.  $\frac{15 \times 10^{-4}}{5 \times 10^2}$

117.  $\frac{180 \times 10^6}{2 \times 10^3}$

119.  $\frac{3 \times 10^4}{12 \times 10^{-3}}$

121.  $(5 \times 10^2)^3$

123.  $(3 \times 10^{-2})^4$

125.  $(4 \times 10^6)^{-1}$

112.  $\frac{20 \times 10^{20}}{10 \times 10^{10}}$

114.  $\frac{18 \times 10^2}{9 \times 10^{-3}}$

116.  $\frac{18 \times 10^{-2}}{9 \times 10^3}$

118.  $\frac{180 \times 10^8}{2 \times 10^4}$

120.  $\frac{5 \times 10^2}{20 \times 10^{-3}}$

122.  $(4 \times 10^3)^2$

124.  $(2 \times 10^{-3})^5$

126.  $(5 \times 10^4)^{-1}$

**Practice PLUS**

In Exercises 127–134, simplify each exponential expression. Assume that variables represent nonzero real numbers.

127.  $\frac{(x^{-2}y)^{-3}}{(x^2y^{-1})^3}$

128.  $\frac{(xy^{-2})^{-2}}{(x^{-2}y)^{-3}}$

129.  $(2x^{-3}yz^{-6})(2x)^{-5}$

130.  $(3x^{-4}yz^{-7})(3x)^{-3}$

131.  $\left(\frac{x^3y^4z^5}{x^{-3}y^{-4}z^{-5}}\right)^{-2}$

132.  $\left(\frac{x^4y^5z^6}{x^{-4}y^{-5}z^{-6}}\right)^{-4}$

133.  $\frac{(2^{-1}x^{-2}y^{-1})^{-2}(2x^{-4}y^3)^{-2}(16x^{-3}y^3)^0}{(2x^{-3}y^{-5})^2}$

134.  $\frac{(2^{-1}x^{-3}y^{-1})^{-2}(2x^{-6}y^4)^{-2}(9x^3y^{-3})^0}{(2x^{-4}y^{-6})^2}$

In Exercises 135–138, perform the indicated computations. Express answers in scientific notation.

135.  $(5 \times 10^3)(1.2 \times 10^{-4}) \div (2.4 \times 10^2)$

136.  $(2 \times 10^2)(2.6 \times 10^{-3}) \div (4 \times 10^3)$

137.  $\frac{(1.6 \times 10^4)(7.2 \times 10^{-3})}{(3.6 \times 10^8)(4 \times 10^{-3})}$

138.  $\frac{(1.2 \times 10^6)(8.7 \times 10^{-2})}{(2.9 \times 10^6)(3 \times 10^{-3})}$

### Application Exercises

We have seen that in 2009, the United States government spent more than it had collected in taxes, resulting in a budget deficit of \$1.35 trillion. In Exercises 139–142, you will use scientific notation to put a number like 1.35 trillion in perspective.

139. a. Express 1.35 trillion in scientific notation.  
b. Express the 2009 U.S. population, 307 million, in scientific notation.  
c. Use your scientific notation answers from parts (a) and (b) to answer this question: If the 2009 budget deficit was evenly divided among every individual in the United States, how much would each citizen have to pay? Express the answer in scientific and decimal notations.
140. a. Express 1.35 trillion in scientific notation.  
b. A trip around the world at the Equator is approximately 25,000 miles. Express this number in scientific notation.  
c. Use your scientific notation answers from parts (a) and (b) to answer this question: How many times can you circle the world at the Equator by traveling 1.35 trillion miles?
141. If there are approximately  $3.2 \times 10^7$  seconds in a year, approximately how many years is 1.35 trillion seconds? (Note: 1.35 trillion seconds would take us back in time to a period when Neanderthals were using stones to make tools.)
142. The Washington Monument, overlooking the U.S. Capitol, stands about 555 feet tall. Stacked end to end, how many monuments would it take to reach 1.35 trillion feet? (Note: That's more than twice the distance from Earth to the sun.)

Use the motion formula  $d = rt$ , distance equals rate times time, and the fact that light travels at the rate of  $1.86 \times 10^5$  miles per second, to solve Exercises 143–144.

143. If the moon is approximately  $2.325 \times 10^5$  miles from Earth, how many seconds does it take moonlight to reach Earth?
144. If the sun is approximately  $9.14 \times 10^7$  miles from Earth, how many seconds, to the nearest tenth of a second, does it take sunlight to reach Earth?
145. Refer to the Blitzer Bonus on page 405. Use scientific notation to verify any three of the bulleted items on ways to spend \$1 trillion.

### Writing in Mathematics

146. Explain the negative exponent rule and give an example.
147. How do you know if an exponential expression is simplified?
148. How do you know if a number is written in scientific notation?

149. Explain how to convert from scientific to decimal notation and give an example.
150. Explain how to convert from decimal to scientific notation and give an example.
151. Describe one advantage of expressing a number in scientific notation over decimal notation.

### Critical Thinking Exercises

**Make Sense?** In Exercises 152–155, determine whether each statement “makes sense” or “does not make sense” and explain your reasoning.

152. There are many exponential expressions that are equal to  $36x^{12}$ , such as  $(6x^6)^2$ ,  $(6x^3)(6x^9)$ ,  $36(x^3)^9$ , and  $6^2(x^2)^6$ .
153. If  $5^{-2}$  is raised to the third power, the result is a number between 0 and 1.
154. The population of Colorado is approximately  $4.6 \times 10^{12}$ .
155. I wrote a number where there is no advantage to using scientific notation instead of decimal notation.

In Exercises 156–163, determine whether each statement is true or false. If the statement is false, make the necessary change(s) to produce a true statement.

156.  $4^{-2} < 4^{-3}$                       157.  $5^{-2} > 2^{-5}$
158.  $(-2)^4 = 2^{-4}$                       159.  $5^2 \cdot 5^{-2} > 2^5 \cdot 2^{-5}$
160.  $534.7 = 5.347 \times 10^3$
161.  $\frac{8 \times 10^{30}}{4 \times 10^{-5}} = 2 \times 10^{25}$
162.  $(7 \times 10^5) + (2 \times 10^{-3}) = 9 \times 10^2$
163.  $(4 \times 10^3) + (3 \times 10^2) = 4.3 \times 10^3$
164. The mad Dr. Frankenstein has gathered enough bits and pieces (so to speak) for  $2^{-1} + 2^{-2}$  of his creature-to-be. Write a fraction that represents the amount of his creature that must still be obtained.

### Technology Exercises

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165. Use a calculator in a fraction mode to check any five of your answers in Exercises 1–22.
166. Use a calculator to check any three of your answers in Exercises 79–90.
167. Use a calculator to check any three of your answers in Exercises 91–106.
168. Use a calculator with an **EE** or **EXP** key to check any four of your computations in Exercises 107–126. Display the result of the computation in scientific notation.



### Review Exercises

169. Solve:  $8 - 6x > 4x - 12$ . (Section 2.7, Example 7)
170. Simplify:  $24 \div 8 \cdot 3 + 28 \div (-7)$ . (Section 1.8, Example 8)
171. List the whole numbers in this set:

$$\left\{-4, -\frac{1}{5}, 0, \pi, \sqrt{16}, \sqrt{17}\right\}.$$

(Section 1.3, Example 5)

### Preview Exercises

Exercises 172–174 will help you prepare for the material covered in the first section of the next chapter. In each exercise, find the product.

172.  $4x^3(4x^2 - 3x + 1)$
173.  $9xy(3xy^2 - y + 9)$
174.  $(x + 3)(x^2 + 5)$

## GROUP PROJECT

# CHAPTER 5

A large number can be put into perspective by comparing it with another number. For example, we put the \$12.3 trillion national debt (Example 10) into perspective by comparing this number to the number of U.S. citizens. In Exercises 139–142, we put the \$1.35 trillion budget deficit into perspective by comparing 1.35 trillion to the number of U.S. citizens, the distance around the world, the number of seconds in a year, and the height of the Washington Monument.

For this project, each group member should consult an almanac, a newspaper, or the Internet to find a number greater than one million. Explain to other members of the group the context in which the large number is used. Express the number in scientific notation. Then put the number into perspective by comparing it with another number.

## Chapter 5 Summary

### Definitions and Concepts

### Examples

#### Section 5.1 Adding and Subtracting Polynomials

A polynomial is a single term or the sum of two or more terms containing variables with whole number exponents. A monomial is a polynomial with exactly one term; a binomial has exactly two terms; a trinomial has exactly three terms. The degree of a polynomial is the highest power of all the terms. The standard form of a polynomial is written in descending powers of the variable.

Polynomials

Monomial:  $2x^5$   
Degree is 5.

Binomial:  $6x^3 + 5x$   
Degree is 3.

Trinomial:  $7x + 4x^2 - 5$   
Degree is 2.

To add polynomials, add like terms.

$$\begin{aligned} &(6x^3 + 5x^2 - 7x) + (-9x^3 + x^2 + 6x) \\ &= (6x^3 - 9x^3) + (5x^2 + x^2) + (-7x + 6x) \\ &= -3x^3 + 6x^2 - x \end{aligned}$$

The opposite, or additive inverse, of a polynomial is that polynomial with the sign of every coefficient changed. To subtract two polynomials, add the first polynomial and the opposite of the polynomial being subtracted.

$$\begin{aligned} &(5y^3 - 9y^2 - 4) - (3y^3 - 12y^2 - 5) \\ &= (5y^3 - 9y^2 - 4) + (-3y^3 + 12y^2 + 5) \\ &= (5y^3 - 3y^3) + (-9y^2 + 12y^2) + (-4 + 5) \\ &= 2y^3 + 3y^2 + 1 \end{aligned}$$



**Definitions and Concepts**

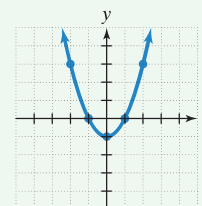
**Examples**

**Section 5.1 Adding and Subtracting Polynomials (continued)**

The graphs of equations defined by polynomials of degree 2, shaped like bowls or inverted bowls, can be obtained using the point-plotting method.

Graph:  $y = x^2 - 1$ .

$x$	$y = x^2 - 1$
-2	$(-2)^2 - 1 = 3$
-1	$(-1)^2 - 1 = 0$
0	$0^2 - 1 = -1$
1	$1^2 - 1 = 0$
2	$2^2 - 1 = 3$



**Section 5.2 Multiplying Polynomials**

**Properties of Exponents**

**Product Rule:**  $b^m \cdot b^n = b^{m+n}$

**Power Rule:**  $(b^m)^n = b^{mn}$

**Products to Powers:**  $(ab)^n = a^n b^n$

$$\begin{aligned} x^3 \cdot x^8 &= x^{3+8} = x^{11} \\ (x^3)^8 &= x^{3 \cdot 8} = x^{24} \\ (-5x^2)^3 &= (-5)^3(x^2)^3 = -125x^6 \end{aligned}$$

To multiply monomials, multiply coefficients and add exponents.

$$(-6x^4)(3x^{10}) = -6 \cdot 3x^{4+10} = -18x^{14}$$

To multiply a monomial and a polynomial, multiply each term of the polynomial by the monomial.

$$\begin{aligned} &2x^4(3x^2 - 6x + 5) \\ &= 2x^4 \cdot 3x^2 - 2x^4 \cdot 6x + 2x^4 \cdot 5 \\ &= 6x^6 - 12x^5 + 10x^4 \end{aligned}$$

To multiply polynomials when neither is a monomial, multiply each term of one polynomial by each term of the other polynomial. Then combine like terms.

$$\begin{aligned} &(2x + 3)(5x^2 - 4x + 2) \\ &= 2x(5x^2 - 4x + 2) + 3(5x^2 - 4x + 2) \\ &= 10x^3 - 8x^2 + 4x + 15x^2 - 12x + 6 \\ &= 10x^3 + 7x^2 - 8x + 6 \end{aligned}$$

**Section 5.3 Special Products**

The FOIL method may be used when multiplying two binomials: First terms multiplied. Outside terms multiplied. Inside terms multiplied. Last terms multiplied.

$$\begin{aligned} (3x + 7)(2x - 5) &= \overset{\text{F}}{3x} \cdot \overset{\text{O}}{2x} + \overset{\text{I}}{3x} \cdot \overset{\text{L}}{(-5)} + \overset{\text{O}}{7} \cdot \overset{\text{I}}{2x} + \overset{\text{L}}{7} \cdot \overset{\text{O}}{(-5)} \\ &= 6x^2 - 15x + 14x - 35 \\ &= 6x^2 - x - 35 \end{aligned}$$

**The Product of the Sum and Difference of Two Terms**

$$(A + B)(A - B) = A^2 - B^2$$

$$\begin{aligned} (4x + 7)(4x - 7) &= (4x)^2 - 7^2 \\ &= 16x^2 - 49 \end{aligned}$$

**Definitions and Concepts**

**Examples**

**Section 5.3 Special Products (continued)**

**The Square of a Binomial Sum**

$$(A + B)^2 = A^2 + 2AB + B^2$$

$$(x^2 + 6)^2 = (x^2)^2 + 2 \cdot x^2 \cdot 6 + 6^2 = x^4 + 12x^2 + 36$$

**The Square of a Binomial Difference**

$$(A - B)^2 = A^2 - 2AB + B^2$$

$$(9x - 3)^2 = (9x)^2 - 2 \cdot 9x \cdot 3 + 3^2 = 81x^2 - 54x + 9$$

**Section 5.4 Polynomials in Several Variables**

To evaluate a polynomial in several variables, substitute the given value for each variable and perform the resulting computation.

Evaluate  $4x^2y + 3xy - 2x$  for  $x = -1$  and  $y = -3$ .

$$\begin{aligned} 4x^2y + 3xy - 2x &= 4(-1)^2(-3) + 3(-1)(-3) - 2(-1) \\ &= 4(1)(-3) + 3(-1)(-3) - 2(-1) \\ &= -12 + 9 + 2 = -1 \end{aligned}$$

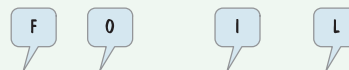
For a polynomial in two variables, the degree of a term is the sum of the exponents on its variables. The degree of the polynomial is the highest degree of all its terms.

$$7x^2y + 12x^4y^3 - 17x^5 + 6$$

Degree of polynomial = 7

Polynomials in several variables are added, subtracted, and multiplied using the same rules for polynomials in one variable.

$$\begin{aligned} (5x^2y^3 - xy + 4y^2) - (8x^2y^3 - 6xy - 2y^2) &= (5x^2y^3 - xy + 4y^2) + (-8x^2y^3 + 6xy + 2y^2) \\ &= (5x^2y^3 - 8x^2y^3) + (-xy + 6xy) + (4y^2 + 2y^2) \\ &= -3x^2y^3 + 5xy + 6y^2 \end{aligned}$$



$$\begin{aligned} (3x - 2y)(x - y) &= 3x \cdot x + 3x(-y) + (-2y)x + (-2y)(-y) \\ &= 3x^2 - 3xy - 2xy + 2y^2 \\ &= 3x^2 - 5xy + 2y^2 \end{aligned}$$

**Section 5.5 Dividing Polynomials**

**Additional Properties of Exponents**

**Quotient Rule:**  $\frac{b^m}{b^n} = b^{m-n}$ ,  $b \neq 0$

**Zero-Exponent Rule:**  $b^0 = 1$ ,  $b \neq 0$

**Quotients to Powers:**  $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$ ,  $b \neq 0$

$$\frac{x^{12}}{x^4} = x^{12-4} = x^8$$

$$(-3)^0 = 1 \quad -3^0 = -(3^0) = -1$$

$$\left(\frac{y^2}{4}\right)^3 = \frac{(y^2)^3}{4^3} = \frac{y^{2 \cdot 3}}{4 \cdot 4 \cdot 4} = \frac{y^6}{64}$$

To divide monomials, divide coefficients and subtract exponents.

$$\frac{-40x^{40}}{20x^{20}} = \frac{-40}{20}x^{40-20} = -2x^{20}$$

**Definitions and Concepts**

**Examples**

**Section 5.5 Dividing Polynomials (continued)**

To divide a polynomial by a monomial, divide each term of the polynomial by the monomial.

$$\begin{aligned} & \frac{8x^6 - 4x^3 + 10x}{2x} \\ &= \frac{8x^6}{2x} - \frac{4x^3}{2x} + \frac{10x}{2x} \\ &= 4x^{6-1} - 2x^{3-1} + 5x^{1-1} = 4x^5 - 2x^2 + 5 \end{aligned}$$

**Section 5.6 Dividing Polynomials by Binomials**

To divide a polynomial by a binomial, begin by arranging the polynomial in descending powers of the variable. If a power of a variable is missing, add that power with a coefficient of 0. Repeat the four steps—divide, multiply, subtract, bring down the next term—until the degree of the remainder is less than the degree of the divisor.

Divide:  $\frac{10x^2 + 13x + 8}{2x + 3}$ .

$$\begin{array}{r} 5x - 1 + \frac{11}{2x + 3} \\ 2x + 3 \overline{)10x^2 + 13x + 8} \\ \underline{10x^2 + 15x} \phantom{+ 8} \\ -2x + 8 \\ \underline{-2x - 3} \\ 11 \end{array}$$

**Section 5.7 Negative Exponents and Scientific Notation**

**Negative Exponents in Numerators and Denominators**

If  $b \neq 0$ ,  $b^{-n} = \frac{1}{b^n}$  and  $\frac{1}{b^{-n}} = b^n$ .

$$6^2 = \frac{1}{6^{-2}} = \frac{1}{\frac{1}{36}}$$

$$\frac{1}{(-2)^{-4}} = (-2)^4 = 16$$

$$\left(\frac{2}{3}\right)^{-3} = \frac{2^{-3}}{3^{-3}} = \frac{3^3}{2^3} = \frac{27}{8}$$

An exponential expression is simplified when

- Each base occurs only once.
- No parentheses appear.
- No powers are raised to powers.
- No negative or zero exponents appear.

Simplify:  $\frac{(2x^4)^3}{x^{18}}$ .

$$\frac{(2x^4)^3}{x^{18}} = \frac{2^3(x^4)^3}{x^{18}} = \frac{8x^{4 \cdot 3}}{x^{18}} = \frac{8x^{12}}{x^{18}} = 8x^{12-18} = 8x^{-6} = \frac{8}{x^6}$$

A positive number in scientific notation is expressed as  $a \times 10^n$ , where  $1 \leq a < 10$  and  $n$  is an integer.

Write  $2.9 \times 10^{-3}$  in decimal notation.

$$2.9 \times 10^{-3} = .0029 = 0.0029$$

Write 16,000 in scientific notation.

$$16,000 = 1.6 \times 10^4$$

Use properties of exponents with base 10

$$10^m \cdot 10^n = 10^{m+n}, \quad \frac{10^m}{10^n} = 10^{m-n}, \quad \text{and} \quad (10^m)^n = 10^{mn}$$

to perform computations with scientific notation.

$$\begin{aligned} & (5 \times 10^3)(4 \times 10^{-8}) \\ &= 5 \cdot 4 \times 10^{3-8} \\ &= 20 \times 10^{-5} \\ &= 2 \times 10^1 \times 10^{-5} = 2 \times 10^{-4} \end{aligned}$$

## CHAPTER 5 REVIEW EXERCISES

**5.1** In Exercises 1–3, identify each polynomial as a monomial, binomial, or trinomial. Give the degree of the polynomial.

- $7x^4 + 9x$
- $3x + 5x^2 - 2$
- $16x$

In Exercises 4–8, add or subtract as indicated.

- $(-6x^3 + 7x^2 - 9x + 3) + (14x^3 + 3x^2 - 11x - 7)$
- $(9y^3 - 7y^2 + 5) + (4y^3 - y^2 + 7y - 10)$
- $(5y^2 - y - 8) - (-6y^2 + 3y - 4)$
- $(13x^4 - 8x^3 + 2x^2) - (5x^4 - 3x^3 + 2x^2 - 6)$
- Subtract  $x^4 + 7x^2 - 11x$  from  $-13x^4 - 6x^2 + 5x$ .

In Exercises 9–11, add or subtract as indicated.

- Add.  $7y^4 - 6y^3 + 4y^2 - 4y$   
 $\quad\quad\quad y^3 - y^2 + 3y - 4$
- Subtract.  $7x^2 - 9x + 2$   
 $\quad\quad\quad -(4x^2 - 2x - 7)$
- Subtract.  $5x^3 - 6x^2 - 9x + 14$   
 $\quad\quad\quad -(-5x^3 + 3x^2 - 11x + 3)$

In Exercises 12–13, graph each equation.

- $y = x^2 + 3$
- $y = 1 - x^2$

**5.2** In Exercises 14–18, simplify each expression.

- $x^{20} \cdot x^3$
- $y \cdot y^5 \cdot y^8$
- $(x^{20})^5$
- $(10y)^2$
- $(-4x^{10})^3$

In Exercises 19–27, find each product.

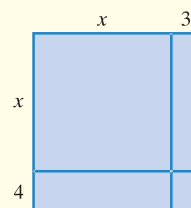
- $(5x)(10x^3)$
- $(-12y^7)(3y^4)$
- $(-2x^3)(-3x^4)(5x^3)$
- $7x(3x^2 + 9)$
- $5x^3(4x^2 - 11x)$
- $3y^2(-7y^2 + 3y - 6)$
- $2y^5(8y^3 - 10y^2 + 1)$
- $(x + 3)(x^2 - 5x + 2)$
- $(3y - 2)(4y^2 + 3y - 5)$

In Exercises 28–29, use a vertical format to find each product.

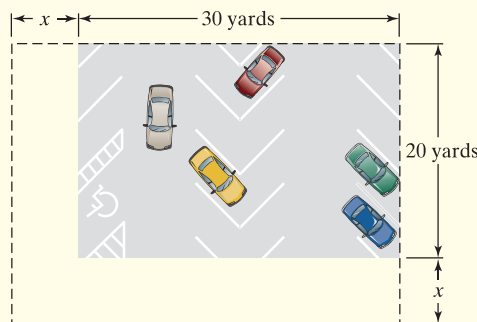
- $y^2 - 4y + 7$   
 $\quad\quad\quad 3y - 5$
- $4x^3 - 2x^2 - 6x - 1$   
 $\quad\quad\quad 2x + 3$

**5.3** In Exercises 30–42, find each product.

- $(x + 6)(x + 2)$
- $(3y - 5)(2y + 1)$
- $(4x^2 - 2)(x^2 - 3)$
- $(5x + 4)(5x - 4)$
- $(7 - 2y)(7 + 2y)$
- $(y^2 + 1)(y^2 - 1)$
- $(x + 3)^2$
- $(3y + 4)^2$
- $(y - 1)^2$
- $(5y - 2)^2$
- $(x^2 + 4)^2$
- $(x^2 + 4)(x^2 - 4)$
- $(x^2 + 4)(x^2 - 5)$
- Write a polynomial in descending powers of  $x$  that represents the area of the shaded region.



- The parking garage shown in the figure measures 30 yards by 20 yards. The length and the width are each increased by a fixed amount,  $x$  yards. Write a trinomial that describes the area of the expanded garage.



**5.4**

- Evaluate  $2x^3y - 4xy^2 + 5y + 6$  for  $x = -1$  and  $y = 2$ .
- Determine the coefficient of each term, the degree of each term, and the degree of the polynomial:  
 $4x^2y + 9x^3y^2 - 17x^4 - 12$ .

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In Exercises 47–56, perform the indicated operations.

47.  $(7x^2 - 8xy + y^2) + (-8x^2 - 9xy + 4y^2)$   
 48.  $(13x^3y^2 - 5x^2y - 9x^2) - (11x^3y^2 - 6x^2y - 3x^2 + 4)$   
 49.  $(-7x^2y^3)(5x^4y^6)$   
 50.  $5ab^2(3a^2b^3 - 4ab)$   
 51.  $(x + 7y)(3x - 5y)$   
 52.  $(4xy - 3)(9xy - 1)$   
 53.  $(3x + 5y)^2$   
 54.  $(xy - 7)^2$   
 55.  $(7x + 4y)(7x - 4y)$   
 56.  $(a - b)(a^2 + ab + b^2)$

**5.5** In Exercises 57–63, simplify each expression.

57.  $\frac{6^{40}}{6^{10}}$                       58.  $\frac{x^{18}}{x^3}$   
 59.  $(-10)^0$                       60.  $-10^0$   
 61.  $400x^0$                       62.  $\left(\frac{x^4}{2}\right)^3$   
 63.  $\left(\frac{-3}{2y^6}\right)^4$

In Exercises 64–68, divide and check each answer.

64.  $\frac{-15y^8}{3y^2}$                       65.  $\frac{40x^8y^6}{5xy^3}$   
 66.  $\frac{18x^4 - 12x^2 + 36x}{6x}$   
 67.  $\frac{30x^8 - 25x^7 - 40x^5}{-5x^3}$   
 68.  $\frac{27x^3y^2 - 9x^2y - 18xy^2}{3xy}$

**5.6** In Exercises 69–72, divide and check each answer.

69.  $\frac{2x^2 + 3x - 14}{x - 2}$   
 70.  $\frac{2x^3 - 5x^2 + 7x + 5}{2x + 1}$   
 71.  $\frac{x^3 - 2x^2 - 33x - 7}{x - 7}$   
 72.  $\frac{y^3 - 27}{y - 3}$

**5.7** In Exercises 73–77, write each expression with positive exponents only and then simplify.

73.  $7^{-2}$                       74.  $(-4)^{-3}$

75.  $2^{-1} + 4^{-1}$

76.  $\frac{1}{5^{-2}}$                       77.  $\left(\frac{2}{5}\right)^{-3}$

In Exercises 78–86, simplify each exponential expression. Assume that variables in denominators do not equal zero.

78.  $\frac{x^3}{x^9}$                       79.  $\frac{30y^6}{5y^8}$   
 80.  $(5x^{-7})(6x^2)$                       81.  $\frac{x^4 \cdot x^{-2}}{x^{-6}}$   
 82.  $\frac{(3y^3)^4}{y^{10}}$                       83.  $\frac{y^{-7}}{(y^4)^3}$   
 84.  $(2x^{-1})^{-3}$                       85.  $\left(\frac{x^7}{x^4}\right)^{-2}$   
 86.  $\frac{(y^3)^4}{(y^{-2})^4}$

In Exercises 87–89, write each number in decimal notation without the use of exponents.

87.  $2.3 \times 10^4$                       88.  $1.76 \times 10^{-3}$   
 89.  $9 \times 10^{-1}$

In Exercises 90–93, write each number in scientific notation.

90. 73,900,000                      91. 0.00062  
 92. 0.38                      93. 3.8

In Exercises 94–96, perform the indicated computation. Write the answers in scientific notation.

94.  $(6 \times 10^{-3})(1.5 \times 10^6)$   
 95.  $\frac{2 \times 10^2}{4 \times 10^{-3}}$   
 96.  $(4 \times 10^{-2})^2$

In Exercises 97–98, use  $10^6$  for one million and  $10^9$  for one billion to rewrite the number in each statement in scientific notation.

97. The 2009 economic stimulus package allocated \$53.6 billion for grants to states for education.  
 98. The population of the United States at the time the economic stimulus package was voted into law was approximately 307 million.  
 99. Use your scientific notation answers from Exercises 97 and 98 to answer this question:

If the cost for grants to states for education was evenly divided among every individual in the United States, how much would each citizen have to pay?

## CHAPTER 5 TEST



Step-by-step test solutions are found on the Chapter Test Prep Videos available in MyMathLab or on YouTube (search “BlitzerIntroAlg” and click on “Channels”).

- Identify  $9x + 6x^2 - 4$  as a monomial, binomial, or trinomial. Give the degree of the polynomial.

In Exercises 2–3, add or subtract as indicated.

- $(7x^3 + 3x^2 - 5x - 11) + (6x^3 - 2x^2 + 4x - 13)$
- $(9x^3 - 6x^2 - 11x - 4) - (4x^3 - 8x^2 - 13x + 5)$

- Graph the equation:  $y = x^2 - 3$ . Select integers for  $x$ , starting with  $-3$  and ending with  $3$ .

In Exercises 5–11, find each product.

- $(-7x^3)(5x^8)$
- $6x^2(8x^3 - 5x - 2)$
- $(3x + 2)(x^2 - 4x - 3)$
- $(3y + 7)(2y - 9)$
- $(7x + 5)(7x - 5)$
- $(x^2 + 3)^2$
- $(5x - 3)^2$
- Evaluate  $4x^2y + 5xy - 6x$  for  $x = -2$  and  $y = 3$ .

In Exercises 13–15, perform the indicated operations.

- $(8x^2y^3 - xy + 2y^2) - (6x^2y^3 - 4xy - 10y^2)$
- $(3a - 7b)(4a + 5b)$
- $(2x + 3y)^2$

In Exercises 16–18, divide and check each answer.

- $\frac{-25x^{16}}{5x^4}$
- $\frac{15x^4 - 10x^3 + 25x^2}{5x}$

$$18. \frac{2x^3 - 3x^2 + 4x + 4}{2x + 1}$$

In Exercises 19–20, write each expression with positive exponents only and then simplify.

$$19. 10^{-2} \qquad 20. \frac{1}{4^{-3}}$$

In Exercises 21–26, simplify each expression.

$$21. (-3x^2)^3 \qquad 22. \frac{20x^3}{5x^8}$$

$$23. (-7x^{-8})(3x^2) \qquad 24. \frac{(2y^3)^4}{y^8}$$

$$25. (5x^{-4})^{-2} \qquad 26. \left(\frac{x^{10}}{x^5}\right)^{-3}$$

27. Write  $3.7 \times 10^{-4}$  in decimal notation.

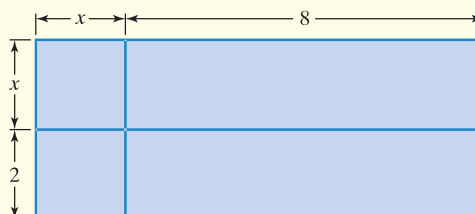
28. Write 7,600,000 in scientific notation.

In Exercises 29–30, perform the indicated computation. Write the answers in scientific notation.

$$29. (4.1 \times 10^2)(3 \times 10^{-5})$$

$$30. \frac{8.4 \times 10^6}{4 \times 10^{-2}}$$

31. Write a polynomial in descending powers of  $x$  that represents the area of the figure.



## CUMULATIVE REVIEW EXERCISES (CHAPTERS 1–5)

In Exercises 1–2, perform the indicated operation or operations.

- $(-7)(-5) \div (12 - 3)$
- $(3 - 7)^2(9 - 11)^3$
- What is the difference in elevation between a plane flying 14,300 feet above sea level and a submarine traveling 750 feet below sea level?

In Exercises 4–5, solve each equation.

$$4. 2(x + 3) + 2x = x + 4$$

$$5. \frac{x}{5} - \frac{1}{3} = \frac{x}{10} - \frac{1}{2}$$

- The length of a rectangular sign is 2 feet less than three times its width. If the perimeter of the sign is 28 feet, what are its dimensions?
- Solve  $7 - 8x \leq -6x - 5$  and graph the solution set on a number line.
- You invested \$6000 in two accounts paying 12% and 14% annual interest. At the end of the year, the total interest from these investments was \$772. How much invested at each rate?

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9. You need to mix a solution that is 70% antifreeze with one that is 30% antifreeze to obtain 20 liters of a mixture that is 60% antifreeze. How many liters of each of the solutions must be used?
10. Graph  $y = -\frac{2}{5}x + 2$  using the slope and  $y$ -intercept.
11. Graph  $x - 2y = 4$  using intercepts.
12. Find the slope of the line passing through the points  $(-3, 2)$  and  $(2, -4)$ . Is the line rising, falling, horizontal, or vertical?
13. The slope of a line is  $-2$  and the line passes through the point  $(3, -1)$ . Write the line's equation in point-slope form and slope-intercept form.

*In Exercises 14–15, solve each system by the method of your choice.*

14. 
$$\begin{cases} 3x + 2y = 10 \\ 4x - 3y = -15 \end{cases}$$

15. 
$$\begin{cases} 2x + 3y = -6 \\ y = 3x - 13 \end{cases}$$

16. You are choosing between two telephone plans. One has a monthly fee of \$15 with a charge of \$0.05 per minute for all calls. The other plan has a monthly fee of \$5 with a charge of \$0.07 per minute for all calls. For how many minutes of calls will the costs for the two plans be the same? What will be the cost for each plan?
17. Graph the solution set for the system of linear inequalities:

$$\begin{cases} 2x + 5y \leq 10 \\ x - y \geq 4. \end{cases}$$

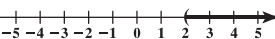
18. Subtract:  $(9x^5 - 3x^3 + 2x - 7) - (6x^5 + 3x^3 - 7x - 9)$ .

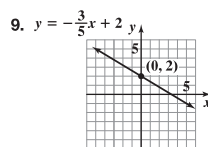
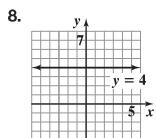
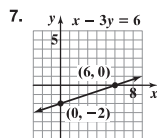
19. Divide:  $\frac{x^3 + 3x^2 + 5x + 3}{x + 1}$ .

20. Simplify:  $\frac{(3x^2)^4}{x^{10}}$ .



### Cumulative Review Exercises

1.  $-36$     2.  $17x - 11$     3.  $-6$  or  $\{-6\}$     4.  $20$  or  $\{20\}$     5.  $t = \frac{A - P}{Pr}$     6.  $(2, \infty)$  or  $\{x | x > 2\}$ ; 

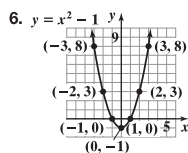


10.  $(0, -2)$  or  $\{(0, -2)\}$     11.  $(\frac{3}{2}, -2)$  or  $\{(\frac{3}{2}, -2)\}$     12. 1    13.  $y - 6 = -4(x + 1)$ ;  $y = -4x + 2$     14. 10 ft  
 15. pen: \$0.80; pad: \$1.20    16.  $-93, 0, \frac{7}{1}, \sqrt{100}$     17. living alone; The percentage of U.S. adults living alone increased from 1960 through 2008.  
 18. married, living with kids; The percentage of U.S. adults married, living with kids decreased from 1960 through 2008.    19. 70 years; 2030  
 20. 2030; In 2030, the same percentage of U.S. adults will be married, living with kids and living alone, namely 19%.

## CHAPTER 5

### Section 5.1 Check Point Exercises

1.  $5x^3 + 4x^2 - 8x - 20$     2.  $5x^3 + 4x^2 - 8x - 20$     3.  $7x^2 + 11x + 4$     4.  $7x^3 + 3x^2 + 12x - 8$     5.  $3y^3 - 10y^2 - 11y - 8$

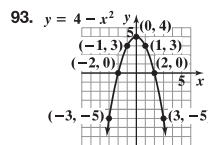
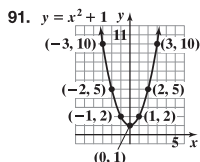
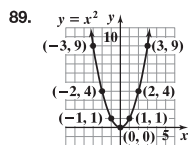


### Concept and Vocabulary Check

1. whole    2. standard    3. monomial    4. binomial    5. trinomial    6.  $n$     7. greatest    8. like    9. opposite

### Exercise Set 5.1

1. binomial, 1    3. binomial, 3    5. monomial, 2    7. monomial, 0    9. trinomial, 2    11. trinomial, 4    13. binomial, 3  
 15. monomial, 23    17.  $-8x + 13$     19.  $12x^2 + 15x - 9$     21.  $10x^2 - 12x$     23.  $5x^2 - 3x + 13$     25.  $4y^3 + 10y^2 + y - 2$   
 27.  $3x^3 + 2x^2 - 9x + 7$     29.  $-2y^3 + 4y^2 + 13y + 13$     31.  $-3y^6 + 8y^4 + y^2$     33.  $10x^3 + 1$     35.  $-\frac{2}{5}x^4 + x^3 - \frac{1}{8}x^2$   
 37.  $0.01x^5 + x^4 - 0.1x^3 + 0.3x + 0.33$     39.  $11y^3 - 3y^2$     41.  $-2x^2 - x + 1$     43.  $-\frac{1}{4}x^4 - \frac{7}{15}x^3 - 0.3$     45.  $-y^3 + 8y^2 - 3y - 14$   
 47.  $-5x^3 - 6x^2 + x - 4$     49.  $7x^4 - 2x^3 + 4x - 2$     51.  $8x^2 + 7x - 5$     53.  $9x^3 - 4.9x^2 + 11.1$     55.  $-2x - 10$     57.  $-5x^2 - 9x - 12$   
 59.  $-5x^2 - x$     61.  $-4x^2 - 4x - 6$     63.  $-2y - 6$     65.  $6y^3 + y^2 + 7y - 20$     67.  $n^3 + 2$     69.  $y^6 - y^3 - y^2 + y$   
 71.  $26x^4 + 9x^2 + 6x$     73.  $\frac{5}{7}x^3 - \frac{9}{20}x$     75.  $4x + 6$     77.  $10x^2 - 7$     79.  $-4y^2 - 7y + 5$     81.  $9x^3 + 11x^2 - 8$   
 83.  $-y^3 + 8y^2 + y + 14$     85.  $7x^4 - 2x^3 + 3x^2 - x + 2$     87.  $0.05x^3 + 0.02x^2 + 1.02x$



95.  $x^2 + 12x$     97.  $y^2 - 19y + 16$     99.  $2x^3 + 3x^2 + 7x - 5$     101.  $-10y^3 + 2y^2 + y + 3$   
 103. a.  $M - W = -35x^3 + 1373x^2 - 15,995x + 63,210$     b. \$12,348    c. \$10,923; overestimates by \$1425  
 105. a. \$56,995; underestimates by \$225    b. (16, 56,995) on the graph for men  
 c. \$42,000, although estimates may vary by  $\pm \$1000$     115. does not make sense    117. does not make sense  
 119. false    121. false    123.  $\frac{2}{3}t^3 - 2t^2 + 4t$     125.  $-10$     126. 5.6    127.  $-4$  or  $\{-4\}$     128. 7    129.  $3x^2 + 15x$     130.  $x^2 + 5x + 6$

**AA22** Answers to Selected Exercises

**Section 5.2** Check Point Exercises

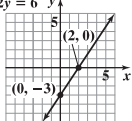
1. a.  $2^6$  or 64    b.  $x^{10}$     c.  $y^8$     d.  $y^9$     2. a.  $3^{20}$     b.  $x^{90}$     c.  $(-5)^{21}$     3. a.  $16x^4$     b.  $-64y^6$     4. a.  $70x^3$     b.  $-20x^9$   
 5. a.  $3x^2 + 15x$     b.  $30x^5 - 12x^3 + 18x^2$     6. a.  $x^2 + 9x + 20$     b.  $10x^2 - 29x - 21$     7.  $5x^3 - 18x^2 + 7x + 6$   
 8.  $6x^5 - 19x^4 + 22x^3 - 8x^2$

Concept and Vocabulary Check

1.  $b^{m+n}$ ; add    2.  $b^{mn}$ ; multiply    3.  $a^n b^n$ ; factor    4. distributive;  $x^2 + 5x + 7; 2x^2$     5.  $4x$ ; 7; like

Exercise Set 5.2

1.  $x^{18}$     3.  $y^{12}$     5.  $x^{11}$     7.  $7^{19}$     9.  $6^{90}$     11.  $x^{45}$     13.  $(-20)^9$     15.  $8x^3$     17.  $25x^2$     19.  $16x^6$     21.  $16y^{24}$     23.  $-32x^{35}$   
 25.  $14x^2$     27.  $24x^3$     29.  $-15y^7$     31.  $\frac{1}{8}a^5$     33.  $-48x^7$     35.  $4x^2 + 12x$     37.  $x^2 - 3x$     39.  $2x^2 - 12x$     41.  $-12y^2 - 20y$   
 43.  $4x^3 + 8x^2$     45.  $2y^4 + 6y^3$     47.  $6y^4 - 8y^3 + 14y^2$     49.  $6x^4 + 8x^3$     51.  $-2x^3 - 10x^2 + 6x$     53.  $12x^4 - 3x^3 + 15x^2$   
 55.  $x^2 + 8x + 15$     57.  $2x^2 + 9x + 4$     59.  $x^2 - 2x - 15$     61.  $x^2 - 2x - 99$     63.  $2x^2 + 3x - 20$     65.  $\frac{3}{16}x^2 + \frac{11}{4}x - 4$   
 67.  $x^3 + 3x^2 + 5x + 3$     69.  $y^3 - 6y^2 + 13y - 12$     71.  $2a^3 - 9a^2 + 19a - 15$     73.  $x^4 + 3x^3 + 5x^2 + 7x + 4$   
 75.  $4x^4 - 4x^3 + 6x^2 - \frac{17}{2}x + 3$     77.  $x^4 + x^3 + x^2 + 3x + 2$     79.  $x^3 + 3x^2 - 37x + 24$     81.  $2x^3 - 9x^2 + 27x - 27$   
 83.  $2x^4 + 9x^3 + 6x^2 + 11x + 12$     85.  $12z^4 - 14z^3 + 19z^2 - 22z + 8$     87.  $21x^5 - 43x^4 + 38x^3 - 24x^2$   
 89.  $4y^6 - 2y^5 - 6y^4 + 5y^3 - 5y^2 + 8y - 3$     91.  $x^4 + 6x^3 - 11x^2 - 4x + 3$     93.  $2x - 2$     95.  $15x^5 + 42x^3 - 8x^2$   
 97.  $2y^3$     99.  $16y + 32$     101.  $2x^2 + 7x - 15 \text{ ft}^2$     103. a.  $(2x + 1)(x + 2)$     b.  $2x^2 + 5x + 2$     c.  $(2x + 1)(x + 2) = 2x^2 + 5x + 2$   
 113. makes sense    115. makes sense    117. false    119. true    121.  $8x + 16$     123.  $-8x^4$     124.  $(-\infty, -1)$  or  $\{x | x < -1\}$   
 125.  $3x - 2y = 6$     126.  $-\frac{2}{3}$     127. a.  $x^2 + 7x + 12$     b.  $x^2 + 25x + 100$     128. a.  $x^2 - 9$     b.  $x^2 - 25$   
 129. a.  $x^2 + 6x + 9$     b.  $x^2 + 10x + 25$



**Section 5.3** Check Point Exercises

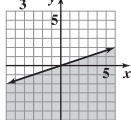
1.  $x^2 + 11x + 30$     2.  $28x^2 - x - 15$     3.  $6x^2 - 22x + 20$     4. a.  $49y^2 - 64$     b.  $16x^2 - 25$     c.  $4a^6 - 9$     5. a.  $x^2 + 20x + 100$   
 b.  $25x^2 + 40x + 16$     6. a.  $x^2 - 18x + 81$     b.  $49x^2 - 42x + 9$

Concept and Vocabulary Check

1.  $2x^2$ ;  $3x$ ;  $10x$ ; 15    2.  $A^2 - B^2$ ; minus    3.  $A^2 + 2AB + B^2$ ; squared; product of the terms; squared  
 4.  $A^2 - 2AB + B^2$ ; minus; product of the terms; squared    5. true    6. false

Exercise Set 5.3

1.  $x^2 + 10x + 24$     3.  $y^2 - 4y - 21$     5.  $2x^2 + 7x - 15$     7.  $4y^2 - y - 3$     9.  $10x^2 - 9x - 9$     11.  $12y^2 - 43y + 35$   
 13.  $-15x^2 - 32x + 7$     15.  $6y^2 - 28y + 30$     17.  $15x^4 - 47x^2 + 28$     19.  $-6x^2 + 17x - 10$     21.  $x^3 + 5x^2 + 3x + 15$   
 23.  $8x^5 + 40x^3 + 3x^2 + 15$     25.  $x^2 - 9$     27.  $9x^2 - 4$     29.  $9r^2 - 16$     31.  $9 - r^2$     33.  $25 - 49x^2$     35.  $4x^2 - \frac{1}{4}$     37.  $y^4 - 1$   
 39.  $r^6 - 4$     41.  $1 - y^8$     43.  $x^{20} - 25$     45.  $x^2 + 4x + 4$     47.  $4x^2 + 20x + 25$     49.  $x^2 - 6x + 9$     51.  $9y^2 - 24y + 16$   
 53.  $16x^4 - 8x^2 + 1$     55.  $49 - 28x + 4x^2$     57.  $4x^2 + 2x + \frac{1}{4}$     59.  $16y^2 - 2y + \frac{1}{16}$     61.  $x^{16} + 6x^8 + 9$     63.  $x^3 - 1$   
 65.  $x^2 - 2x + 1$     67.  $9y^2 - 49$     69.  $12x^4 + 3x^3 + 27x^2$     71.  $70y^2 + 2y - 12$     73.  $x^4 + 2x^2 + 1$     75.  $x^4 + 3x^2 + 2$   
 77.  $x^4 - 16$     79.  $4 - 12x^5 + 9x^{10}$     81.  $\frac{3}{16}x^4 + 7x^2 - 96$     83.  $x^2 + 2x + 1$     85.  $4x^2 - 9$     87.  $6x + 22$   
 89.  $16x^4 - 72x^2 + 81$     91.  $16x^4 - 1$     93.  $x^3 + 6x^2 + 12x + 8$     95.  $x^2 + 6x + 9 - y^2$     97.  $(x + 1)(x + 2) \text{ yd}^2$     99.  $56 \text{ yd}^2$ ; (6, 56)  
 101.  $(x^2 + 4x + 4) \text{ in}^2$     109. makes sense    111. makes sense, although answers may vary    113. true    115. false    117.  $4x^3 - 36x^2 + 80x$   
 119. Change  $x^2 + 1$  to  $x^2 + 2x + 1$ .    121. Graphs coincide.    123.  $(2, -1)$  or  $\{(2, -1)\}$     124.  $(1, 1)$  or  $\{(1, 1)\}$   
 125.  $y \leq \frac{1}{3}x$     126. -72    127.  $11xy$     128.  $3x^2 + 11xy + 10y^2$



**Section 5.4** Check Point Exercises

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1. -9    2. polynomial degree: 9;

Term	Coefficient	Degree
$8x^4y^5$	8	9
$-7x^3y^2$	-7	5
$-x^2y$	-1	3
$-5x$	-5	1
11	11	0

3.  $2x^2y + 2xy - 4$     4.  $3x^3 + 2x^2y + 5xy^2 - 10$     5.  $60x^5y^5$     6.  $60x^5y^7 - 12x^3y^3 + 18xy^2$   
 7. a.  $21x^2 - 25xy + 6y^2$     b.  $4x^2 + 16xy + 16y^2$     8. a.  $36x^2y^4 - 25x^2$     b.  $x^3 - y^3$

**Concept and Vocabulary Check**

1. -18    2. 6    3.  $a; n + m$     4. 5; 9; 9    5. false    6. true

**Exercise Set 5.4**

1. 1    3. -47    5. -6    7. polynomial degree: 9;

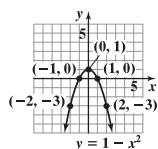
Term	Coefficient	Degree
$x^3y^2$	1	5
$-5x^2y^7$	-5	9
$6y^2$	6	2
-3	-3	0

9.  $7x^2y - 4xy$     11.  $2x^2y + 13xy + 13$     13.  $-11x^4y^2 - 11x^2y^2 + 2xy$     15.  $-5x^3 + 8xy - 9y^2$     17.  $x^4y^2 + 8x^3y + y - 6x$   
 19.  $5x^3 + x^2y - xy^2 - 4y^3$     21.  $-3x^2y^2 + xy^2 + 5y^2$     23.  $8a^2b^4 + 3ab^2 + 8ab$     25.  $-30x + 37y$     27.  $40x^3y^2$     29.  $-24x^5y^9$   
 31.  $45x^2y + 18xy^2$     33.  $50x^3y^2 - 15xy^3$     35.  $28a^3b^5 + 8a^2b^3$     37.  $-a^2b + ab^2 - b^3$     39.  $7x^2 + 38xy + 15y^2$     41.  $2x^2 + xy - 21y^2$   
 43.  $15x^2y^2 + xy - 2$     45.  $4x^2 + 12xy + 9y^2$     47.  $x^2y^2 - 6xy + 9$     49.  $x^4 + 2x^2y^2 + y^4$     51.  $x^4 - 4x^2y^2 + 4y^4$     53.  $9x^2 - y^2$   
 55.  $a^2b^2 - 1$     57.  $x^2 - y^4$     59.  $9a^4b^2 - a^2$     61.  $9x^2y^4 - 16y^2$     63.  $a^3 - ab^2 + a^2b - b^3$     65.  $x^3 + 4x^2y + 4xy^2 + y^3$   
 67.  $x^3 - 4x^2y + 4xy^2 - y^3$     69.  $x^2y^2 - a^2b^2$     71.  $x^6y + x^4y + x^4 + 2x^2 + 1$     73.  $x^4y^4 - 6x^2y^2 + 9$     75.  $x^2 + 2xy + y^2 - 1$   
 77.  $3x^2 + 8xy + 5y^2$     79.  $2xy + y^2$     81.  $x^{12}y^{12} - 2x^6y^6 + 1$     83.  $x^4y^4 - 18x^2y^2 + 81$     85.  $x^2 - y^2 - 2yz - z^2$   
 87. no; need 120 more board feet    89. 192 ft    91. 0 ft; The ball hits the ground.    93. 2.5 to 6 sec    95. (2, 192)  
 97. 2.5 sec; 196 ft    101. makes sense    103. does not make sense    105. false    107. true    109.  $-x^2 + 18xy + 80y^2$   
 112.  $W = \frac{2R - L}{3}$     113. 3.8    114. -1.6 or {-1.6}    115. 4    116.  $\frac{x^6}{125}$     117.  $\frac{32a^{15}}{b^{20}}$

**Mid-Chapter Check Point Exercises**

1.  $-55x^4y^6$     2.  $6x^2y^3$     3.  $12x^2 - x - 35$     4.  $-x + 12$     5.  $2x^3 - 11x^2 + 17x - 5$     6.  $x^2 - x - 4$     7.  $64x^2 - 48x + 9$     8.  $70x^9$   
 9.  $x^4 - 4$     10.  $x^4 + 4x^2 + 4$     11.  $18a^2 - 11ab - 10b^2$     12.  $70x^5 - 14x^3 + 21x^2$     13.  $5a^2b^3 + 2ab - b^2$     14.  $18y^2 - 50$   
 15.  $2x^3 - x^2 + x$     16.  $10x^2 - 5xy - 3y^2$     17.  $-4x^5 + 7x^4 - 10x + 23$     18.  $x^3 + 27y^3$     19.  $10x^7 - 5x^4 + 8x^3 - 4$   
 20.  $y^2 - 12yz + 36z^2$     21.  $-21x^2 + 7$

22.



**Section 5.5** Check Point Exercises

1. a.  $5^8$     b.  $x^7$     c.  $y^{19}$     2. a. 1    b. 1    c. -1    d. 20    e. 1    3. a.  $\frac{x^2}{25}$     b.  $\frac{x^{12}}{8}$     c.  $\frac{16a^{40}}{b^{12}}$   
 4. a.  $-2x^8$     b.  $\frac{1}{5}$     c.  $3x^5y^3$     5.  $-5x^7 + 2x^3 - 3x$     6.  $5x^6 - \frac{7}{5}x + 2$     7.  $3x^6y^4 - xy + 10$

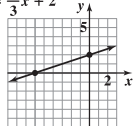
**Concept and Vocabulary Check**

1.  $b^{m-n}$ ; subtract    2. 1    3.  $\frac{a^n}{b^m}$ ; numerator; denominator    4. divide; subtract    5. dividend; divisor; quotient  
 6. divisor; quotient; dividend    7.  $20x^8 - 10x^4 + 6x^3; 2x^3$

**AA24** Answers to Selected Exercises

**Exercise Set 5.5**

1.  $3^{15}$    3.  $x^4$    5.  $y^8$    7.  $5^3 \cdot 2^4$    9.  $x^{75}y^{40}$    11. 1   13. 1   15. -1   17. 100   19. 1   21. 0   23. -2   25.  $\frac{x^2}{9}$    27.  $\frac{x^6}{64}$   
 29.  $\frac{4x^6}{25}$    31.  $-\frac{64}{27a^9}$    33.  $-\frac{32a^{35}}{b^{20}}$    35.  $\frac{x^8y^{12}}{16z^4}$    37.  $3x^5$    39.  $-2x^{20}$    41.  $-\frac{1}{2}y^3$    43.  $\frac{7}{5}y^{12}$    45.  $6x^5y^4$    47.  $-\frac{1}{2}x^{12}$    49.  $\frac{9}{7}$   
 51.  $-\frac{1}{10}x^8y^9z^4$    53.  $5x^4 + x^3$    55.  $2x^3 - x^2$    57.  $y^6 - 9y + 1$    59.  $-8x^2 + 5x$    61.  $6x^3 + 2x^2 + 3x$    63.  $3x^3 - 2x^2 + 10x$   
 65.  $4x - 6$    67.  $-6z^2 - 2z$    69.  $4x^2 + 3x - 1$    71.  $5x^4 - 3x^2 - x$    73.  $-9x^3 + \frac{9}{2}x^2 - 10x + 5$    75.  $4xy + 2x - 5y$   
 77.  $-4x^5y^3 + 3xy + 2$    79.  $4x^2 - x + 6$    81.  $-xy^2$    83.  $y + 5$    85.  $3x^{12n} - 6x^{9n} + 2$   
 87. a. \$5.39   b.  $\frac{3.6x^2 + 158x + 2790}{-0.2x^2 + 21x + 1015}$    c. \$5.45; overestimates by \$0.06   d. no; The divisor is not a monomial.  
 97. does not make sense   99. does not make sense   101. false   103. true   105.  $\frac{9x^8 - 12x^6}{3x^3}$    107. 20.3   108. 0.875  
 109.  $y = \frac{1}{3}x + 2$    110. quotient: 26; remainder: 0   111. quotient: 123; remainder: 6   112. quotient: 257; remainder: 1



**Section 5.6** Check Point Exercises

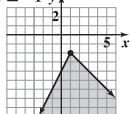
1.  $x + 5$    2.  $4x - 3 - \frac{3}{2x + 3}$    3.  $x^2 + x + 1$

**Concept and Vocabulary Check**

1.  $10x^3 + 4x^2 + 0x + 9$    2.  $8x^2; 2x; 4x; 10x$    3.  $5x; 3x - 2; 15x^2 - 10x; 15x^2 - 22x$    4.  $6x^2 - 10x; 6x^2 + 8x; 18x; -4; 18x - 4$   
 5.  $14; x - 12; 14; x - 12 + \frac{14}{x - 5}$

**Exercise Set 5.6**

1.  $x + 4$    3.  $2x + 5$    5.  $x - 2$    7.  $2y + 1$    9.  $x - 5 + \frac{14}{x + 2}$    11.  $y + 3 + \frac{4}{y + 2}$    13.  $x^2 - 5x + 2$    15.  $6y - 1$   
 17.  $2a + 3$    19.  $y^2 - y + 2$    21.  $3x + 5 - \frac{5}{2x - 5}$    23.  $x^2 + 2x + 8 + \frac{13}{x - 2}$    25.  $2y^2 + y + 1 + \frac{6}{2y + 3}$   
 27.  $2y^2 - 3y + 2 + \frac{1}{3y + 2}$    29.  $9x^2 + 3x + 1$    31.  $y^3 - 9y^2 + 27y - 27$    33.  $2y + 4 + \frac{4}{2y - 1}$    35.  $y^3 + y^2 - y - 1 + \frac{4}{y - 1}$   
 37.  $4x - 3 + \frac{-7x + 7}{x^2 + 2}$    39.  $x^2 + ax + a^2$    41.  $2x^2 - 3x + 2$    43.  $x^2 + 2x + 3$    45.  $x^2 + 2x + 3$  units   47. a.  $\frac{30,000x^3 - 30,000}{x - 1}$   
 b.  $30,000x^2 + 30,000x + 30,000$    c. \$94,575   53. makes sense   55. does not make sense   57. false   59. true   61.  $2x^2 - 2x + 5$   
 63. Answers will vary;  $x^6 - x^5 + x^4 - x^3 + x^2 - x + 1 - \frac{2}{x + 1}$    65.  $x - 5$  should be  $x + 5$ .   67.  $2x - 4$  should be  $2x + 4$ .  
 69.  $\frac{2x - y \geq 4}{x + y \leq -1}$    70. 1.2   71. -6 or {-6}   72. a. 2; -2   b.  $\frac{1}{7^2} = 7^{-2}$    73.  $16x^2$    74.  $x^9$



**Section 5.7** Check Point Exercises

1. a.  $\frac{1}{6^2} = \frac{1}{36}$    b.  $\frac{1}{5^3} = \frac{1}{125}$    c.  $\frac{1}{(-3)^4} = \frac{1}{81}$    d.  $-\frac{1}{3^4} = -\frac{1}{81}$    e.  $\frac{1}{8^1} = \frac{1}{8}$    2. a.  $\frac{7^2}{2^3} = \frac{49}{8}$    b.  $\frac{5^2}{4^2} = \frac{25}{16}$    c.  $\frac{y^2}{7}$    d.  $\frac{y^8}{x^1} = \frac{y^8}{x}$   
 3.  $\frac{1}{x^{10}}$    4. a.  $\frac{1}{x^8}$    b.  $\frac{15}{x^6}$    c.  $-\frac{2}{y^6}$    5.  $\frac{36}{x^3}$    6.  $\frac{1}{x^{20}}$    7. a. 7,400,000,000   b. 0.000003017   8. a.  $7.41 \times 10^9$    b.  $9.2 \times 10^{-8}$   
 9. a.  $6 \times 10^{10}$    b.  $2.1 \times 10^{11}$    c.  $6.4 \times 10^{-5}$    10. \$2560

**Concept and Vocabulary Check**

1.  $\frac{1}{b^n}$    2. false   3. true   4.  $b^n$    5. true   6. false   7. a number greater than or equal to 1 and less than 10; integer   8. true   9. false

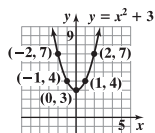
Exercise Set 5.7

1.  $\frac{1}{8^2} = \frac{1}{64}$     3.  $\frac{1}{5^3} = \frac{1}{125}$     5.  $\frac{1}{(-6)^2} = \frac{1}{36}$     7.  $\frac{1}{-6^2} = -\frac{1}{36}$     9.  $\frac{1}{4^1} = \frac{1}{4}$     11.  $\frac{1}{2^1} + \frac{1}{3^1} = \frac{1}{2} + \frac{1}{3} = \frac{5}{6}$     13.  $3^2 = 9$     15.  $(-3)^2 = 9$   
 17.  $\frac{8^2}{2^3} = 8$     19.  $\frac{4^2}{1^2} = 16$     21.  $\frac{5^3}{3^3} = \frac{125}{27}$     23.  $\frac{x^5}{6}$     25.  $\frac{y^1}{x^8} = \frac{y}{x^8}$     27.  $3 \cdot (-5)^3 = -375$     29.  $\frac{1}{x^5}$     31.  $\frac{8}{x^3}$     33.  $\frac{1}{x^6}$     35.  $\frac{1}{y^{99}}$   
 37.  $\frac{3}{z^5}$     39.  $\frac{4}{x^4}$     41.  $\frac{1}{3a^3}$     43.  $\frac{7}{5w^8}$     45.  $\frac{1}{x^5}$     47.  $\frac{1}{y^{11}}$     49.  $\frac{16}{x^2}$     51.  $216y^{17}$     53.  $\frac{1}{x^6}$     55.  $\frac{1}{16x^{12}}$     57.  $\frac{x^2}{9}$     59.  $\frac{y^3}{8}$   
 61.  $\frac{2x^6}{5}$     63.  $x^8$     65.  $16y^6$     67.  $\frac{1}{y^2}$     69.  $\frac{1}{y^{50}}$     71.  $\frac{1}{a^{12}b^{15}}$     73.  $\frac{a^8}{b^{24}}$     75.  $\frac{4}{x^4}$     77.  $\frac{y^9}{x^6}$     79. 870    81. 923,000    83. 3.4  
 85. 0.79    87. 0.0215    89. 0.000786    91.  $3.24 \times 10^4$     93.  $2.2 \times 10^8$     95.  $7.13 \times 10^2$     97.  $6.751 \times 10^3$     99.  $2.7 \times 10^{-3}$   
 101.  $2.02 \times 10^{-5}$     103.  $5 \times 10^{-3}$     105.  $3.14159 \times 10^0$     107.  $6 \times 10^5$     109.  $1.6 \times 10^9$     111.  $3 \times 10^4$     113.  $3 \times 10^6$     115.  $3 \times 10^{-6}$   
 117.  $9 \times 10^4$     119.  $2.5 \times 10^6$     121.  $1.25 \times 10^8$     123.  $8.1 \times 10^{-7}$     125.  $2.5 \times 10^{-7}$     127. 1    129.  $\frac{y}{16x^8z^6}$     131.  $\frac{1}{x^{12}y^{16}z^{20}}$   
 133.  $\frac{x^{18}y^6}{4}$     135.  $2.5 \times 10^{-3}$     137.  $8 \times 10^{-5}$     139. a.  $1.35 \times 10^{12}$     b.  $3.07 \times 10^8$     c.  $\$4.40 \times 10^3$ ; \$4400    141. 42,000 years  
 143. 1.25 sec    153. makes sense    155. makes sense    157. true    159. false    161. false    163. true    169.  $(-\infty, 2)$  or  $\{x|x < 2\}$   
 170. 5    171. 0,  $\sqrt{16}$     172.  $16x^5 - 12x^4 + 4x^3$     173.  $27x^2y^3 - 9xy^2 + 81xy$     174.  $x^3 + 3x^2 + 5x + 15$

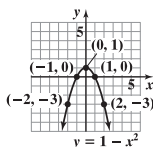
Review Exercises

1. binomial, 4    2. trinomial, 2    3. monomial, 1    4.  $8x^3 + 10x^2 - 20x - 4$     5.  $13y^3 - 8y^2 + 7y - 5$     6.  $11y^2 - 4y - 4$   
 7.  $8x^4 - 5x^3 + 6$     8.  $-14x^4 - 13x^2 + 16x$     9.  $7y^4 - 5y^3 + 3y^2 - y - 4$     10.  $3x^2 - 7x + 9$     11.  $10x^3 - 9x^2 + 2x + 11$

12.



13.



14.  $x^{23}$     15.  $y^{14}$     16.  $x^{100}$     17.  $100y^2$     18.  $-64x^{30}$     19.  $50x^4$     20.  $-36y^{11}$     21.  $30x^{12}$     22.  $21x^3 + 63x$     23.  $20x^5 - 55x^4$   
 24.  $-21y^4 + 9y^3 - 18y^2$     25.  $16y^8 - 20y^7 + 2y^5$     26.  $x^3 - 2x^2 - 13x + 6$     27.  $12y^3 + y^2 - 21y + 10$     28.  $3y^3 - 17y^2 + 41y - 35$   
 29.  $8x^4 + 8x^3 - 18x^2 - 20x - 3$     30.  $x^2 + 8x + 12$     31.  $6y^2 - 7y - 5$     32.  $4x^4 - 14x^2 + 6$     33.  $25x^2 - 16$     34.  $49 - 4y^2$   
 35.  $y^4 - 1$     36.  $x^2 + 6x + 9$     37.  $9y^2 + 24y + 16$     38.  $y^2 - 2y + 1$     39.  $25y^2 - 20y + 4$     40.  $x^4 + 8x^2 + 16$     41.  $x^4 - 16$   
 42.  $x^4 - x^2 - 20$     43.  $x^2 + 7x + 12$     44.  $x^2 + 50x + 600 \text{ yd}^2$     45. 28

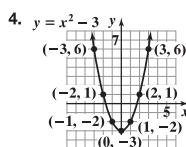
46. polynomial degree: 5;

Term	Coefficient	Degree
$4x^2y$	4	3
$9x^3y^2$	9	5
$-17x^4$	-17	4
-12	-12	0

47.  $-x^2 - 17xy + 5y^2$     48.  $2x^3y^2 + x^2y - 6x^2 - 4$     49.  $-35x^6y^9$     50.  $15a^3b^5 - 20a^2b^3$     51.  $3x^2 + 16xy - 35y^2$   
 52.  $36x^2y^2 - 31xy + 3$     53.  $9x^2 + 30xy + 25y^2$     54.  $x^2y^2 - 14xy + 49$     55.  $49x^2 - 16y^2$     56.  $a^3 - b^3$     57.  $6^{30}$     58.  $x^{15}$   
 59. 1    60. -1    61. 400    62.  $\frac{x^{12}}{8}$     63.  $\frac{81}{16y^{24}}$     64.  $-5y^6$     65.  $8x^7y^3$     66.  $3x^3 - 2x + 6$     67.  $-6x^5 + 5x^4 + 8x^2$   
 68.  $9x^2y - 3x - 6y$     69.  $2x + 7$     70.  $x^2 - 3x + 5$     71.  $x^2 + 5x + 2 + \frac{7}{x-7}$     72.  $y^2 + 3y + 9$     73.  $\frac{1}{7^2} = \frac{1}{49}$     74.  $\frac{1}{(-4)^3} = -\frac{1}{64}$   
 75.  $\frac{1}{2^1} + \frac{1}{4^1} = \frac{1}{2} + \frac{1}{4} = \frac{3}{4}$     76.  $5^2 = 25$     77.  $\frac{5^3}{2^3} = \frac{125}{8}$     78.  $\frac{1}{x^6}$     79.  $\frac{6}{y^2}$     80.  $\frac{30}{x^5}$     81.  $x^8$     82.  $81y^2$     83.  $\frac{1}{y^{19}}$   
 84.  $\frac{x^3}{8}$     85.  $\frac{1}{x^6}$     86.  $y^{20}$     87. 23,000    88. 0.00176    89. 0.9    90.  $7.39 \times 10^7$     91.  $6.2 \times 10^{-4}$     92.  $3.8 \times 10^{-1}$     93.  $3.8 \times 10^0$   
 94.  $9 \times 10^3$     95.  $5 \times 10^4$     96.  $1.6 \times 10^{-3}$     97.  $\$5.36 \times 10^{10}$     98.  $3.07 \times 10^8$     99.  $\approx \$175$

Chapter Test

1. trinomial, 2    2.  $13x^3 + x^2 - x - 24$     3.  $5x^3 + 2x^2 + 2x - 9$

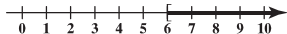


5.  $-35x^{11}$     6.  $48x^5 - 30x^3 - 12x^2$     7.  $3x^3 - 10x^2 - 17x - 6$     8.  $6y^2 - 13y - 63$     9.  $49x^2 - 25$     10.  $x^4 + 6x^2 + 9$   
 11.  $25x^2 - 30x + 9$     12. 30    13.  $2x^2y^3 + 3xy + 12y^2$     14.  $12a^2 - 13ab - 35b^2$     15.  $4x^2 + 12xy + 9y^2$     16.  $-5x^{12}$

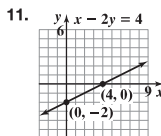
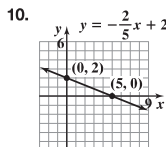
**AA26** Answers to Selected Exercises

17.  $3x^3 - 2x^2 + 5x$     18.  $x^2 - 2x + 3 + \frac{1}{2x + 1}$     19.  $\frac{1}{10^2} = \frac{1}{100}$     20.  $4^3 = 64$     21.  $-27x^6$     22.  $\frac{4}{x^5}$     23.  $-\frac{21}{x^6}$   
 24.  $16y^4$     25.  $\frac{x^8}{25}$     26.  $\frac{1}{x^{15}}$     27. 0.00037    28.  $7.6 \times 10^6$     29.  $1.23 \times 10^{-2}$     30.  $2.1 \times 10^8$     31.  $x^2 + 10x + 16$

**Cumulative Review Exercises**

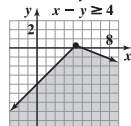
1.  $\frac{35}{9}$     2. -128    3. 15,050 ft    4.  $-\frac{2}{3}$  or  $\left\{-\frac{2}{3}\right\}$     5.  $-\frac{5}{3}$  or  $\left\{-\frac{5}{3}\right\}$     6. 4 ft by 10 ft  
 7.  $[6, \infty)$  or  $\{x | x \geq 6\}$ ;     8. \$3400 at 12% and \$2600 at 14%

9. 15 liters of 70% and 5 liters of 30%



12.  $-\frac{6}{5}$  falling    13.  $y + 1 = -2(x - 3); y = -2x + 5$     14. (0, 5) or  $\{(0, 5)\}$     15. (3, -4) or  $\{(3, -4)\}$     16. 500 min; \$40

17.  $2x + 5y \leq 10$



18.  $3x^5 - 6x^3 + 9x + 2$     19.  $x^2 + 2x + 3$     20.  $\frac{81}{x^2}$

## CHAPTER 6

### Section 6.1 Check Point Exercises

1. a.  $3x^2$     b.  $4x^2$     c.  $x^2y$     2.  $6(x^2 + 3)$     3.  $5x^2(5 + 7x)$     4.  $3x^3(5x^2 + 4x - 9)$     5.  $2xy(4x^2y - 7x + 1)$   
 6.  $-4ab^2(4a^3b^3 - 6a^2b^2 + 5)$     7. a.  $(x + 1)(x^2 + 7)$     b.  $(y + 4)(x - 7)$     8.  $(x + 5)(x^2 + 2)$     9.  $(y + 3)(x - 5)$

#### Concept and Vocabulary Check

1. factoring    2. greatest common factor; smallest/least    3. false    4. false

#### Exercise Set 6.1

1. 4    3.  $4x$     5.  $2x^3$     7.  $3y$     9.  $xy$     11.  $4x^4y^3$     13.  $8(x + 1)$     15.  $4(y - 1)$     17.  $5(x + 6)$     19.  $6(5x - 2)$     21.  $x(x + 5)$   
 23.  $6(3y^2 + 2)$     25.  $7x^2(2x + 3)$     27.  $y(13y - 25)$     29.  $9y^4(1 + 3y^2)$     31.  $4x^2(2 - x^2)$     33.  $4(3y^2 + 4y - 2)$     35.  $3x^2(3x^2 + 6x + 2)$   
 37.  $50y^2(2y^3 - y + 2)$     39.  $5x(2 - 4x + x^2)$     41. cannot be factored    43.  $3xy(2x^2y + 3)$     45.  $10xy(3xy^2 - y + 2)$   
 47.  $8x^2y(4xy - 3x - 2)$     49.  $-6(2x^2 - 3)$     51.  $-8x^2(x^2 - 4x - 2)$     53.  $-2ab(2a^2b - 3)$     55.  $-6x^2y(2xy + 3x - 4)$     57.  $(x + 5)(x + 3)$   
 59.  $(x + 2)(x - 4)$     61.  $(y + 6)(x - 7)$     63.  $(x + y)(3x - 1)$     65.  $(3x + 1)(4x + 1)$     67.  $(5x + 4)(7x^2 + 1)$     69.  $(x + 2)(x + 4)$   
 71.  $(x - 5)(x + 3)$     73.  $(x^2 + 5)(x - 2)$     75.  $(x^2 + 2)(x - 1)$     77.  $(y + 5)(x + 9)$     79.  $(y - 1)(x + 5)$     81.  $(x - 2y)(3x + 5y)$   
 83.  $(3x - 2)(x^2 - 2)$     85.  $(x - a)(x - b)$     87.  $6x^2yz(4xy^2z^2 + 5y + 3z)$     89.  $(x^3 - 4)(1 + 3y)$   
 91.  $2x^2(x + 1)(2x^3 - 3x - 4)$     93.  $(x - 1)(3x^4 + x^2 + 5)$     95.  $36x^2 - 4\pi x^2; 4x^2(9 - \pi)$     97. a. 48 ft    b.  $16x(4 - x)$   
 c. 48 ft; yes; no; Answers will vary.    99.  $x^3 - 2$     107. makes sense    109. does not make sense  
 111. false    113. false    115.  $4(x + 150)$     119.  $(x - 2)(x - 5)$  should be  $(x - 2)(x + 5)$     121.  $x^2 + 17x + 70$     122.  $(-3, -2)$  or  $\{(-3, -2)\}$   
 123.  $y - 2 = 1(x + 7)$  or  $y - 5 = 1(x + 4); y = x + 9$     124. 2 and 4    125. -3 and -2    126. -5 and 7

### Section 6.2 Check Point Exercises

1.  $(x + 2)(x + 3)$     2.  $(x - 2)(x - 4)$     3.  $(x + 5)(x - 2)$     4.  $(y - 9)(y + 3)$     5. cannot factor over the integers; prime  
 6.  $(x - 3y)(x - y)$     7.  $2x(x - 4)(x + 7)$     8.  $-2(y - 2)(y + 7)$

#### Concept and Vocabulary Check

1. 20; -12    2. completely    3. + 10    4. - 6    5. + 5    6. - 7    7. - 2y

#### Exercise Set 6.2

1.  $(x + 6)(x + 1)$     3.  $(x + 2)(x + 5)$     5.  $(x + 1)(x + 10)$     7.  $(x - 4)(x - 3)$     9.  $(x - 6)(x - 6)$     11.  $(y - 3)(y - 5)$   
 13.  $(x + 5)(x - 2)$     15.  $(y + 13)(y - 3)$     17.  $(x - 5)(x + 3)$     19.  $(x - 4)(x + 2)$     21. prime    23.  $(y - 4)(y - 12)$     25. prime  
 27.  $(w - 32)(w + 2)$     29.  $(y - 5)(y - 13)$     31.  $(r + 3)(r + 9)$     33. prime    35.  $(x + 6y)(x + y)$     37.  $(x - 3y)(x - 5y)$   
 39.  $(x - 6y)(x + 3y)$     41.  $(a - 15b)(a - 3b)$     43.  $3(x + 2)(x + 3)$     45.  $4(y - 2)(y + 1)$     47.  $10(x - 10)(x + 6)$