# A Review of Basic Algebra

# 0



## CAREERS AND MATHEMATICS: Pharmacist

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Pharmacists distribute prescription drugs to individuals. They also advise patients, physicians, and other health-care workers on the selection, dosages, interactions, and side effects of medications. They also monitor patients to ensure that they are using their medications safely and effectively. Some pharmacists specialize in oncology, nuclear pharmacy, geriatric pharmacy, or psychiatric pharmacy.

### Education and Mathematics Required

- Pharmacists are required to possess a Pharm.D. degree from an accredited college or school of pharmacy. This degree generally takes four years to complete. To be admitted to a Pharm.D. program, at least two years of college must be completed, which includes courses in the natural sciences, mathematics, humanities, and the social sciences. A series of examinations must also be passed to obtain a license to practice pharmacy.
- College Algebra, Trigonometry, Statistics, and Calculus I are courses required for admission to a Pharm.D. program.

### How Pharmacists Use Math and Who Employs Them

- Pharmacists use math throughout their work to calculate dosages of various drugs. These dosages are based on weight and whether the medication is given in pill form, by infusion, or intravenously.
- Most pharmacists work in a community setting, such as a retail drugstore, or in a healthcare facility, such as a hospital.

### Career Outlook and Salary

- Employment of pharmacists is expected to grow by 17 percent between 2008 and 2018, which is faster than the average for all occupations.
- Median annual wages of wage and salary pharmacists is approximately $106,410.

For more information see: www.bls.gov/oco

In this chapter, we review many concepts and skills learned in previous algebra courses. Be sure to master this material now, because it is the basis for the rest of this course.





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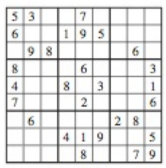
# 0.1 Sets of Real Numbers

In this section, we will learn to

1. Identify sets of real numbers.
2. Identify properties of real numbers.
3. Graph subsets of real numbers on the number line.
4. Graph intervals on the number line.
5. Define absolute value.
6. Find distances on the number line.



SUDOKU

Sudoku, a game that involves number placement, is very popular. The objective is to fill a 9 by 9 grid so that each column, each row and each of the 3 by 3 blocks contains the numbers from 1 to 9. A partially completed Sudoku grid is shown in the margin.

To solve Sudoku puzzles, logic and the set of numbers, {1, 2, 3, 4, 5, 6, 7, 8, 9} are used. Sets of numbers are important in mathematics, and we begin our study of algebra with this topic.

A **set** is a collection of objects, such as a set of dishes or a set of golf clubs. The set of vowels in the English language can be denoted as {a, e, i, o, u}, where the braces { } are read as "the set of."

If every member of one set $B$ is also a member of another set $A$, we say that $B$ is a **subset** of $A$. We can denote this by writing $B \subset A$, where the symbol $\subset$ is read as "is a subset of." (See Figure 0.1 below.) If set $B$ equals set $A$, we can write $B \subseteq A$.

If A and B are two sets, we can form a new set consisting of all members that are in set A or set B or both. This set is called the **union** of A and B. We can denote this set by writing, $A \cup B$ where the symbol $\cup$ is read as "union." (See Figure 0.1 below.)

We can also form the set consisting of all members that are in both set A and set B. This set is called the **intersection** of A and B. We can denote this set by writing $A \cap B$, where the symbol $\cap$ is read as "intersection." (See Figure 0.1 below.)

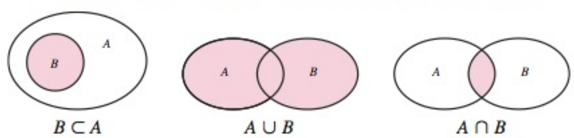



FIGURE 0-1

**EXAMPLE 1** **Understanding Subsets and Finding the Union and Intersection of Two Sets**

Let $A = \{a, e, i\}$, $B = \{c, d, e\}$, and $V = \{a, e, i, o, u\}$.

**a.** Is $A \subset V$? **b.** Find $A \cup B$. **c.** Find $A \cap B$.

**SOLUTION**

**a.** Since each member of set $A$ is also a member of set $V$, $A \subset V$.

**b.** The union of set $A$ and set $B$ contains the members of set $A$, set $B$, or both. Thus, $A \cup B = \{a, c, d, e, i\}$.

**c.** The intersection of set $A$ and set $B$ contains the members that are in both set $A$ and set $B$. Thus, $A \cap B = \{e\}$.

**Self Check 1** **a.** Is $B \subset V$? **b.** Find $B \cup V$
**c.** Find $A \cap V$

**Now Try Exercise 33.**

## 1. Identify Sets of Real Numbers

There are several sets of numbers that we use in everyday life.

**Basic Sets of Numbers**

**Natural numbers**
The numbers that we use for counting: $\{1, 2, 3, 4, 5, 6, \ldots\}$

**Whole numbers**
The set of natural numbers including 0: $\{0, 1, 2, 3, 4, 5, 6, \ldots\}$

**Integers**
The set of whole numbers and their negatives:
$\{\ldots, -5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5, \ldots\}$

In the definitions above, each group of three dots (called an *ellipsis*) indicates that the numbers continue forever in the indicated direction.

Two important subsets of the natural numbers are the *prime* and *composite* numbers. A **prime number** is a natural number greater than 1 that is divisible only by itself and 1. A **composite number** is a natural number greater than 1 that is not prime.

- **The set of prime numbers**: $\{2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, \ldots\}$
- **The set of composite numbers**: $\{4, 6, 8, 9, 10, 12, 14, 15, 16, 18, 20, 21, \ldots\}$

Two important subsets of the set of integers are the *even* and *odd integers*. The **even integers** are the integers that are exactly divisible by 2. The **odd integers** are the integers that are not exactly divisible by 2.

- **The set of even integers:** $\{\ldots, -10, -8, -6, -4, -2, 0, 2, 4, 6, 8, 10, \ldots\}$
- **The set of odd integers:** $\{\ldots, -9, -7, -5, -3, -1, 1, 3, 5, 7, 9, \ldots\}$

So far, we have listed numbers inside braces to specify sets. This method is called the **roster method**. When we give a rule to determine which numbers are in a set, we are using **set-builder notation**. To use set-builder notation to denote the set of prime numbers, we write

$$\{\underbrace{x}_{\text{variable}} \underbrace{|}_{\text{such that}} \underbrace{x \text{ is a prime number}}_{\text{rule that determines membership in the set}}\}$$

Read as "the set of all numbers $x$ such that $x$ is a prime number." Recall that when a letter stands for a number, it is called a variable.

**Caution**

Remember that the denominator of a fraction can **never** be 0.

The fractions of arithmetic are called *rational numbers*.

**Rational Numbers**

**Rational numbers** are fractions that have an integer numerator and a nonzero integer denominator. Using set-builder notation, the rational numbers are

$$\left\{\frac{a}{b} \mid a \text{ is an integer and } b \text{ is a nonzero integer}\right\}$$

Rational numbers can be written as fractions or decimals. Some examples of rational numbers are

$$5 = \frac{5}{1}, \quad \frac{3}{4} = 0.75, \quad -\frac{1}{3} = -0.333\ldots, \quad -\frac{5}{11} = -0.454545\ldots$$

The = sign indicates that two quantities are equal.

These examples suggest that the decimal forms of all rational numbers are either *terminating decimals* or *repeating decimals.*

**EXAMPLE 2** **Determining whether the Decimal Form of a Fraction Terminates or Repeats**

Determine whether the decimal form of each fraction terminates or repeats:

**a.** $\frac{7}{16}$ **b.** $\frac{65}{99}$

**SOLUTION** In each case, we perform a long division and write the quotient as a decimal.

**a.** To change $\frac{7}{16}$ to a decimal, we perform a long division to get $\frac{7}{16} = 0.4375$. Since 0.4375 terminates, we can write $\frac{7}{16}$ as a terminating decimal.

**b.** To change $\frac{65}{99}$ to a decimal, we perform a long division to get $\frac{65}{99} = 0.656565\ldots$ . Since 0.656565 ... repeats, we can write $\frac{65}{99}$ as a repeating decimal.

We can write repeating decimals in compact form by using an overbar. For example, $0.656565\ldots = 0.\overline{65}$.

**Self Check 2** Determine whether the decimal form of each fraction terminates or repeats:

**a.** $\frac{38}{99}$ **b.** $\frac{7}{8}$

**Now Try Exercise 35.**

Some numbers have decimal forms that neither terminate nor repeat. These nonterminating, nonrepeating decimals are called **irrational numbers**. Three examples of irrational numbers are

$$1.010010001000010\ldots \qquad \sqrt{2} = 1.414213562\ldots \qquad \text{and} \qquad \pi = 3.141592654\ldots$$

The union of the set of rational numbers (the terminating and repeating decimals) and the set of irrational numbers (the nonterminating, nonrepeating decimals) is the set of *real numbers* (the set of all decimals).

**Real Numbers** A **real number** is any number that is rational or irrational. Using set-builder notation, the set of real numbers is

$$\{x \mid x \text{ is a rational or an irrational number}\}$$

**EXAMPLE 3** **Classifying Real Numbers**

In the set $\{-3, -2, 0, \frac{1}{2}, 1, \sqrt{5}, 2, 4, 5, 6\}$, list all

**a.** even integers **b.** prime numbers **c.** rational numbers

**SOLUTION** We will check to see whether each number is a member of the set of even integers, the set of prime numbers, and the set of rational numbers.

**a.** even integers: $-2, 0, 2, 4, 6$

**b.** prime numbers: 2, 5

**c.** rational numbers: $-3, -2, 0, \frac{1}{2}, 1, 2, 4, 5, 6$

**Self Check 3** In the set in Example 3, list all **a.** odd integers **b.** composite numbers **c.** irrational numbers.

**Now Try Exercise 43.**

Figure 0-2 shows how the previous sets of numbers are related.

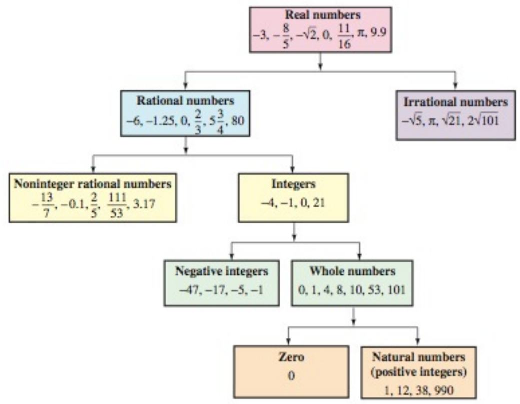



FIGURE 0-2

## 2. Identify Properties of Real Numbers

When we work with real numbers, we will use the following properties.

**Properties of Real Numbers** If $a$, $b$, and $c$ are real numbers,

**The Commutative Properties for Addition and Multiplication**

$$a + b = b + a \qquad ab = ba$$

**The Associative Properties for Addition and Multiplication**

$$(a + b) + c = a + (b + c) \qquad (ab)c = a(bc)$$

**The Distributive Property of Multiplication over Addition or Subtraction**

$$a(b + c) = ab + ac \quad \text{or} \quad a(b - c) = ab - ac$$

**The Double Negative Rule**

$$-(-a) = a$$

**Caution** When the Associative Property is used, the order of the real numbers does **not** change. The real numbers that occur within the parentheses change.

The Distributive Property also applies when more than two terms are within parentheses.

**EXAMPLE 4** **Identifying Properties of Real Numbers**

Determine which property of real numbers justifies each statement.

**a.** $(9 + 2) + 3 = 9 + (2 + 3)$ **b.** $3(x + y + 2) = 3x + 3y + 3 \cdot 2$

**SOLUTION** We will compare the form of each statement to the forms listed in the properties of real numbers box.

**a.** This form matches the Associative Property of Addition.

**b.** This form matches the Distributive Property.

**Self Check 4** Determine which property of real numbers justifies each statement:

**a.** $mn = nm$ **b.** $(xy)z = x(yz)$ **c.** $p + q = q + p$

**Now Try Exercise 17.**

## 3. Graph Subsets of Real Numbers on the Number Line

We can graph subsets of real numbers on the **number line**. The number line shown in Figure 0-3 continues forever in both directions. The **positive numbers** are represented by the points to the right of 0, and the **negative numbers** are represented by the points to the left of 0.

**Comment**

Zero is neither positive nor negative.

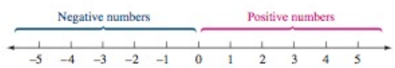



FIGURE 0-3

Figure 0-4(a) shows the graph of the natural numbers from 1 to 5. The point associated with each number is called the *graph* of the number, and the number is called the *coordinate* of its point.

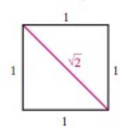
Figure 0-4(b) shows the graph of the prime numbers that are less than 10.

Figure 0-4(c) shows the graph of the integers from −4 to 3.

Figure 0-4(d) shows the graph of the real numbers $-\frac{7}{3}$, $-\frac{3}{4}$, $0.\overline{3}$, and $\sqrt{2}$.

**Comment**

$\sqrt{2}$ can be shown as the diagonal of a square with sides of length 1.



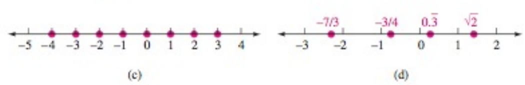



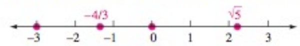

FIGURE 0-4

The graphs in Figure 0-4 suggest that there is a **one-to-one correspondence** between the set of real numbers and the points on a number line. This means that to each real number there corresponds exactly one point on the number line, and to each point on the number line there corresponds exactly one real-number coordinate.

**EXAMPLE 5** **Graphing a Set of Numbers on a Number Line**

Graph the set $\left\{-3, -\frac{4}{3}, 0, \sqrt{5}\right\}$.

**SOLUTION** We will mark (plot) each number on the number line. To the nearest tenth, $\sqrt{5} = 2.2$.




**Self Check 5** Graph the set $\left\{-2, \frac{3}{4}, \sqrt{3}\right\}$. (*Hint*: To the nearest tenth, $\sqrt{3} = 1.7$.)

**Now Try Exercise 51.**

## 4. Graph Intervals on the Number Line

To show that two quantities are not equal, we can use an **inequality symbol.**

| Symbol | Read as | Examples | | |
|---|---|---|---|---|
| $\neq$ | "is not equal to" | $5 \neq 8$ | and | $0.25 \neq \frac{1}{3}$ |
| $<$ | "is less than" | $12 < 20$ | and | $0.17 < 1.1$ |
| $>$ | "is greater than" | $15 > 9$ | and | $\frac{1}{2} > 0.2$ |
| $\leq$ | "is less than or equal to" | $25 \leq 25$ | and | $1.7 \leq 2.3$ |
| $\geq$ | "is greater than or equal to" | $19 \geq 19$ | and | $15.2 \geq 13.7$ |
| $\approx$ | "is approximately equal to" | $\sqrt{2} \approx 1.414$ | and | $\sqrt{3} \approx 1.732$ |

It is possible to write an inequality with the inequality symbol pointing in the opposite direction. For example,

- $12 < 20$ is equivalent to $20 > 12$
- $2.3 \geq -1.7$ is equivalent to $-1.7 \leq 2.3$

In Figure 0-3, the coordinates of points get larger as we move from left to right on a number line. Thus, if $a$ and $b$ are the coordinates of two points, the one to the right is the greater. This suggests the following facts:

- If $a > b$, point $a$ lies to the right of point $b$ on a number line.
- If $a < b$, point $a$ lies to the left of point $b$ on a number line.

Figure 0-5(a) shows the graph of the *inequality* $x > -2$ (or $-2 < x$). This graph includes all real numbers $x$ that are greater than $-2$. The parenthesis at $-2$ indicates that $-2$ is not included in the graph. Figure 0-5(b) shows the graph of $x \leq 5$ (or $5 \geq x$). The bracket at 5 indicates that 5 is included in the graph.


(a) (b)

**FIGURE 0-5**

Sometimes two inequalities can be written as a single expression called a **compound inequality**. For example, the compound inequality

$$5 < x < 12$$

is a combination of the inequalities $5 < x$ and $x < 12$. It is read as "5 is less than $x$, and $x$ is less than 12," and it means that $x$ is between 5 and 12. Its graph is shown in Figure 0-6.

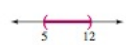



**FIGURE 0-6**

The graphs shown in Figures 0-5 and 0-6 are portions of a number line called **intervals**. The interval shown in Figure 0-7(a) is denoted by the inequality $-2 < x < 4$, or in **interval notation** as $(-2, 4)$. The parentheses indicate that the endpoints are not included. The interval shown in Figure 0-7(b) is denoted by the inequality $x > 1$, or as $(1, \infty)$ in interval notation.


$(-2, 4)$
$-2 < x < 4$
$x$ is between $-2$ and 4.

(a)

$(1, \infty)$
$x > 1$
$x$ is greater than 1.

(b)

**FIGURE 0-7**

**Caution**

The symbol $\infty$ (infinity) is **not** a real number. It is used to indicate that the graph in Figure 0-7(b) extends infinitely far to the right.

A compound inequality such as $-2 < x < 4$ can be written as two separate inequalities:

$x > -2$ and $x < 4$

This expression represents the **intersection** of two intervals. In interval notation, this expression can be written as

$(-2, \infty) \cap (-\infty, 4)$ Read the symbol $\cap$ as "intersection."

Since the graph of $-2 < x < 4$ will include all points whose coordinates satisfy both $x > -2$ and $x < 4$ at the same time, its graph will include all points that are larger than $-2$ but less than 4. This is the interval $(-2, 4)$, whose graph is shown in Figure 0-7(a).

**EXAMPLE 6 Writing an Inequality in Interval Notation and Graphing the Inequality**

Write the inequality $-3 < x < 5$ in interval notation and graph it.

**SOLUTION** This is the interval $(-3, 5)$. Its graph includes all real numbers between $-3$ and 5, as shown in Figure 0-8.

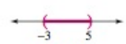



**FIGURE 0-8**

**Self Check 6** Write the inequality $-2 < x \leq 5$ in interval notation and graph it.

**Now Try Exercise 63.**

If an interval extends forever in one direction, it is called an **unbounded interval**.

**Unbounded Intervals**

| Interval | Inequality | Graph |
|---|---|---|
| $(a, \infty)$ | $x > a$ |  |
| $(2, \infty)$ | $x > 2$ |  |
| $[a, \infty)$ | $x \geq a$ |  |
| $[2, \infty)$ | $x \geq 2$ |  |
| $(-\infty, a)$ | $x < a$ |  |
| $(-\infty, 2)$ | $x < 2$ |  |
| $(-\infty, a]$ | $x \leq a$ |  |
| $(-\infty, 2]$ | $x \leq 2$ |  |
| $(-\infty, \infty)$ | $-\infty < x < \infty$ |  |

$(-3, 2)$
$-3 < x < 2$

(a)

$[-2, 3)$
$-2 \leq x < 3$

(b)

**FIGURE 0-9**

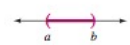
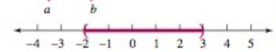
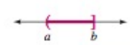
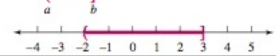
A bounded interval with no endpoints is called an **open interval**. Figure 0-9(a) shows the open interval between $-3$ and 2. A bounded interval with one endpoint is called a **half-open interval**. Figure 0-9(b) shows the half-open interval between $-2$ and 3, including $-2$.

Intervals that include two endpoints are called **closed intervals**. Figure 0-10 shows the graph of a closed interval from $-2$ to 4.

$[-2, 4]$
$-2 \leq x \leq 4$

**FIGURE 0-10**

**Open Intervals, Half-Open Intervals, and Closed Intervals**

| Interval | Inequality | Graph |
|---|---|---|
| **Open** | | |
| $(a, b)$ | $a < x < b$ |  |
| $(-2, 3)$ | $-2 < x < 3$ |  |
| **Half-Open** | | |
| $[a, b)$ | $a \leq x < b$ |  |
| $[-2, 3)$ | $-2 \leq x < 3$ |  |
| **Half-Open** | | |
| $(a, b]$ | $a < x \leq b$ |  |
| $(-2, 3]$ | $-2 < x \leq 3$ |  |
| **Closed** | | |
| $[a, b]$ | $a \leq x \leq b$ |  |
| $[-2, 3]$ | $-2 \leq x \leq 3$ |  |

**EXAMPLE 7** **Writing an Inequality in Interval Notation and Graphing the Inequality**

Write the inequality $3 \leq x$ in interval notation and graph it.

**SOLUTION** The inequality $3 \leq x$ can be written in the form $x \geq 3$. This is the interval $[3, \infty)$. Its graph includes all real numbers greater than or equal to 3, as shown in Figure 0-11.


FIGURE 0-11

**Self Check 7** Write the inequality $5 > x$ in interval notation and graph it.

**Now Try Exercise 65.**

**EXAMPLE 8** **Writing an Inequality in Interval Notation and Graphing the Inequality**

Write the inequality $5 \geq x \geq -1$ in interval notation and graph it.

**SOLUTION** The inequality $5 \geq x \geq -1$ can be written in the form

$$-1 \leq x \leq 5$$

This is the interval $[-1, 5]$. Its graph includes all real numbers from $-1$ to 5. The graph is shown in Figure 0-12.


FIGURE 0-12

**Self Check 8** Write the inequality $0 \leq x \leq 3$ in interval notation and graph it.

**Now Try Exercise 69.**

The expression

$$x < -2 \text{ or } x \geq 3$$ Read as "$x$ is less than $-2$ or $x$ is greater than or equal to 3."

represents the *union* of two intervals. In interval notation, it is written as

$$(-\infty, -2) \cup [3, \infty)$$ Read the symbol $\cup$ as "union."

Its graph is shown in Figure 0-13.

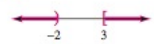



FIGURE 0-13

## 5. Define Absolute Value

The **absolute value** of a real number $x$ (denoted as $|x|$) is the distance on a number line between 0 and the point with a coordinate of $x$. For example, points with coordinates of 4 and $-4$ both lie four units from 0, as shown in Figure 0-14. Therefore, it follows that

$$|-4| = |4| = 4$$

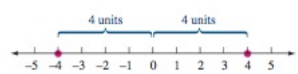



FIGURE 0-14

In general, for any real number $x$,

$$|-x| = |x|$$

We can define absolute value algebraically as follows.

**Absolute Value** If $x$ is a real number, then

$$|x| = x \qquad \text{when } x \geq 0$$
$$|x| = -x \qquad \text{when } x < 0$$

**Caution**

Remember that $x$ is **not** always positive and $-x$ is **not** always negative.

This definition indicates that when $x$ is positive or 0, then $x$ is its own absolute value. However, when $x$ is negative, then $-x$ (which is positive) is its absolute value. Thus, $|x|$ is always nonnegative.

$$|x| \geq 0 \qquad \text{for all real numbers } x$$

### EXAMPLE 9 Using the Definition of Absolute Value

Write each number without using absolute value symbols:

**a.** $|3|$ **b.** $|-4|$ **c.** $|0|$ **d.** $-|-8|$

**SOLUTION** In each case, we will use the definition of absolute value.

**a.** $|3| = 3$ **b.** $|-4| = 4$ **c.** $|0| = 0$ **d.** $-|-8| = -(8) = -8$

**Self Check 9** Write each number without using absolute value symbols:
**a.** $|-10|$ **b.** $|12|$ **c.** $-|6|$

**Now Try Exercise 85.**

In Example 10, we must determine whether the number inside the absolute value is positive or negative.

### EXAMPLE 10 Simplifying an Expression with Absolute Value Symbols

Write each number without using absolute value symbols:

**a.** $|\pi - 1|$ **b.** $|2 - \pi|$ **c.** $|2 - x|$ if $x \geq 5$

**SOLUTION** **a.** Since $\pi \approx 3.1416$, $\pi - 1$ is positive, and $\pi - 1$ is its own absolute value.

$$|\pi - 1| = \pi - 1$$

**b.** Since $2 - \pi$ is negative, its absolute value is $-(2 - \pi)$.

$$|2 - \pi| = -(2 - \pi) = -2 - (-\pi) = -2 + \pi = \pi - 2$$

**c.** Since $x \geq 5$, the expression $2 - x$ is negative, and its absolute value is $-(2 - x)$.

$$|2 - x| = -(2 - x) = -2 + x = x - 2 \text{ provided } x \geq 5$$

**Self Check 10** Write each number without using absolute value symbols. (*Hint:* $\sqrt{5} \approx 2.236$.)
**a.** $|2 - \sqrt{5}|$ **b.** $|2 - x|$ if $x \leq 1$

**Now Try Exercise 89.**

## 6. Find Distances on the Number Line

On the number line shown in Figure 0-15, the distance between the points with coordinates of 1 and 4 is $4 - 1$, or 3 units. However, if the subtraction were done in the other order, the result would be $1 - 4$, or $-3$ units. To guarantee that the distance between two points is always positive, we can use absolute value symbols. Thus, the distance $d$ between two points with coordinates of 1 and 4 is

$$d = |4 - 1| = |1 - 4| = 3$$

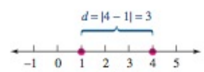



**FIGURE 0-15**

In general, we have the following definition for the distance between two points on the number line.

**Distance between Two Points** If $a$ and $b$ are the coordinates of two points on the number line, the distance between the points is given by the formula

$$d = |b - a|$$

**EXAMPLE 11** **Finding the Distance between Two Points on a Number Line**

Find the distance on a number line between points with coordinates of **a.** 3 and 5 **b.** $-2$ and 3 **c.** $-5$ and $-1$

**SOLUTION** We will use the formula for finding the distance between two points.

**a.** $d = |5 - 3| = |2| = 2$

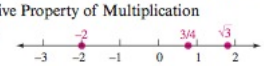
**b.** $d = |3 - (-2)| = |3 + 2| = |5| = 5$

**c.** $d = |-1 - (-5)| = |-1 + 5| = |4| = 4$

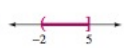
**Self Check 11** Find the distance on a number line between points with coordinates of **a.** 4 and 10 **b.** $-2$ and $-7$

**Now Try Exercise 99.**

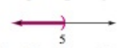
**Self Check Answers** **1. a.** no **b.** {a, c, d, e, i, o, u} **c.** {a, e, i} **2. a.** repeats **b.** terminates **3. a.** $-3, 1, 5$ **b.** 4, 6 **c.** $\sqrt{5}$ **4. a.** Commutative Property of Multiplication **b.** Associative Property of Multiplication **c.** Commutative Property of Addition **5.**



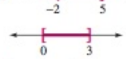

**6.** $(-2, 5]$




**7.** $(-\infty, 5)$




**8.** $[0, 3]$




**9. a.** 10 **b.** 12 **c.** $-6$

**10. a.** $\sqrt{5} - 2$ **b.** $2 - x$ **11. a.** 6 **b.** 5

## Exercises 0.1

### Getting Ready

***You should be able to complete these vocabulary and concept statements before you proceed to the practice exercises.***

***Fill in the blanks.***

**1.** A _____ is a collection of objects.

**2.** If every member of one set $B$ is also a member of a second set $A$, then $B$ is called a ______ of $A$.

**3.** If $A$ and $B$ are two sets, the set that contains all members that are in sets $A$ and $B$ or both is called the ______ of $A$ and $B$.

**4.** If $A$ and $B$ are two sets, the set that contains all members that are in both sets is called the __________ of $A$ and $B$.

**5.** A real number is any number that can be expressed as a _______ .

**6.** A _______ is a letter that is used to represent a number.

**7.** The smallest prime number is ___ .

**8.** All integers that are exactly divisible by 2 are called _____ integers.

**9.** Natural numbers greater than 1 that are not prime are called _________ numbers.

**10.** Fractions such as $\frac{2}{3}$, $\frac{8}{2}$, and $-\frac{7}{9}$ are called _______ numbers.

**11.** Irrational numbers are ________ that don't terminate and don't repeat.

**12.** The symbol _____ is read as "is less than or equal to."

**13.** On a number line, the ________ numbers are to the left of 0.

**14.** The only integer that is neither positive nor negative is ___ .

**15.** The Associative Property of Addition states that $(x + y) + z =$ __________ .

**16.** The Commutative Property of Multiplication states that $xy =$ ____ .

**17.** Use the Distributive Property to complete the statement: $5(m + 2) =$ __________ .

**18.** The statement $(m + n)p = p(m + n)$ illustrates the ___________ Property of ___________ .

**19.** The graph of an _______ is a portion of a number line.

**20.** The graph of an open interval has ____ endpoints.

**21.** The graph of a closed interval has _____ endpoints.

**22.** The graph of a ________ interval has one endpoint.

**23.** Except for 0, the absolute value of every number is _______ .

**24.** The _______ between two distinct points on a number line is always positive.

***Let***

$\mathbf{N}$ = *the set of natural numbers*
$\mathbf{W}$ = *the set of whole numbers*
$\mathbf{Z}$ = *the set of integers*
$\mathbf{Q}$ = *the set of rational numbers*
$\mathbf{R}$ = *the set of real numbers*

***Determine whether each statement is true or false. Read the symbol $\subset$ as "is a subset of."***

**25.** $\mathrm{N} \subset \mathrm{W}$

**26.** $\mathrm{Q} \subset \mathrm{R}$

**27.** $\mathrm{Q} \subset \mathrm{N}$

**28.** $\mathrm{Z} \subset \mathrm{Q}$

**29.** $\mathrm{W} \subset \mathrm{Z}$

**30.** $\mathrm{R} \subset \mathrm{Z}$

### Practice

***Let $A = \{a, b, c, d, e\}$, $B = \{d, e, f, g\}$, and $C = \{a, c, e, f\}$. Find each set.***

**31.** $A \cup B$

**32.** $A \cap B$

**33.** $A \cap C$

**34.** $B \cup C$

***Determine whether the decimal form of each fraction terminates or repeats.***

**35.** $\frac{9}{16}$

**36.** $\frac{3}{8}$

**37.** $\frac{3}{11}$

**38.** $\frac{5}{12}$

***Consider the following set:***
$\{-5, -4, -\frac{2}{3}, 0, 1, \sqrt{2}, 2, 2.75, 6, 7\}$.

**39.** Which numbers are natural numbers?

**40.** Which numbers are whole numbers?

**41.** Which numbers are integers?

**42.** Which numbers are rational numbers?

**43.** Which numbers are irrational numbers?

**44.** Which numbers are prime numbers?

**45.** Which numbers are composite numbers?

**46.** Which numbers are even integers?

**47.** Which numbers are odd integers?

**48.** Which numbers are negative numbers?

***Graph each subset of the real numbers on a number line.***

**49.** The natural numbers between 1 and 5

**50.** The composite numbers less than 10

**51.** The prime numbers between 10 and 20

**52.** The integers from –2 to 4

**53.** The integers between –5 and 0

**54.** The even integers between –9 and –1

**55.** The odd integers between –6 and 4

**56.** $-0.7$, 1.75, and $3\frac{7}{8}$

***Write each inequality in interval notation and graph the interval.***

**57.** $x > 2$ **58.** $x < 4$

**59.** $0 < x < 5$ **60.** $-2 < x < 3$

**61.** $x > -4$ **62.** $x < 3$

**63.** $-2 \le x < 2$ **64.** $-4 < x \le 1$

**65.** $x \le 5$ **66.** $x \ge -1$

**67.** $-5 < x \le 0$ **68.** $-3 \le x < 4$

**69.** $-2 \le x \le 3$ **70.** $-4 \le x \le 4$

**71.** $6 \ge x \ge 2$ **72.** $3 \ge x \ge -2$

***Write each pair of inequalities as the intersection of two intervals and graph the result.***

**73.** $x > -5$ and $x < 4$

**74.** $x \ge -3$ and $x < 6$

**75.** $x \ge -8$ and $x \le -3$

**76.** $x > 1$ and $x \le 7$

***Write each inequality as the union of two intervals and graph the result.***

**77.** $x < -2$ or $x > 2$

**78.** $x \le -5$ or $x > 0$

**79.** $x \le -1$ or $x \ge 3$

**80.** $x < -3$ or $x \ge 2$

***Write each expression without using absolute value symbols.***

**81.** $|13|$ **82.** $|-17|$

**83.** $|0|$ **84.** $-|63|$

**85.** $-|-8|$ **86.** $|-25|$

**87.** $-|32|$ **88.** $-|-6|$

**89.** $|\pi - 5|$ **90.** $|8 - \pi|$

**91.** $|\pi - \pi|$ **92.** $|2\pi|$

**93.** $|x + 1|$ and $x \ge 2$ **94.** $|x + 1|$ and $x \le -2$

**95.** $|x - 4|$ and $x < 0$ **96.** $|x - 7|$ and $x > 10$

***Find the distance between each pair of points on the number line.***

**97.** 3 and 8 **98.** $-5$ and 12

**99.** $-8$ and $-3$ **100.** 6 and $-20$

## Applications

**101.** What subset of the real numbers would you use to describe the populations of several cities?

**102.** What subset of the real numbers would you use to describe the subdivisions of an inch on a ruler?

**103.** What subset of the real numbers would you use to report temperatures in several cities?

**104.** What subset of the real numbers would you use to describe the financial condition of a business?

## Discovery and Writing

**105.** Explain why $-x$ could be positive.

**106.** Explain why every integer is a rational number.

**107.** Is the statement $|ab| = |a| \cdot |b|$ always true? Explain.

**108.** Is the statement $\left|\frac{a}{b}\right| = \frac{|a|}{|b|}$ $(b \neq 0)$ always true? Explain.

**109.** Is the statement $|a + b| = |a| + |b|$ always true? Explain.

**110.** Under what conditions will the statement given in Exercise 109 be true?

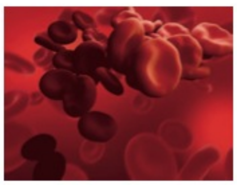
**111.** Explain why it is incorrect to write $a < b > c$ if $a < b$ and $b > c$.

**112.** Explain why $|b - a| = |a - b|$.

# 0.2 Integer Exponents and Scientific Notation

In this section, we will learn to

1. Define natural-number exponents.
2. Apply the rules of exponents.
3. Apply the rules for order of operations to evaluate expressions.
4. Express numbers in scientific notation.
5. Use scientific notation to simplify computations.



The number of cells in the human body is approximated to be one hundred trillion or 100,000,000,000,000. One hundred trillion is $(10)(10)(10)\cdots(10)$, where ten occurs fourteen times. Fourteen factors of ten can be written as $10^{14}$.

In this section, we will use integer exponents to represent repeated multiplication of numbers.

## 1. Define Natural-Number Exponents

When two or more quantities are multiplied together, each quantity is called a *factor* of the product. The exponential expression $x^4$ indicates that $x$ is to be used as a factor four times.

$$x^4 = \overbrace{x \cdot x \cdot x \cdot x}^{4 \text{ factors of } x}$$

In general, the following is true.

**Natural-Number Exponents** For any natural number $n$,

$$x^n = \overbrace{x \cdot x \cdot x \cdot \cdots \cdot x}^{n \text{ factors of } x}$$

In the **exponential expression** $x^n$, $x$ is called the **base**, and $n$ is called the **exponent** or the **power** to which the base is raised. The expression $x^n$ is called a **power of $x$**. From the definition, we see that a natural-number exponent indicates how many times the base of an exponential expression is to be used as a factor in a product. If an exponent is 1, the 1 is usually not written:

$$x^1 = x$$

**EXAMPLE 1** **Using the Definition of Natural-Number Exponents**

Write each expression without using exponents:

**a.** $4^2$ **b.** $(-4)^2$ **c.** $-5^3$ **d.** $(-5)^3$ **e.** $3x^4$ **f.** $(3x)^4$

**SOLUTION** In each case, we apply the definition of natural-number exponents.

**a.** $4^2 = 4 \cdot 4 = 16$ Read $4^2$ as "four squared."

**b.** $(-4)^2 = (-4)(-4) = 16$ Read $(-4)^2$ as "negative four squared."

**c.** $-5^3 = -5(5)(5) = -125$ Read $-5^3$ as "the negative of five cubed."

**d.** $(-5)^3 = (-5)(-5)(-5) = -125$ Read $(-5)^3$ as "negative five cubed."

**e.** $3x^4 = 3 \cdot x \cdot x \cdot x \cdot x$ Read $3x^4$ as "3 times $x$ to the fourth power."

**f.** $(3x)^4 = (3x)(3x)(3x)(3x) = 81 \cdot x \cdot x \cdot x \cdot x$ Read $(3x)^4$ as "3$x$ to the fourth power."

**Self Check 1** Write each expression without using exponents:
**a.** $7^3$ **b.** $(-3)^2$ **c.** $5a^3$ **d.** $(5a)^4$

**Now Try Exercise 19.**

**Comment** Note the distinction between $ax^n$ and $(ax)^n$:

$$ax^n = a \cdot \underbrace{x \cdot x \cdot x \cdot \cdots \cdot x}_{n \text{ factors of } x} \qquad (ax)^n = \underbrace{(ax)(ax)(ax) \cdot \cdots \cdot (ax)}_{n \text{ factors of } ax}$$

Also note the distinction between $-x^n$ and $(-x)^n$:

$$-x^n = -\underbrace{(x \cdot x \cdot x \cdot \cdots \cdot x)}_{n \text{ factors of } x} \qquad (-x)^n = \underbrace{(-x)(-x)(-x) \cdot \cdots \cdot (-x)}_{n \text{ factors of } -x}$$

**ACCENT ON TECHNOLOGY**

## Using a Calculator to Find Powers

We can use a graphing calculator to find powers of numbers. For example, consider $2.35^3$.

- Input 2.35 and press the ^ key.
- Input 3 and press ENTER.

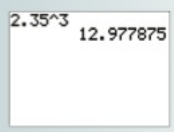



FIGURE 0-16

**Comment**

To find powers on a scientific calculator use the $y^x$ key.

We see that $2.35^3 = 12.977875$ as the figure shows above.

## 2. Apply the Rules of Exponents

We begin the review of the rules of exponents by considering the product $x^m x^n$. Since $x^m$ indicates that $x$ is to be used as a factor $m$ times, and since $x^n$ indicates that $x$ is to be used as a factor $n$ times, there are $m + n$ factors of $x$ in the product $x^m x^n$.

$$x^m x^n = \overbrace{\underbrace{x \cdot x \cdot x \cdot \cdots \cdot x}_{m \text{ factors of } x} \cdot \underbrace{x \cdot x \cdot x \cdot \cdots \cdot x}_{n \text{ factors of } x}}^{m+n \text{ factors of } x} = x^{m+n}$$

This suggests that to multiply exponential expressions with the same base, we *keep the base and add the exponents.*

**Product Rule for Exponents** If $m$ and $n$ are natural numbers, then

$$x^m x^n = x^{m+n}$$

**Comment**

The Product Rule applies to exponential expressions with the same base. A product of two powers with different bases, such as $x^4y^3$, cannot be simplified.

To find another property of exponents, we consider the exponential expression $(x^m)^n$. In this expression, the exponent $n$ indicates that $x^m$ is to be used as a factor $n$ times. This implies that $x$ is to be used as a factor $mn$ times.

$$(x^m)^n = \overbrace{\overbrace{(x^m)(x^m)(x^m) \cdot \cdots \cdot (x^m)}^{n \text{ factors of } x^m}}^{mn \text{ factors of } x} = x^{mn}$$

This suggests that to raise an exponential expression to a power, we *keep the base and multiply the exponents.*

To raise a product to a power, we raise each factor to that power.

$$(xy)^n = \overbrace{(xy)(xy)(xy) \cdot \cdots \cdot (xy)}^{n \text{ factors of } xy} = \overbrace{(x \cdot x \cdot x \cdot \cdots \cdot x)}^{n \text{ factors of } x}\overbrace{(y \cdot y \cdot y \cdot \cdots \cdot y)}^{n \text{ factors of } y} = x^n y^n$$

To raise a fraction to a power, we raise both the numerator and the denominator to that power. If $y \neq 0$, then

$$\left(\frac{x}{y}\right)^n = \overbrace{\left(\frac{x}{y}\right)\left(\frac{x}{y}\right)\left(\frac{x}{y}\right) \cdot \cdots \cdot \left(\frac{x}{y}\right)}^{n \text{ factors of } \frac{x}{y}}$$

$$= \frac{\overbrace{xxx \cdot \cdots \cdot x}^{n \text{ factors of } x}}{\underbrace{yyy \cdot \cdots \cdot y}_{n \text{ factors of } y}}$$

$$= \frac{x^n}{y^n}$$

The previous three results are called the *Power Rules of Exponents.*

**Power Rules of Exponents** If $m$ and $n$ are natural numbers, then

$$(x^m)^n = x^{mn} \qquad (xy)^n = x^n y^n \qquad \left(\frac{x}{y}\right)^n = \frac{x^n}{y^n} \quad (y \neq 0)$$

**EXAMPLE 2 Using Exponent Rules to Simplify Expressions with Natural-Number Exponents**

Simplify: **a.** $x^5x^7$ **b.** $x^2y^3x^5y$ **c.** $(x^4)^9$ **d.** $(x^2x^5)^3$

**e.** $\left(\frac{x}{y^2}\right)^5$ **f.** $\left(\frac{5x^2y}{z^3}\right)^2$

**SOLUTION** In each case, we will apply the appropriate rule of exponents.

**a.** $x^5x^7 = x^{5+7} = x^{12}$ **b.** $x^2y^3x^5y = x^{2+5}y^{3+1} = x^7y^4$

**c.** $(x^4)^9 = x^{4\cdot 9} = x^{36}$ **d.** $(x^2x^5)^3 = (x^7)^3 = x^{21}$

**e.** $\left(\frac{x}{y^2}\right)^5 = \frac{x^5}{(y^2)^5} = \frac{x^5}{y^{10}} \quad (y \neq 0)$

**f.** $\left(\frac{5x^2y}{z^3}\right)^2 = \frac{5^2(x^2)^2y^2}{(z^3)^2} = \frac{25x^4y^2}{z^6} \quad (z \neq 0)$

**Self Check 2** Simplify:

**a.** $(y^5)^2$ **b.** $(a^2a^4)^3$ **c.** $(x^2)^3(x^3)^2$ **d.** $\left(\frac{3a^3b^2}{c^3}\right)^3 \quad (c \neq 0)$

**Now Try Exercise 49.**

If we assume that the rules for natural-number exponents hold for exponents of 0, we can write

$$x^0x^n = x^{0+n} = x^n = 1x^n$$

Since $x^0x^n = 1x^n$, it follows that if $x \neq 0$, then $x^0 = 1$.

**Zero Exponent** $x^0 = 1 \quad (x \neq 0)$

If we assume that the rules for natural-number exponents hold for exponents that are negative integers, we can write

$$x^{-n}x^n = x^{-n+n} = x^0 = 1 \quad (x \neq 0)$$

However, we know that

$$\frac{1}{x^n} \cdot x^n = 1 \quad (x \neq 0) \qquad \frac{1}{x^n} \cdot x^n = \frac{x^n}{x^n}\text{, and any nonzero number divided by itself is 1.}$$

Since $x^{-n}x^n = \frac{1}{x^n} \cdot x^n$, it follows that $x^{-n} = \frac{1}{x^n}$ $(x \neq 0)$.

**Negative Exponents** If $n$ is an integer and $x \neq 0$, then

$$x^{-n} = \frac{1}{x^n} \quad \text{and} \quad \frac{1}{x^{-n}} = x^n$$

Because of the previous definitions, all of the rules for natural-number exponents will hold for integer exponents.

**EXAMPLE 3** **Simplifying Expressions with Integer Exponents**

Simplify and write all answers without using negative exponents:

**a.** $(3x)^0$ **b.** $3(x^0)$ **c.** $x^{-4}$ **d.** $\frac{1}{x^{-6}}$ **e.** $x^{-3}x$ **f.** $(x^{-4}x^8)^{-5}$

**SOLUTION** We will use the definitions of zero exponent and negative exponents to simplify each expression.

**a.** $(3x)^0 = 1$ **b.** $3(x^0) = 3(1) = 3$ **c.** $x^{-4} = \frac{1}{x^4}$ **d.** $\frac{1}{x^{-6}} = x^6$

**e.** $x^{-3}x = x^{-3+1} = x^{-2} = \frac{1}{x^2}$

**f.** $(x^{-4}x^{8})^{-5} = (x^4)^{-5} = x^{-20} = \frac{1}{x^{20}}$

**Self Check 3** Simplify and write all answers without using negative exponents:

**a.** $7a^0$ **b.** $3a^{-2}$ **c.** $a^{-4}a^2$ **d.** $(a^3a^{-7})^3$

**Now Try Exercise 59.**

To develop the Quotient Rule for Exponents, we proceed as follows:

$$\frac{x^m}{x^n} = x^m\left(\frac{1}{x^n}\right) = x^mx^{-n} = x^{m+(-n)} = x^{m-n} \quad (x \neq 0)$$

This suggests that to divide two exponential expressions with the same nonzero base, we *keep the base and subtract the exponent in the denominator from the exponent in the numerator*.

**Quotient Rule for Exponents** If $m$ and $n$ are integers, then

$$\frac{x^m}{x^n} = x^{m-n} \quad (x \neq 0)$$

**EXAMPLE 4 Simplifying Expressions with Integer Exponents**

Simplify and write all answers without using negative exponents:

**a.** $\frac{x^8}{x^5}$ **b.** $\frac{x^2x^4}{x^{-5}}$

**SOLUTION** We will apply the Product and Quotient Rules of Exponents.

**a.** $\frac{x^8}{x^5} = x^{8-5} = x^3$

**b.** $\frac{x^2x^4}{x^{-5}} = \frac{x^6}{x^{-5}} = x^{6-(-5)} = x^{11}$

**Self Check 4** Simplify and write all answers without using negative exponents:

**a.** $\frac{x^{-6}}{x^2}$ **b.** $\frac{x^4x^{-3}}{x^2}$

**Now Try Exercise 69.**

**EXAMPLE 5 Simplifying Expressions with Integer Exponents**

Simplify and write all answers without using negative exponents:

**a.** $\left(\frac{x^3y^{-2}}{x^{-2}y^3}\right)^{-2}$ **b.** $\left(\frac{x}{y}\right)^{-n}$

**SOLUTION** We will apply the appropriate rules of exponents.

**a.** $$\left(\frac{x^3y^{-2}}{x^{-2}y^3}\right)^{-2} = (x^{3-(-2)}y^{-2-3})^{-2}$$
$$= (x^5y^{-5})^{-2}$$
$$= x^{-10}y^{10}$$
$$= \frac{y^{10}}{x^{10}}$$

**b.** $$\left(\frac{x}{y}\right)^{-n} = \frac{x^{-n}}{y^{-n}}$$
$$= \frac{x^{-n}x^ny^n}{y^{-n}x^ny^n} \quad \text{Multiply numerator and denominator by 1 in the form } \frac{x^ny^n}{x^ny^n}.$$
$$= \frac{x^0y^n}{y^0x^n} \quad x^{-n}x^n = x^0 \text{ and } y^{-n}y^n = y^0.$$
$$= \frac{y^n}{x^n} \quad x^0 = 1 \text{ and } y^0 = 1.$$
$$= \left(\frac{y}{x}\right)^n$$

**Self Check 5** Simplify and write all answers without using negative exponents:

**a.** $\left(\frac{x^4y^{-3}}{x^{-3}y^2}\right)^2$ **b.** $\left(\frac{2a}{3b}\right)^{-3}$

**Now Try Exercise 75.**

Part b of Example 5 establishes the following rule.

**A Fraction to a Negative Power** If $n$ is a natural number, then

$$\left(\frac{x}{y}\right)^{-n} = \left(\frac{y}{x}\right)^n \quad (x \neq 0 \quad \text{and} \quad y \neq 0)$$

## 3. Apply the Rules for Order of Operations to Evaluate Expressions

When several operations occur in an expression, we must perform the operations in the following order to get the correct result.

**Strategy for Evaluating Expressions Using Order of Operations** If an expression does not contain grouping symbols such as parentheses or brackets, follow these steps:

1. Find the values of any exponential expressions.
2. Perform all multiplications and/or divisions, working from left to right.
3. Perform all additions and/or subtractions, working from left to right.

- If an expression contains grouping symbols such as parentheses, brackets, or braces, use the rules above to perform the calculations within each pair of grouping symbols, working from the innermost pair to the outermost pair.
- In a fraction, simplify the numerator and the denominator of the fraction separately. Then simplify the fraction, if possible.

**Comment** Many students remember the Order of Operations Rule with the acronym **PEMDAS:**

- **P**arentheses
- **E**xponents
- **M**ultiplication
- **D**ivision
- **A**ddition
- **S**ubtraction

For example, to simplify $\frac{3[4-(6+10)]}{2^2-(6+7)}$, we proceed as follows:

$$\frac{3[4-(6+10)]}{2^2-(6+7)} = \frac{3(4-16)}{2^2-(6+7)}$$ Simplify within the inner parentheses: $6 + 10 = 16$.

$$= \frac{3(-12)}{2^2-13}$$ Simplify within the parentheses: $4 - 16 = -12$, and $6 + 7 = 13$.

$$= \frac{3(-12)}{4-13}$$ Evaluate the power: $2^2 = 4$.

$$= \frac{-36}{-9}$$ $3(-12) = -36$. $4 - 13 = -9$.

$$= 4$$

**EXAMPLE 6 Evaluating Algebraic Expressions**

If $x = -2$, $y = 3$, and $z = -4$, evaluate

**a.** $-x^2 + y^2z$ **b.** $\frac{2z^3 - 3y^2}{5x^2}$

**SOLUTION** In each part, we will substitute the numbers for the variables, apply the rules of order of operations, and simplify.

**a.** $-x^2 + y^2z = -(-2)^2 + 3^2(-4)$

$= -(4) + 9(-4)$ Evaluate the powers.

$= -4 + (-36)$ Do the multiplication.

$= -40$ Do the addition.

**b.** $$\frac{2z^3 - 3y^2}{5x^2} = \frac{2(-4)^3 - 3(3)^2}{5(-2)^2}$$

$$= \frac{2(-64) - 3(9)}{5(4)}$$ Evaluate the powers.

$$= \frac{-128 - 27}{20}$$ Do the multiplications.

$$= \frac{-155}{20}$$ Do the subtraction.

$$= -\frac{31}{4}$$ Simplify the fraction.

**Self Check 6** If $x = 3$ and $y = -2$, evaluate $\frac{2x^2 - 3y^2}{x - y}$.

**Now Try Exercise 91.**

## 4. Express Numbers in Scientific Notation

Scientists often work with numbers that are very large or very small. These numbers can be written compactly by expressing them in *scientific notation.*

**Scientific Notation** A number is written in *scientific notation* when it is written in the form

$$N \times 10^n$$

where $1 \leq |N| < 10$ and $n$ is an integer.

olling/Shutterstock.com

Light travels 29,980,000,000 centimeters per second.

To express this number in scientific notation, we must write it as the product of a number between 1 and 10 and some integer power of 10. The number 2.998 lies between 1 and 10. To get 29,980,000,000, the decimal point in 2.998 must be moved ten places to the right. This is accomplished by multiplying 2.998 by $10^{10}$.

$$\text{Standard notation} \longrightarrow 29{,}980{,}000{,}000 = 2.998 \times 10^{10} \longleftarrow \text{Scientific notation}$$

One meter is approximately 0.0006214 mile. To express this number in scientific notation, we must write it as the product of a number between 1 and 10 and some integer power of 10. The number 6.214 lies between 1 and 10. To get 0.0006214, the decimal point in 6.214 must be moved four places to the left. This is accomplished by multiplying 6.214 by $\frac{1}{10^4}$ or by multiplying 6.214 by $10^{-4}$.

$$\text{Standard notation} \longrightarrow 0.0006214 = 6214 \times 10^{-4} \longleftarrow \text{Scientific notation}$$

To write each of the following numbers in scientific notation, we start to the right of the first nonzero digit and count to the decimal point. The exponent gives the number of places the decimal point moves, and the sign of the exponent indicates the direction in which it moves.

**a.** $3\,7\,2\,0\,0\,0 = 3.72 \times 10^5$ 5 places to the right.

**b.** $0\,.\,0\,0\,0\,5\,3\,7 = 5.37 \times 10^{-4}$ 4 places to the left.

**c.** $7.36 = 7.36 \times 10^0$ No movement of the decimal point.

**EXAMPLE 7 Writing Numbers in Scientific Notation**

Write each number in scientific notation: **a.** 62,000 **b.** $-0.0027$

**SOLUTION**

**a.** We must express 62,000 as a product of a number between 1 and 10 and some integer power of 10. This is accomplished by multiplying 6.2 by $10^4$.

$$62{,}000 = 6.2 \times 10^4$$

**b.** We must express $-0.0027$ as a product of a number whose absolute value is between 1 and 10 and some integer power of 10. This is accomplished by multiplying $-2.7$ by $10^{-3}$.

$$-0.0027 = -2.7 \times 10^{-3}$$

**Self Check 7** Write each number in scientific notation:

**a.** $-93{,}000{,}000$ **b.** 0.0000087

**Now Try Exercise 103.**

**EXAMPLE 8 Writing Numbers in Standard Notation**

Write each number in standard notation:

**a.** $7.35 \times 10^2$ **b.** $3.27 \times 10^{-5}$

**SOLUTION** **a.** The factor of $10^2$ indicates that 7.35 must be multiplied by 2 factors of 10. Because each multiplication by 10 moves the decimal point one place to the right, we have

$$7.35 \times 10^2 = 735$$

**b.** The factor of $10^{-5}$ indicates that 3.27 must be divided by 5 factors of 10. Because each division by 10 moves the decimal point one place to the left, we have

$$3.27 \times 10^{-5} = 0.0000327$$

**Self Check 8** Write each number in standard notation: **a.** $6.3 \times 10^3$ **b.** $9.1 \times 10^{-4}$

**Now Try Exercise 111.**

## 5. Use Scientific Notation to Simplify Computations

Another advantage of scientific notation becomes evident when we multiply and divide very large and very small numbers.

**EXAMPLE 9** **Using Scientific Notation to Simplify Computations**

Use scientific notation to calculate $\dfrac{(3{,}400{,}000)(0.00002)}{170{,}000{,}000}$.

**SOLUTION** After changing each number to scientific notation, we can do the arithmetic on the numbers and the exponential expressions separately.

$$\begin{aligned}\frac{(3{,}400{,}000)(0.00002)}{170{,}000{,}000} &= \frac{(3.4 \times 10^6)(2.0 \times 10^{-5})}{1.7 \times 10^8}\\ &= \frac{6.8}{1.7} \times 10^{6+(-5)-8}\\ &= 4.0 \times 10^{-7}\\ &= 0.0000004\end{aligned}$$

**Self Check 9** Use scientific notation to simplify $\dfrac{(192{,}000)(0.0015)}{(0.0032)(4{,}500)}$.

**Now Try Exercise 119.**

**ACCENT ON TECHNOLOGY**

### Scientific Notation

Graphing calculators will often give an answer in scientific notation. For example, consider $21^8$.

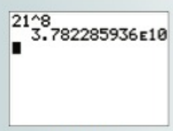



FIGURE 0-17

We see in the figure that the answer given is 3.782285936E10, which means $3.782285936 \times 10^{10}$.

**Comment** To calculate an expression like $\frac{21^8}{0.00000000061}$ on a scientific calculator, it is necessary to convert the denominator to scientific notation because the number has too many digits to fit on the screen.

- For scientific notation, we enter these numbers and press these keys: 6.1 $\boxed{\text{EXP}}$ 11 $\boxed{+/-}$.
- To evaluate the expression above, we enter these numbers and press these keys:

  21 $\boxed{y^x}$ 8 $\boxed{=}$ $\boxed{\div}$ 6.1 $\boxed{\text{EXP}}$ 11 $\boxed{+/-}$ $\boxed{=}$

The display will read $\boxed{6.200468748^{20}}$. In standard notation, the answer is approximately 620,046,874,800,000,000,000.

**Self Check Answers** **1. a.** $7 \cdot 7 \cdot 7 = 343$ **b.** $(-3)(-3) = 9$ **c.** $5 \cdot a \cdot a \cdot a$ **d.** $(5a)(5a)(5a)(5a) = 625 \cdot a \cdot a \cdot a \cdot a$ **2. a.** $y^6$ **b.** $a^{18}$ **c.** $x^{12}$ **d.** $\frac{27a^9b^6}{c^9}$ **3. a.** 7 **b.** $\frac{3}{a^2}$ **c.** $\frac{1}{a^2}$ **d.** $\frac{1}{a^{12}}$ **4. a.** $\frac{1}{x^8}$ **b.** $\frac{1}{x}$ **5. a.** $\frac{x^{14}}{y^{10}}$ **b.** $\frac{27b^3}{8a^3}$ **6.** $\frac{6}{5}$ **7. a.** $-9.3 \times 10^7$ **b.** $8.7 \times 10^{-6}$ **8. a.** 6,300 **b.** 0.00091 **9.** 20

## Exercises 0.2

### Getting Ready

***You should be able to complete these vocabulary and concept statements before you proceed to the practice exercises.***

***Fill in the blanks.***

1. Each quantity in a product is called a ______ of the product.
2. A ______ number exponent tells how many times a base is used as a factor.
3. In the expression $(2x)^3$, ___ is the exponent and ____ is the base.
4. The expression $x^n$ is called an __________ expression.
5. A number is in ________ notation when it is written in the form $N \times 10^n$, where $1 \le |N| < 10$ and $n$ is an _______.
6. Unless __________ indicate otherwise, ____________ are performed before additions.

***Complete each exponent rule. Assume $x \neq 0$.***

7. $x^m x^n =$ _____
8. $(x^m)^n =$ _____
9. $(xy)^n =$ _____
10. $\frac{x^m}{x^n} =$ _____
11. $x^0 =$ __
12. $x^{-n} = \frac{}{\_\_}$

### Practice

***Write each number or expression without using exponents.***

13. $13^2$
14. $10^3$
15. $-5^2$
16. $(-5)^2$
17. $4x^3$
18. $(4x)^3$
19. $(-5x)^4$
20. $-6x^2$
21. $-8x^4$
22. $(-8x)^4$

***Write each expression using exponents.***

23. $7xxx$
24. $-8yyyy$
25. $(-x)(-x)$
26. $(2a)(2a)(2a)$
27. $(3t)(3t)(-3t)$
28. $-(2b)(2b)(2b)(2b)$
29. $xxxyy$
30. $aaabbbb$

***Use a calculator to simplify each expression.***

31. $2.2^3$
32. $7.1^4$
33. $-0.5^4$
34. $(-0.2)^4$

***Simplify each expression. Write all answers without using negative exponents. Assume that all variables are restricted to those numbers for which the expression is defined.***

35. $x^2x^3$
36. $y^3y^4$
37. $(z^2)^3$
38. $(t^6)^7$
39. $(y^5y^2)^3$
40. $(a^3a^6)a^4$
41. $(z^2)^3(z^4)^5$
42. $(t^3)^4(t^5)^2$
43. $(a^2)^3(a^4)^2$
44. $(a^2)^4(a^3)^3$
45. $(3x)^3$
46. $(-2y)^4$
47. $(x^2y)^3$
48. $(x^3z^4)^6$

**49.** $\left(\frac{a^2}{b}\right)^3$

**50.** $\left(\frac{x}{y^3}\right)^4$

**51.** $(-x)^0$

**52.** $4x^0$

**53.** $(4x)^0$

**54.** $-2x^0$

**55.** $z^{-4}$

**56.** $\frac{1}{t^{-2}}$

**57.** $y^{-2}y^{-3}$

**58.** $-m^{-2}m^3$

**59.** $(x^3x^{-4})^{-2}$

**60.** $(y^{-2}y^3)^{-4}$

**61.** $\frac{x^7}{x^3}$

**62.** $\frac{r^5}{r^2}$

**63.** $\frac{a^{21}}{a^{17}}$

**64.** $\frac{t^{13}}{t^4}$

**65.** $\frac{(x^2)^2}{x^2x}$

**66.** $\frac{s^9s^3}{(s^2)^2}$

**67.** $\left(\frac{m^3}{n^2}\right)^3$

**68.** $\left(\frac{t^4}{t^3}\right)^3$

**69.** $\frac{(a^3)^{-2}}{aa^2}$

**70.** $\frac{r^9r^{-3}}{(r^{-2})^3}$

**71.** $\left(\frac{a^{-3}}{b^{-1}}\right)^{-4}$

**72.** $\left(\frac{t^{-4}}{t^{-3}}\right)^{-2}$

**73.** $\left(\frac{r^4r^{-6}}{r^3r^{-3}}\right)^2$

**74.** $\frac{(x^{-3}x^2)^2}{(x^2x^{-5})^{-3}}$

**75.** $\left(\frac{x^5y^{-2}}{x^{-3}y^2}\right)^4$

**76.** $\left(\frac{x^{-7}y^5}{x^7y^{-4}}\right)^3$

**77.** $\left(\frac{5x^{-3}y^{-2}}{3x^2y^{-3}}\right)^{-2}$

**78.** $\left(\frac{3x^2y^{-5}}{2x^{-2}y^{-6}}\right)^{-3}$

**79.** $\left(\frac{3x^5y^{-3}}{6x^{-5}y^3}\right)^{-2}$

**80.** $\left(\frac{12x^{-4}y^3z^{-5}}{4x^4y^{-3}z^5}\right)^3$

**81.** $\frac{(8^{-2}z^{-3}y)^{-1}}{(5y^2z^{-2})^3(5yz^{-2})^{-1}}$

**82.** $\frac{(m^{-2}n^3p^4)^{-2}(mn^{-2}p^3)^4}{(mn^{-2}p^3)^{-4}(mn^2p)^{-1}}$

***Simplify each expression.***

**83.** $-\frac{5[6^2 + (9 - 5)]}{4(2 - 3)^2}$

**84.** $\frac{6[3 - (4 - 7)^2]}{-5(2 - 4^2)}$

***Let $x = -2$, $y = 0$, and $z = 3$ and evaluate each expression.***

**85.** $x^2$

**86.** $-x^2$

**87.** $x^3$

**88.** $-x^3$

**89.** $(-xz)^3$

**90.** $-xz^3$

**91.** $\frac{-(x^2z^3)}{z^2 - y^2}$

**92.** $\frac{z^2(x^2 - y^2)}{x^3z}$

**93.** $5x^2 - 3y^3z$

**94.** $3(x - z)^2 + 2(y - z)^3$

**95.** $\frac{-3x^{-3}z^{-2}}{6x^2z^{-3}}$

**96.** $\frac{(-5x^2z^{-3})^2}{5xz^{-2}}$

***Express each number in scientific notation.***

**97.** 372,000

**98.** 89,500

**99.** −177,000,000

**100.** −23,470,000,000

**101.** 0.007

**102.** 0.00052

**103.** −0.000000693

**104.** −0.000000089

**105.** one trillion

**106.** one millionth

***Express each number in standard notation.***

**107.** $9.37 \times 10^5$

**108.** $4.26 \times 10^9$

**109.** $2.21 \times 10^{-5}$

**110.** $2.774 \times 10^{-2}$

**111.** $0.00032 \times 10^4$

**112.** $9{,}300.0 \times 10^{-4}$

**113.** $-3.2 \times 10^{-3}$

**114.** $-7.25 \times 10^3$

***Use the method of Example 9 to do each calculation. Write all answers in scientific notation.***

**115.** $\frac{(65{,}000)(45{,}000)}{250{,}000}$

**116.** $\frac{(0.000000045)(0.00000012)}{45{,}000{,}000}$

**117.** $\frac{(0.00000035)(170{,}000)}{0.00000085}$

**118.** $\frac{(0.0000000144)(12{,}000)}{600{,}000}$

**119.** $\frac{(45{,}000{,}000{,}000)(212{,}000)}{0.00018}$

**120.** $\frac{(0.00000000275)(4{,}750)}{500{,}000{,}000{,}000}$

## Applications

***Use scientific notation to compute each answer. Write all answers in scientific notation.***

**121. Speed of sound** The speed of sound in air is $3.31 \times 10^4$ centimeters per second. Compute the speed of sound in meters per minute.

**122. Volume of a box** Calculate the volume of a box that has dimensions of 6,000 by 9,700 by 4,700 millimeters.

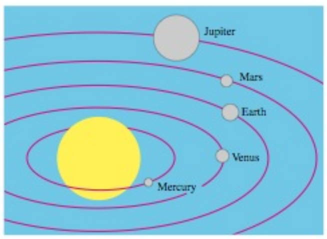
**123. Mass of a proton** The mass of one proton is 0.00000000000000000000000167248 gram. Find the mass of one billion protons.

**124.** **Speed of light** The speed of light in a vacuum is approximately 30,000,000,000 centimeters per second. Find the speed of light in miles per hour. (160,934.4 cm = 1 mile.)

**125.** **Astronomy** The distance $d$, in miles, of the $n$th planet from the sun is given by the formula

$$d = 9{,}275{,}200[3(2^{n-2}) + 4]$$

To the nearest million miles, find the distance of Earth and the distance of Mars from the sun. Give each answer in scientific notation.




**126.** **License Plates** The number of different license plates of the form three digits followed by three letters, as in the illustration, is $10 \cdot 10 \cdot 10 \cdot 26 \cdot 26 \cdot 26$. Write this expression using exponents. Then evaluate it and express the result in scientific notation.


## Discovery and Writing

**127.** **New way to the center of the earth** The spectacular "blue marble' image is the most detailed true-color image of the entire Earth to date. A new NASA-developed technique estimates Earth's center of mass within 1 millimeter (0.04 inch) a year by using a combination of four space-based techniques.

NASA Goddard Space Flight Center

The distance from the Earth's center to the North Pole (the **polar radius**) measures approximately 6,356.750 km, and the distance from the center to the equator (the **equatorial radius** ) measures approximately 6,378.135 km. Express each distance using scientific notation.

**128.** Refer to Exercise 127. Given that 1 km is approximately equal to 0.62 miles, use scientific notation to express each distance in miles.

***Write each expression with a single base.***

**129.** $x^n x^2$

**130.** $\dfrac{x^m}{x^3}$

**131.** $\dfrac{x^m x^2}{x^3}$

**132.** $\dfrac{x^{3m+5}}{x^2}$

**133.** $x^{m+1}x^3$

**134.** $a^{n-3}a^3$

**135.** Explain why $-x^4$ and $(-x)^4$ represent different numbers.

**136.** Explain why $32 \times 10^2$ is not in scientific notation.

## Review

**137.** Graph the interval $(-2,4)$.

**138.** Graph the interval $(-\infty,-3] \cup [3,\infty)$.

**139.** Evaluate $|\pi - 5|$.

**140.** Find the distance between $-7$ and $-5$ on the number line.

# 0.3 Rational Exponents and Radicals

In this section, we will learn to

1. Define rational exponents whose numerators are 1.
2. Define rational exponents whose numerators are not 1.
3. Define radical expressions.
4. Simplify and combine radicals.
5. Rationalize denominators and numerators.

MedioImages/Photodisc/Jupiter Images

"Dead Man's Curve" is a 1964 hit song by the rock and roll duo Jan Berry and Dean Torrence. The song details a teenage drag race that ends in an accident. Today, dead man's curve is a commonly used expression given to dangerous curves on our roads. Every curve has a "critical speed." If we exceed this speed, regardless of how skilled a driver we are, we will lose control of the vehicle.

The radical expression $3.9\sqrt{r}$ gives the critical speed in miles per hour when we travel a curved road with a radius of $r$ feet. A knowledge of square roots and radicals is important and used in the construction of safe highways and roads. We will study the topic of radicals in this section.

## 1. Define Rational Exponents Whose Numerators Are 1

If we apply the rule $(x^m)^n = x^{mn}$ to $(25^{1/2})^2$, we obtain

$$\begin{aligned}(25^{1/2})^2 &= 25^{(1/2)2} && \text{Keep the base and multiply the exponents.}\\ &= 25^1 && \tfrac{1}{2}\cdot 2 = 1\\ &= 25\end{aligned}$$

**Caution**

In the expression $a^{1/n}$, there is **no** real-number $n$th root of $a$ when $n$ is even and $a < 0$. For example, $(-64)^{1/2}$ is **not** a real number, because the square of **no** real number is $-64$.

Thus, $25^{1/2}$ is a real number whose square is 25. Although both $5^2 = 25$ and $(-5)^2 = 25$, we define $25^{1/2}$ to be the positive real number whose square is 25:

$$25^{1/2} = 5 \qquad \text{Read } 25^{1/2} \text{ as "the square root of 25."}$$

In general, we have the following definition.

**Rational Exponents**

If $a \geq 0$ and $n$ is a natural number, then $a^{1/n}$ (read as "the $n$th root of $a$") is the nonnegative real number $b$ such that

$$b^n = a$$

Since $b = a^{1/n}$, we have $b^n = (a^{1/n})^n = a$.

**EXAMPLE 1 Simplifying Expressions with Rational Exponents**

In each case, we will apply the definition of rational exponents.

**a.** $16^{1/2} = 4$ Because $4^2 = 16$. Read $16^{1/2}$ as "the square root of 16."

**b.** $27^{1/3} = 3$ Because $3^3 = 27$. Read $27^{1/3}$ as "the cube root of 27."

**c.** $\left(\frac{1}{81}\right)^{1/4} = \frac{1}{3}$ Because $\left(\frac{1}{3}\right)^4 = \frac{1}{81}$. Read $\left(\frac{1}{81}\right)^{1/4}$ as "the fourth root of $\frac{1}{81}$."



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**d.** $-32^{1/5} = -(32^{1/5})$ Read $32^{1/5}$ as "the fifth root of 32."
$= -(2)$ Because $2^5 = 32$.
$= -2$

**Self Check 1** Simplify: **a.** $100^{1/2}$ **b.** $243^{1/5}$

**Now Try Exercise 15.**

If $n$ is even in the expression $a^{1/n}$ and the base contains variables, we often use absolute value symbols to guarantee that an even root is nonnegative.

$(49x^2)^{1/2} = 7|x|$ Because $(7|x|)^2 = 49x^2$. Since $x$ could be negative, absolute value symbols are necessary to guarantee that the square root is nonnegative.

$(16x^4)^{1/4} = 2|x|$ Because $(2|x|)^4 = 16x^4$. Since $x$ could be negative, absolute value symbols are necessary to guarantee that the fourth root is nonnegative.

$(729x^{12})^{1/6} = 3x^2$ Because $(3x^2)^6 = 729x^{12}$. Since $x^2$ is always nonnegative, no absolute value symbols are necessary. Read $(729x^{12})^{1/6}$ as "the sixth root of $729x^{12}$."

If $n$ is an odd number in the expression $a^{1/n}$, the base $a$ can be negative.

**EXAMPLE 2 Simplifying Expressions with Rational Exponents**

Simplify by using the definition of rational exponent.

**a.** $(-8)^{1/3} = -2$ Because $(-2)^3 = -8$.

**b.** $(-3{,}125)^{1/5} = -5$ Because $(-5)^5 = -3{,}125$.

**c.** $\left(-\frac{1}{1{,}000}\right)^{1/3} = -\frac{1}{10}$ Because $\left(-\frac{1}{10}\right)^3 = -\frac{1}{1{,}000}$.

**Self Check 2** Simplify: **a.** $(-125)^{1/3}$ **b.** $(-100{,}000)^{1/5}$

**Now Try Exercise 19.**

**Caution**

If $n$ is odd in the expression $a^{1/n}$, we **don't** need to use absolute value symbols, because odd roots can be negative.

$(-27x^3)^{1/3} = -3x$ Because $(-3x)^3 = -27x^3$.

$(-128a^7)^{1/7} = -2a$ Because $(-2a)^7 = -128a^7$.

We summarize the definitions concerning $a^{1/n}$ as follows.

**Summary of Definitions of $a^{1/n}$**

If $n$ is a natural number and $a$ is a real number in the expression $a^{1/n}$, then

If $a \geq 0$, then $a^{1/n}$ is the nonnegative real number $b$ such that $b^n = a$.

$$\text{If } a < 0 \begin{cases} \text{and } n \text{ is odd, then } a^{1/n} \text{ is the real number } b \text{ such that } b^n = a. \\ \text{and } n \text{ is even, then } a^{1/n} \text{ is not a real number.} \end{cases}$$

The following chart also shows the possibilities that can occur when simplifying $a^{1/n}$.

**Strategy for Simplifying Expressions of the Form $a^{1/n}$**

| $a$ | $n$ | $a^{1/n}$ | Examples |
|---|---|---|---|
| $a = 0$ | $n$ is a natural number. | $0^{1/n}$ is the real number 0 because $0^n = 0$. | $0^{1/2} = 0$ because $0^2 = 0$.<br>$0^{1/5} = 0$ because $0^5 = 0$. |
| $a > 0$ | $n$ is a natural number. | $a^{1/n}$ is the nonnegative real number such that $(a^{1/n})^n = a$. | $16^{1/2} = 4$ because $4^2 = 16$.<br>$27^{1/3} = 3$ because $3^3 = 27$. |
| $a < 0$ | $n$ is an odd natural number. | $a^{1/n}$ is the real number such that $(a^{1/n})^n = a$. | $(-32)^{1/5} = -2$ because $(-2)^5 = -32$.<br>$(-125)^{1/3} = -5$ because $(-5)^3 = -125$. |
| $a < 0$ | $n$ is an even natural number. | $a^{1/n}$ is not a real number. | $(-9)^{1/2}$ is not a real number.<br>$(-81)^{1/4}$ is not a real number. |

## 2. Define Rational Exponents Whose Rational Exponents Are Not 1

The definition of $a^{1/n}$ can be extended to include rational exponents whose numerators are not 1. For example, $4^{3/2}$ can be written as either

$(4^{1/2})^3$ or $(4^3)^{1/2}$ Because of the Power Rule, $(x^m)^n = x^{mn}$.

This suggests the following rule.

**Rule for Rational Exponents** If $m$ and $n$ are positive integers, the fraction $\frac{m}{n}$ is in lowest terms, and $a^{1/n}$ is a real number, then

$$a^{m/n} = (a^{1/n})^m = (a^m)^{1/n}$$

In the previous rule, we can view the expression $a^{m/n}$ in two ways:

1. $(a^{1/n})^m$: the $m$th power of the $n$th root of $a$
2. $(a^m)^{1/n}$: the $n$th root of the $m$th power of $a$

For example, $(-27)^{2/3}$ can be simplified in two ways:

$$(-27)^{2/3} = [(-27)^{1/3}]^2 = (-3)^2 = 9 \quad \text{or} \quad (-27)^{2/3} = [(-27)^2]^{1/3} = (729)^{1/3} = 9$$

As this example suggests, it is usually easier to take the root of the base first to avoid large numbers.

**Comment** It is helpful to think of the phrase *power over root* when we see a rational exponent. The numerator of the fraction represents the *power* and the denominator represents the *root*. Begin with the *root* when simplifying to avoid large numbers.

**Negative Rational Exponents** If $m$ and $n$ are positive integers, the fraction $\frac{m}{n}$ is in lowest terms and $a^{1/n}$ is a real number, then

$$a^{-m/n} = \frac{1}{a^{m/n}} \quad \text{and} \quad \frac{1}{a^{-m/n}} = a^{m/n} \qquad (a \neq 0)$$

**EXAMPLE 3 Simplifying Expressions with Rational Exponents**

We will apply the rules for rational exponents.

**a.** $25^{3/2} = (25^{1/2})^3 = 5^3 = 125$

**b.** $\left(-\frac{x^6}{1{,}000}\right)^{2/3} = \left[\left(-\frac{x^6}{1{,}000}\right)^{1/3}\right]^2 = \left(-\frac{x^2}{10}\right)^2 = \frac{x^4}{100}$

**c.** $32^{-2/5} = \frac{1}{32^{2/5}} = \frac{1}{(32^{1/5})^2} = \frac{1}{2^2} = \frac{1}{4}$

**d.** $\frac{1}{81^{-3/4}} = 81^{3/4} = (81^{1/4})^3 = 3^3 = 27$

**Self Check 3** Simplify: **a.** $49^{3/2}$ **b.** $16^{-3/4}$ **c.** $\frac{1}{(27x^3)^{-2/3}}$

**Now Try Exercise 43.**

Because of the definition, rational exponents follow the same rules as integer exponents.

**EXAMPLE 4 Using Exponent Rules to Simplify Expressions with Rational Exponents**

Simplify each expression. Assume that all variables represent positive numbers, and write answers without using negative exponents.

**a.** $(36x)^{1/2} = 36^{1/2}x^{1/2} = 6x^{1/2}$

**b.** $\frac{(a^{1/3}b^{2/3})^6}{(y^3)^2} = \frac{a^{6/3}b^{12/3}}{y^6} = \frac{a^2b^4}{y^6}$

**c.** $\dfrac{a^{x/2}a^{x/4}}{a^{x/6}} = a^{x/2 + x/4 - x/6}$

$= a^{6x/12 + 3x/12 - 2x/12}$

$= a^{7x/12}$

**d.** $\left[\dfrac{-c^{-2/5}}{c^{4/5}}\right]^{5/3} = (-c^{-2/5-4/5})^{5/3}$

$= [(-1)(c^{-6/5})]^{5/3}$

$= (-1)^{5/3}(c^{-6/5})^{5/3}$

$= -1c^{-30/15}$

$= -c^{-2}$

$= -\dfrac{1}{c^2}$

**Self Check 4** Use the directions for Example 4:

**a.** $\left(\dfrac{y^2}{49}\right)^{1/2}$ **b.** $\dfrac{b^{3/7}b^{2/7}}{b^{4/7}}$ **c.** $\dfrac{(9r^2s)^{1/2}}{rs^{-3/2}}$

**Now Try Exercise 59.**

## 3. Define Radical Expressions

Radical signs can also be used to express roots of numbers.

**Definition of $\sqrt[n]{a}$** If $n$ is a natural number greater than 1 and if $a^{1/n}$ is a real number, then

$$\sqrt[n]{a} = a^{1/n}$$

In the **radical expression** $\sqrt[n]{a}$, the symbol $\sqrt{\ }$ is the **radical sign**, $a$ is the **radicand**, and $n$ is the **index** (or the **order**) of the radical expression. If the order is 2, the expression is a **square root**, and we do not write the index.

$$\sqrt{a} = \sqrt[2]{a}$$

If the index of a radical is 3, we call the radical a **cube root**.

**$n$th Root of a Nonnegative Number** If $n$ is a natural number greater than 1 and $a \geq 0$, then $\sqrt[n]{a}$ is the nonnegative number whose $n$th power is $a$.

$$\left(\sqrt[n]{a}\right)^n = a$$

**Caution** In the expression $\sqrt[n]{a}$, there is **no** real-number $n$th root of $a$ when $n$ is even and $a < 0$. For example, $\sqrt{-64}$ is **not** a real number, because the square of **no** real number is $-64$.

If 2 is substituted for $n$ in the equation $\left(\sqrt[n]{a}\right)^n = a$, we have

$$\left(\sqrt[2]{a}\right)^2 = \left(\sqrt{a}\right)^2 = \sqrt{a}\sqrt{a} = a \text{ for } a \geq 0$$

This shows that if a number $a$ can be factored into two equal factors, either of those factors is a square root of $a$. Furthermore, if $a$ can be factored into $n$ equal factors, any one of those factors is an $n$th root of $a$.

If $n$ is an odd number greater than 1 in the expression $\sqrt[n]{a}$, the radicand can be negative.

**EXAMPLE 5 Finding *n*th Roots of Real Numbers**

We apply the definitions of cube root and fifth root.

a. $\sqrt[3]{-27} = -3$ Because $(-3)^3 = -27$.

b. $\sqrt[3]{-8} = -2$ Because $(-2)^3 = -8$.

c. $\sqrt[3]{-\frac{27}{1{,}000}} = -\frac{3}{10}$ Because $\left(-\frac{3}{10}\right)^3 = -\frac{27}{1{,}000}$.

d. $-\sqrt[5]{-243} = -\left(\sqrt[5]{-243}\right)$
$= -(-3)$
$= 3$

**Self Check 5** Find each root: **a.** $\sqrt[3]{216}$ **b.** $\sqrt[5]{-\frac{1}{32}}$

**Now Try Exercise 69.**

We summarize the definitions concerning $\sqrt[n]{a}$ as follows.

**Summary of Definitions of $\sqrt[n]{a}$**

If $n$ is a natural number greater than 1 and $a$ is a real number, then

If $a \geq 0$, then $\sqrt[n]{a}$ is the nonnegative real number such that $\left(\sqrt[n]{a}\right)^n = a$.

If $a < 0$ $\begin{cases} \text{and } n \text{ is odd, then } \sqrt[n]{a} \text{ is the real number such that } \left(\sqrt[n]{a}\right)^n = a. \\ \text{and } n \text{ is even, then } \sqrt[n]{a} \text{ is not a real number.} \end{cases}$

The following chart also shows the possibilities that can occur when simplifying $\sqrt[n]{a}$.

**Strategy for Simplifying Expressions of the Form $\sqrt[n]{a}$**

| $a$ | $n$ | $\sqrt[n]{a}$ | Examples |
|---|---|---|---|
| $a = 0$ | $n$ is a natural number greater than 1. | $\sqrt[n]{0}$ is the real number 0 because $0^n = 0$. | $\sqrt[3]{0} = 0$ because $0^3 = 0$. $\sqrt[5]{0} = 0$ because $0^5 = 0$. |
| $a > 0$ | $n$ is a natural number greater than 1. | $\sqrt[n]{a}$ is the nonnegative real number such that $\left(\sqrt[n]{a}\right)^n = a$. | $\sqrt{16} = 4$ because $4^2 = 16$. $\sqrt[3]{27} = 3$ because $3^3 = 27$. |
| $a < 0$ | $n$ is an odd natural number greater than 1. | $\sqrt[n]{a}$ is the real number such that $\left(\sqrt[n]{a}\right)^n = a$. | $\sqrt[5]{-32} = -2$ because $(-2)^5 = -32$. $\sqrt[3]{-125} = -5$ because $(-5)^3 = -125$. |
| $a < 0$ | $n$ is an even natural number. | $\sqrt[n]{a}$ is not a real number. | $\sqrt{-9}$ is not a real number. $\sqrt[4]{-81}$ is not a real number. |

We have seen that if $a^{1/n}$ is real, then $a^{m/n} = (a^{1/n})^m = (a^m)^{1/n}$. This same fact can be stated in radical notation.

$$a^{m/n} = (\sqrt[n]{a})^m = \sqrt[n]{a^m}$$

Thus, *the mth power of the nth root of a* is the same as *the nth root of the mth power of a*. For example, to find $\sqrt[3]{27^2}$, we can proceed in either of two ways:

$$\sqrt[3]{27^2} = \left(\sqrt[3]{27}\right)^2 = 3^2 = 9 \text{ or } \sqrt[3]{27^2} = \sqrt[3]{729} = 9$$

By definition, $\sqrt{a^2}$ represents a nonnegative number. If $a$ could be negative, we must use absolute value symbols to guarantee that $\sqrt{a^2}$ will be nonnegative. Thus, if $a$ is unrestricted,

$$\sqrt{a^2} = |a|$$

A similar argument holds when the index is any even natural number. The symbol $\sqrt[4]{a^4}$, for example, means the *positive* fourth root of $a^4$. Thus, if $a$ is unrestricted,

$$\sqrt[4]{a^4} = |a|$$

**EXAMPLE 6 Simplifying Radical Expressions**

If $x$ is unrestricted, simplify **a.** $\sqrt[6]{64x^6}$ **b.** $\sqrt[3]{x^3}$ **c.** $\sqrt{9x^8}$

**SOLUTION** We apply the definitions of sixth roots, cube roots, and square roots.

**a.** $\sqrt[6]{64x^6} = 2|x|$ Use absolute value symbols to guarantee that the result will be nonnegative.

**b.** $\sqrt[3]{x^3} = x$ Because the index is odd, no absolute value symbols are needed.

**c.** $\sqrt{9x^8} = 3x^4$ Because $3x^4$ is always nonnegative, no absolute value symbols are needed. ■

**Self Check 6** Use the directions for Example 6:

**a.** $\sqrt[4]{16x^4}$ **b.** $\sqrt[3]{27y^3}$ **c.** $\sqrt[4]{x^8}$

**Now Try Exercise 73.**

## 4. Simplify and Combine Radicals

Many properties of exponents have counterparts in radical notation. For example, since $a^{1/n}b^{1/n} = (ab)^{1/n}$ and $\frac{a^{1/n}}{b^{1/n}} = \left(\frac{a}{b}\right)^{1/n}$ and $(b \neq 0)$, we have the following.

**Multiplication and Division Properties of Radicals** If all expressions represent real numbers,

$$\sqrt[n]{a}\sqrt[n]{b} = \sqrt[n]{ab} \qquad \frac{\sqrt[n]{a}}{\sqrt[n]{b}} = \sqrt[n]{\frac{a}{b}} \quad (b \neq 0)$$

In words, we say

*The product of two nth roots is equal to the nth root of their product.*

*The quotient of two nth roots is equal to the nth root of their quotient.*

**Caution** These properties involve the $n$th root of the product of two numbers or the $n$th root of the quotient of two numbers. There is **no** such property for sums or differences. For example,

$\sqrt{9+4} \neq \sqrt{9} + \sqrt{4}$, because

$$\sqrt{9+4} = \sqrt{13} \quad \text{but} \quad \sqrt{9} + \sqrt{4} = 3 + 2 = 5$$

and $\sqrt{13} \neq 5$. In general,

$$\sqrt{a+b} \neq \sqrt{a} + \sqrt{b} \quad \text{and} \quad \sqrt{a-b} \neq \sqrt{a} - \sqrt{b}$$

Numbers that are squares of positive integers, such as

1, 4, 9, 16, 25, and 36

are called **perfect squares**. Expressions such as $4x^2$ and $\frac{1}{9}x^6$ are also perfect squares, because each one is the square of another expression with integer exponents and rational coefficients.

$$4x^2 = (2x)^2 \text{ and } \frac{1}{9}x^6 = \left(\frac{1}{3}x^3\right)^2$$

Numbers that are cubes of positive integers, such as

1, 8, 27, 64, 125, and 216

are called **perfect cubes**. Expressions such as $64x^3$ and $\frac{1}{27}x^9$ are also perfect cubes, because each one is the cube of another expression with integer exponents and rational coefficients.

$$64x^3 = (4x)^3 \text{ and } \frac{1}{27}x^9 = \left(\frac{1}{3}x^3\right)^3$$

There are also perfect fourth powers, perfect fifth powers, and so on.

We can use perfect powers and the Multiplication Property of Radicals to simplify many radical expressions. For example, to simplify $\sqrt{12x^5}$, we factor $12x^5$ so that one factor is the largest perfect square that divides $12x^5$. In this case, it is $4x^4$. We then rewrite $12x^5$ as $4x^4 \cdot 3x$ and simplify.

$$\begin{aligned} \sqrt{12x^5} &= \sqrt{4x^4 \cdot 3x} && \text{Factor } 12x^5 \text{ as } 4x^4 \cdot 3x. \\ &= \sqrt{4x^4}\sqrt{3x} && \text{Use the Multiplication Property of Radicals: } \sqrt{ab} = \sqrt{a}\sqrt{b}. \\ &= 2x^2\sqrt{3x} && \sqrt{4x^4} = 2x^2 \end{aligned}$$

To simplify $\sqrt[3]{432x^9y}$, we find the largest perfect-cube factor of $432x^9y$ (which is $216x^9$) and proceed as follows:

$$\begin{aligned} \sqrt[3]{432x^9y} &= \sqrt[3]{216x^9 \cdot 2y} && \text{Factor } 432x^9y \text{ as } 216x^9 \cdot 2y. \\ &= \sqrt[3]{216x^9}\sqrt[3]{2y} && \text{Use the Multiplication Property of Radicals: } \sqrt[3]{ab} = \sqrt[3]{a}\sqrt[3]{b}. \\ &= 6x^3\sqrt[3]{2y} && \sqrt[3]{216x^9} = 6x^3 \end{aligned}$$

Radical expressions with the same index and the same radicand are called **like** or **similar radicals**. We can combine the like radicals in $3\sqrt{2} + 2\sqrt{2}$ by using the Distributive Property.

$$\begin{aligned} 3\sqrt{2} + 2\sqrt{2} &= (3+2)\sqrt{2} \\ &= 5\sqrt{2} \end{aligned}$$

This example suggests that to combine like radicals, we *add their numerical coefficients and keep the same radical.*

When radicals have the same index but different radicands, we can often change them to equivalent forms having the same radicand. We can then combine them. For example, to simplify $\sqrt{27} - \sqrt{12}$, we simplify both radicals and combine like radicals.

$$\begin{aligned}\sqrt{27} - \sqrt{12} &= \sqrt{9 \cdot 3} - \sqrt{4 \cdot 3} && \text{Factor 27 and 12.}\\ &= \sqrt{9}\sqrt{3} - \sqrt{4}\sqrt{3} && \sqrt{ab} = \sqrt{a}\sqrt{b}\\ &= 3\sqrt{3} - 2\sqrt{3} && \sqrt{9} = 3 \text{ and } \sqrt{4} = 2.\\ &= \sqrt{3} && \text{Combine like radicals.}\end{aligned}$$

**EXAMPLE 7** **Adding and Subtracting Radical Expressions**

Simplify: **a.** $\sqrt{50} + \sqrt{200}$ **b.** $3z\sqrt[5]{64z} - 2\sqrt[5]{2z^6}$

**SOLUTION** We will simplify each radical expression and then combine like radicals.

**a.** $$\begin{aligned}\sqrt{50} + \sqrt{200} &= \sqrt{25 \cdot 2} + \sqrt{100 \cdot 2}\\ &= \sqrt{25}\sqrt{2} + \sqrt{100}\sqrt{2}\\ &= 5\sqrt{2} + 10\sqrt{2}\\ &= 15\sqrt{2}\end{aligned}$$

**b.** $$\begin{aligned}3z\sqrt[5]{64z} - 2\sqrt[5]{2z^6} &= 3z\sqrt[5]{32 \cdot 2z} - 2\sqrt[5]{z^5 \cdot 2z}\\ &= 3z\sqrt[5]{32}\sqrt[5]{2z} - 2\sqrt[5]{z^5}\sqrt[5]{2z}\\ &= 3z(2)\sqrt[5]{2z} - 2z\sqrt[5]{2z}\\ &= 6z\sqrt[5]{2z} - 2z\sqrt[5]{2z}\\ &= 4z\sqrt[5]{2z}\end{aligned}$$

**Self Check 7** Simplify: **a.** $\sqrt{18} - \sqrt{8}$ **b.** $2\sqrt[3]{81a^4} + a\sqrt[3]{24a}$

**Now Try Exercise 85.**

## 5. Rationalize Denominators and Numerators

By **rationalizing the denominator**, we can write a fraction such as

$$\frac{\sqrt{5}}{\sqrt{3}}$$

as a fraction with a rational number in the denominator. All that we must do is multiply both the numerator and the denominator by $\sqrt{3}$. (Note that $\sqrt{3}\sqrt{3}$ is the rational number 3.)

$$\frac{\sqrt{5}}{\sqrt{3}} = \frac{\sqrt{5}\sqrt{3}}{\sqrt{3}\sqrt{3}} = \frac{\sqrt{15}}{3}$$

To rationalize the numerator, we multiply both the numerator and the denominator by $\sqrt{5}$. (Note that $\sqrt{5}\sqrt{5}$ is the rational number 5.)

$$\frac{\sqrt{5}}{\sqrt{3}} = \frac{\sqrt{5}\sqrt{5}}{\sqrt{3}\sqrt{5}} = \frac{5}{\sqrt{15}}$$

**EXAMPLE 8** **Rationalizing the Denominator of a Radical Expression**

Rationalize each denominator and simplify. Assume that all variables represent positive numbers.

**a.** $\frac{1}{\sqrt{7}}$ **b.** $\sqrt[3]{\frac{3}{4}}$ **c.** $\sqrt{\frac{3}{x}}$ **d.** $\sqrt{\frac{3a^3}{5x^5}}$

**SOLUTION** We will multiply both the numerator and the denominator by a radical that will make the denominator a rational number.

**a.** $\frac{1}{\sqrt{7}} = \frac{1\sqrt{7}}{\sqrt{7}\sqrt{7}}$

$= \frac{\sqrt{7}}{7}$

**b.** $\sqrt[3]{\frac{3}{4}} = \frac{\sqrt[3]{3}}{\sqrt[3]{4}}$

$= \frac{\sqrt[3]{3}\sqrt[3]{2}}{\sqrt[3]{4}\sqrt[3]{2}}$ Multiply numerator and denominator by $\sqrt[3]{2}$, because $\sqrt[3]{4}\sqrt[3]{2} = \sqrt[3]{8} = 2$.

$= \frac{\sqrt[3]{6}}{\sqrt[3]{8}}$

$= \frac{\sqrt[3]{6}}{2}$

**c.** $\sqrt{\frac{3}{x}} = \frac{\sqrt{3}}{\sqrt{x}}$

$= \frac{\sqrt{3}\sqrt{x}}{\sqrt{x}\sqrt{x}}$

$= \frac{\sqrt{3x}}{x}$

**d.** $\sqrt{\frac{3a^3}{5x^5}} = \frac{\sqrt{3a^3}}{\sqrt{5x^5}}$

$= \frac{\sqrt{3a^3}\sqrt{5x}}{\sqrt{5x^5}\sqrt{5x}}$ Multiply numerator and denominator by $\sqrt{5x}$, because $\sqrt{5x^5}\sqrt{5x} = \sqrt{25x^6} = 5x^3$.

$= \frac{\sqrt{15a^3x}}{\sqrt{25x^6}}$

$= \frac{\sqrt{a^2}\sqrt{15ax}}{5x^3}$

$= \frac{a\sqrt{15ax}}{5x^3}$

**Self Check 8** Use the directions for Example 8:

**a.** $\frac{6}{\sqrt{6}}$ **b.** $\sqrt[3]{\frac{2}{5x}}$

**Now Try Exercise 101.**

**EXAMPLE 9** **Rationalizing the Numerator of a Radical Expression**

Rationalize each numerator and simplify. Assume that all variables represent positive numbers: **a.** $\frac{\sqrt{x}}{7}$ **b.** $\frac{2\sqrt[3]{9x}}{3}$

**SOLUTION** We will multiply both the numerator and the denominator by a radical that will make the numerator a rational number.

**a.** $$\frac{\sqrt{x}}{7} = \frac{\sqrt{x}\cdot\sqrt{x}}{7\sqrt{x}} = \frac{x}{7\sqrt{x}}$$

**b.** $$\frac{2\sqrt[3]{9x}}{3} = \frac{2\sqrt[3]{9x}\cdot\sqrt[3]{3x^2}}{3\sqrt[3]{3x^2}} = \frac{2\sqrt[3]{27x^3}}{3\sqrt[3]{3x^2}} = \frac{2(3x)}{3\sqrt[3]{3x^2}} = \frac{2x}{\sqrt[3]{3x^2}}$$ Divide out the 3's.

**Self Check 9** Use the directions for Example 9:

**a.** $\frac{\sqrt{2x}}{5}$ **b.** $\frac{3\sqrt[3]{2y^2}}{6}$

**Now Try Exercise 111.**

After rationalizing denominators, we often can simplify an expression.

**EXAMPLE 10 Rationalizing Denominators and Simplifying**

Simplify: $\sqrt{\frac{1}{2}} + \sqrt{\frac{1}{8}}$.

**SOLUTION** We will rationalize the denominators of each radical and then combine like radicals.

$$\sqrt{\frac{1}{2}} + \sqrt{\frac{1}{8}} = \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{8}}$$ $\sqrt{\frac{1}{2}} = \frac{\sqrt{1}}{\sqrt{2}} = \frac{1}{\sqrt{2}}$; $\sqrt{\frac{1}{8}} = \frac{\sqrt{1}}{\sqrt{8}} = \frac{1}{\sqrt{8}}$

$$= \frac{1\sqrt{2}}{\sqrt{2}\sqrt{2}} + \frac{1\sqrt{2}}{\sqrt{8}\sqrt{2}}$$

$$= \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{\sqrt{16}}$$

$$= \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{4}$$

$$= \frac{3\sqrt{2}}{4}$$

**Self Check 10** Simplify: $\sqrt[3]{\frac{x}{2}} - \sqrt[3]{\frac{x}{16}}$.

**Now Try Exercise 115.**

Another property of radicals can be derived from the properties of exponents. If all of the expressions represent real numbers,

$$\sqrt[n]{\sqrt[m]{x}} = \sqrt[n]{x^{1/m}} = (x^{1/m})^{1/n} = x^{1/(mn)} = \sqrt[mn]{x}$$

$$\sqrt[m]{\sqrt[n]{x}} = \sqrt[m]{x^{1/n}} = (x^{1/n})^{1/m} = x^{1/(nm)} = \sqrt[nm]{x}$$

These results are summarized in the following *theorem* (a fact that can be proved).

**Theorem** If all of the expressions involved represent real numbers, then

$$\sqrt[m]{\sqrt[n]{x}} = \sqrt[n]{\sqrt[m]{x}} = \sqrt[mn]{x}$$

We can use the previous theorem to simplify many radicals. For example,

$$\sqrt[3]{\sqrt{8}} = \sqrt{\sqrt[3]{8}} = \sqrt{2}$$

Rational exponents can be used to simplify many radical expressions, as shown in the following example.

### EXAMPLE 11 Simplifying Radicals Using the Previously Stated Theorem

Simplify. Assume that $x$ and $y$ are positive numbers.

**a.** $\sqrt[6]{4}$ **b.** $\sqrt[12]{x^3}$ **c.** $\sqrt[9]{8y^3}$

**SOLUTION** In each case, we will write the radical as an exponential expression, simplify the resulting expression, and write the final result as a radical.

**a.** $\sqrt[6]{4} = 4^{1/6} = (2^2)^{1/6} = 2^{2/6} = 2^{1/3} = \sqrt[3]{2}$

**b.** $\sqrt[12]{x^3} = x^{3/12} = x^{1/4} = \sqrt[4]{x}$

**c.** $\sqrt[9]{8y^3} = (2^3y^3)^{1/9} = (2y)^{3/9} = (2y)^{1/3} = \sqrt[3]{2y}$

**Self Check 11** Simplify: **a.** $\sqrt[4]{4}$ **b.** $\sqrt[9]{27x^3}$

**Now Try Exercise 117.**

**Self Check Answers** **1. a.** 10 **b.** 3 **2. a.** $-5$ **b.** $-10$ **3. a.** 343 **b.** $\frac{1}{8}$ **c.** $9x^2$ **4. a.** $\frac{y}{7}$ **b.** $b^{1/7}$ **c.** $3x^2$ **5. a.** 6 **b.** $-\frac{1}{2}$ **6. a.** $2|x|$ **b.** $3y$ **c.** $x^2$ **7. a.** $\sqrt{2}$ **b.** $8a\sqrt[3]{3a}$ **8. a.** $\sqrt{6}$ **b.** $\frac{\sqrt[3]{50x^2}}{5x}$ **9. a.** $\frac{2x}{5\sqrt{2x}}$ **b.** $\frac{y}{\sqrt[3]{4y}}$ **10.** $\frac{\sqrt[3]{4x}}{4}$ **11. a.** $\sqrt{2}$ **b.** $\sqrt[3]{3x}$

## Exercises 0.3

### Getting Ready

***You should be able to complete these vocabulary and concept statements before you proceed to the practice exercises.***

***Fill in the blanks.***

1. If $a = 0$ and $n$ is a natural number, then $a^{1/n} =$ ___.
2. If $a > 0$ and $n$ is a natural number, then $a^{1/n}$ is a _______ number.
3. If $a < 0$ and $n$ is an even number, then $a^{1/n}$ is _____ a real number.
4. $6^{2/3}$ can be written as ______ or ______.
5. $\sqrt[n]{a} =$ ____
6. $\sqrt{a^2} =$ ____
7. $\sqrt[n]{a}\sqrt[n]{b} =$ _____
8. $\sqrt[n]{\frac{a}{b}} =$ _____
9. $\sqrt{x+y}$ ____ $\sqrt{x} + \sqrt{y}$
10. $\sqrt[m]{\sqrt[n]{x}}$ or $\sqrt[n]{\sqrt[m]{x}}$ can be written as _____.

## Practice

*Simplify each expression.*

**11.** $9^{1/2}$ **12.** $8^{1/3}$

**13.** $\left(\frac{1}{25}\right)^{1/2}$ **14.** $\left(\frac{16}{625}\right)^{1/4}$

**15.** $-81^{1/4}$ **16.** $-\left(\frac{8}{27}\right)^{1/3}$

**17.** $(10{,}000)^{1/4}$ **18.** $1{,}024^{1/5}$

**19.** $\left(-\frac{27}{8}\right)^{1/3}$ **20.** $-64^{1/3}$

**21.** $(-64)^{1/2}$ **22.** $(-125)^{1/3}$

*Simplify each expression. Use absolute value symbols when necessary.*

**23.** $(16a^2)^{1/2}$ **24.** $(25a^4)^{1/2}$

**25.** $(16a^4)^{1/4}$ **26.** $(-64a^3)^{1/3}$

**27.** $(-32a^5)^{1/5}$ **28.** $(64a^6)^{1/6}$

**29.** $(-216b^6)^{1/3}$ **30.** $(256t^8)^{1/4}$

**31.** $\left(\frac{16a^4}{25b^2}\right)^{1/2}$ **32.** $\left(-\frac{a^5}{32b^{10}}\right)^{1/5}$

**33.** $\left(-\frac{1{,}000x^6}{27y^3}\right)^{1/3}$ **34.** $\left(\frac{49t^2}{100z^4}\right)^{1/2}$

*Simplify each expression. Write all answers without using negative exponents.*

**35.** $4^{3/2}$ **36.** $8^{2/3}$

**37.** $-16^{3/2}$ **38.** $(-8)^{2/3}$

**39.** $-1{,}000^{2/3}$ **40.** $100^{3/2}$

**41.** $64^{-1/2}$ **42.** $25^{-1/2}$

**43.** $64^{-3/2}$ **44.** $49^{-3/2}$

**45.** $-9^{-3/2}$ **46.** $(-27)^{-2/3}$

**47.** $\left(\frac{4}{9}\right)^{5/2}$ **48.** $\left(\frac{25}{81}\right)^{3/2}$

**49.** $\left(-\frac{27}{64}\right)^{-2/3}$ **50.** $\left(\frac{125}{8}\right)^{-4/3}$

*Simplify each expression. Assume that all variables represent positive numbers. Write all answers without using negative exponents.*

**51.** $(100s^4)^{1/2}$ **52.** $(64u^6v^3)^{1/3}$

**53.** $(32y^{10}z^5)^{-1/5}$ **54.** $(625a^4b^8)^{-1/4}$

**55.** $(x^{10}y^5)^{3/5}$ **56.** $(64a^6b^{12})^{5/6}$

**57.** $(r^8s^{16})^{-3/4}$ **58.** $(-8x^9y^{12})^{-2/3}$

**59.** $\left(-\frac{8a^6}{125b^9}\right)^{2/3}$ **60.** $\left(\frac{16x^4}{625y^8}\right)^{3/4}$

**61.** $\left(\frac{27r^6}{1{,}000s^{12}}\right)^{-2/3}$ **62.** $\left(-\frac{32m^{10}}{243n^{15}}\right)^{-2/5}$

**63.** $\frac{a^{2/5}a^{4/5}}{a^{1/5}}$ **64.** $\frac{x^{6/7}x^{3/7}}{x^{2/7}x^{5/7}}$

*Simplify each radical expression.*

**65.** $\sqrt{49}$ **66.** $\sqrt{81}$

**67.** $\sqrt[3]{125}$ **68.** $\sqrt[3]{-64}$

**69.** $\sqrt[3]{-125}$ **70.** $\sqrt[5]{-243}$

**71.** $\sqrt[5]{-\frac{32}{100{,}000}}$ **72.** $\sqrt[4]{\frac{256}{625}}$

*Simplify each expression, using absolute value symbols when necessary. Write answers without using negative exponents.*

**73.** $\sqrt{36x^2}$ **74.** $-\sqrt{25y^2}$

**75.** $\sqrt{9y^4}$ **76.** $\sqrt{a^4b^8}$

**77.** $\sqrt[3]{8y^3}$ **78.** $\sqrt[3]{-27z^9}$

**79.** $\sqrt[4]{\frac{x^4y^8}{z^{12}}}$ **80.** $\sqrt[5]{\frac{a^{10}b^5}{c^{15}}}$

*Simplify each expression. Assume that all variables represent positive numbers so that no absolute value symbols are needed.*

**81.** $\sqrt{8}-\sqrt{2}$ **82.** $\sqrt{75}-2\sqrt{27}$

**83.** $\sqrt{200x^2}+\sqrt{98x^2}$ **84.** $\sqrt{128a^3}-a\sqrt{162a}$

**85.** $2\sqrt{48y^5}-3y\sqrt{12y^3}$ **86.** $y\sqrt{112y}+4\sqrt{175y^3}$

**87.** $2\sqrt[3]{81}+3\sqrt[3]{24}$ **88.** $3\sqrt[4]{32}-2\sqrt[4]{162}$

**89.** $\sqrt[4]{768z^5}+\sqrt[4]{48z^5}$ **90.** $-2\sqrt[5]{64y^2}+3\sqrt[5]{486y^2}$

**91.** $\sqrt{8x^2y}-x\sqrt{2y}+\sqrt{50x^2y}$

**92.** $3x\sqrt{18x}+2\sqrt{2x^3}-\sqrt{72x^3}$

**93.** $\sqrt[3]{16xy^4}+y\sqrt[3]{2xy}-\sqrt[3]{54xy^4}$

**94.** $\sqrt[4]{512x^5}-\sqrt[4]{32x^5}+\sqrt[4]{1{,}250x^5}$

*Rationalize each denominator and simplify. Assume that all variables represent positive numbers.*

**95.** $\frac{3}{\sqrt{3}}$ **96.** $\frac{6}{\sqrt{5}}$

**97.** $\frac{2}{\sqrt{x}}$ **98.** $\frac{8}{\sqrt{y}}$

**99.** $\frac{2}{\sqrt[3]{2}}$ **100.** $\frac{4d}{\sqrt[3]{9}}$

**101.** $\frac{5a}{\sqrt[3]{25a}}$ **102.** $\frac{7}{\sqrt[3]{36c}}$

**103.** $\frac{2b}{\sqrt[4]{3a^2}}$ **104.** $\sqrt{\frac{x}{2y}}$

**105.** $\sqrt[3]{\frac{2u^4}{9v}}$ **106.** $\sqrt[3]{-\frac{3s^5}{4r^2}}$

***Rationalize each numerator and simplify. Assume that all variables are positive numbers.***

**107.** $\dfrac{\sqrt{5}}{10}$

**108.** $\dfrac{\sqrt{y}}{3}$

**109.** $\dfrac{\sqrt[3]{9}}{3}$

**110.** $\dfrac{\sqrt[3]{16b^2}}{16}$

**111.** $\dfrac{\sqrt[5]{16b^3}}{64a}$

**112.** $\sqrt{\dfrac{3x}{57}}$

***Rationalize each denominator and simplify.***

**113.** $\sqrt{\dfrac{1}{3}} - \sqrt{\dfrac{1}{27}}$

**114.** $\sqrt[3]{\dfrac{1}{2}} + \sqrt[3]{\dfrac{1}{16}}$

**115.** $\sqrt{\dfrac{x}{8}} - \sqrt{\dfrac{x}{2}} + \sqrt{\dfrac{x}{32}}$

**116.** $\sqrt[3]{\dfrac{y}{4}} + \sqrt[3]{\dfrac{y}{32}} - \sqrt[3]{\dfrac{y}{500}}$

***Simplify each radical expression.***

**117.** $\sqrt[4]{9}$

**118.** $\sqrt[6]{27}$

**119.** $\sqrt[10]{16x^6}$

**120.** $\sqrt[6]{27x^9}$

## Discovery and Writing

***We often can multiply and divide radicals with different indices. For example, to multiply $\sqrt{3}$ by $\sqrt[3]{5}$, we first write each radical as a sixth root***

$$\sqrt{3} = 3^{1/2} = 3^{3/6} = \sqrt[6]{3^3} = \sqrt[6]{27}$$

$$\sqrt[3]{5} = 5^{1/3} = 5^{2/6} = \sqrt[6]{5^2} = \sqrt[6]{25}$$

***and then multiply the sixth roots.***

$$\sqrt{3}\sqrt[3]{5} = \sqrt[6]{27}\sqrt[6]{25} = \sqrt[6]{(27)(25)} = \sqrt[6]{675}$$

***Division is similar.***

***Use this idea to write each of the following expressions as a single radical.***

**121.** $\sqrt{2}\sqrt[3]{2}$

**122.** $\sqrt{3}\sqrt[3]{5}$

**123.** $\dfrac{\sqrt[4]{3}}{\sqrt{2}}$

**124.** $\dfrac{\sqrt[3]{2}}{\sqrt{5}}$

**125.** For what values of $x$ does $\sqrt[4]{x^4} = x$? Explain.

**126.** If all of the radicals involved represent real numbers and $y \neq 0$, explain why

$$\sqrt[n]{\frac{x}{y}} = \frac{\sqrt[n]{x}}{\sqrt[n]{y}}$$

**127.** If all of the radicals involved represent real numbers and there is no division by 0, explain why

$$\left(\frac{x}{y}\right)^{-m/n} = \sqrt[n]{\frac{y^m}{x^m}}$$

**128.** The definition of $x^{m/n}$ requires that $\sqrt[n]{x}$ be a real number. Explain why this is important. (Hint: Consider what happens when $n$ is even, $m$ is odd, and $x$ is negative.)

## Review

**129.** Write $-2 < x \leq 5$ using interval notation.

**130.** Write the expression $|3 - x|$ without using absolute value symbols. Assume that $x > 4$.

***Evaluate each expression when $x = -2$ and $y = 3$.***

**131.** $x^2 - y^2$

**132.** $\dfrac{xy + 4y}{x}$

**133.** Write 617,000,000 in scientific notation.

**134.** Write $0.00235 \times 10^4$ in standard notation.

# 0.4 Polynomials

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In this section, we will learn to

1. Define polynomials.
2. Add and subtract polynomials.
3. Multiply polynomials.
4. Rationalize denominators.
5. Divide polynomials.

Football is one of the most popular sports in the United States. Brett Favre, one of the most talented quarterbacks ever to play the game, holds the record for the most career NFL touchdown passes, the most NFL pass completions, and the most passing yards. He led the Green Bay Packers to the Super Bowl and most recently played for the Minnesota Vikings. His nickname is Gunslinger.

An algebraic expression can be used to model the trajectory or path of a football when passed by Brett Favre. Suppose the height in feet, $t$ seconds after the football leaves Brett's hand, is given by the algebraic expression

$$-0.1t^2 + t + 5.5$$

At a time of $t = 3$ seconds, we see that the height of the football is

$$-0.1(3)^2 + 3 + 5.5 = 7.6 \text{ ft.}$$

Algebraic expressions like $-0.1t^2 + t + 5.5$ are called **polynomials,** and we will study them in this section.

## 1. Define Polynomials

A **monomial** is a real number or the product of a real number and one or more variables with whole-number exponents. The number is called the **coefficient** of the variables. Some examples of monomials are

$$3x, \quad 7ab^2, \quad -5ab^3c^4, \quad x^3, \quad \text{and} \quad -12$$

with coefficients of 3, 7, −5, 1, and −12, respectively.

The **degree** of a monomial is the sum of the exponents of its variables. All nonzero constants (except 0) have a degree of 0.

The degree of $3x$ is 1. The degree of $7ab^2$ is 3.

The degree of $-5ab^3c^4$ is 7. The degree of $x^3$ is 3.

The degree of $-12$ is 0 (since $-12 = -12x^0$). 0 has no defined degree.

A monomial or a sum of monomials is called a **polynomial**. Each monomial in that sum is called a **term** of the polynomial. A polynomial with two terms is called a **binomial**, and a polynomial with three terms is called a **trinomial**.

| Monomials | Binomials | Trinomials |
|---|---|---|
| $3x^2$ | $2a + 3b$ | $x^2 + 7x - 4$ |
| $-25xy$ | $4x^3 - 3x^2$ | $4y^4 - 2y + 12$ |
| $a^2b^3c$ | $-2x^3 - 4y^2$ | $12x^3y^2 - 8xy - 24$ |

The **degree of a polynomial** is the degree of the term in the polynomial with highest degree. The only polynomial with no defined degree is 0, which is called the **zero polynomial**. Here are some examples.

- $3x^2y^3 + 5xy^2 + 7$ is a trinomial of 5th degree, because its term with highest degree (the first term) is 5.
- $3ab + 5a^2b$ is a binomial of degree 3.
- $5x + 3y^2 + \sqrt[4]{3}z^4 - \sqrt{7}$ is a polynomial, because its variables have whole-number exponents. It is of degree 4.
- $-7y^{1/2} + 3y^2 + \sqrt[4]{3z}$ is not a polynomial, because one of its variables ($y$ in the first term) does not have a whole-number exponent.

If two terms of a polynomial have the same variables with the same exponents, they are **like** or **similar** terms. To combine the like terms in the sum $3x^2y + 5x^2y$ or the difference $7xy^2 - 2xy^2$, we use the Distributive Property:

$$3x^2y + 5x^2y = (3 + 5)x^2y = 8x^2y \qquad 7xy^2 - 2xy^2 = (7 - 2)xy^2 = 5xy^2$$

This illustrates that *to combine like terms, we add (or subtract) their coefficients and keep the same variables and the same exponents.*

## 2. Add and Subtract Polynomials

Recall that we can use the Distributive Property to remove parentheses enclosing the terms of a polynomial. When the sign preceding the parentheses is +, we simply drop the parentheses:

$$\begin{aligned} +(a + b - c) &= +1(a + b - c) \\ &= 1a + 1b - 1c \\ &= a + b - c \end{aligned}$$

When the sign preceding the parentheses is −, we drop the parentheses and the − sign and change the sign of each term within the parentheses.

$$\begin{aligned} -(a + b - c) &= -1(a + b - c) \\ &= -1a + (-1)b - (-1)c \\ &= -a - b + c \end{aligned}$$

We can use these facts to add and subtract polynomials. *To add (or subtract) polynomials, we remove parentheses (if necessary) and combine like terms.*

**EXAMPLE 1 Adding Polynomials**

Add: $(3x^3y + 5x^2 - 2y) + (2x^3y - 5x^2 + 3x)$.

**SOLUTION** To add the polynomials, we remove parentheses and combine like terms.

$$\begin{aligned} &(3x^3y + 5x^2 - 2y) + (2x^3y - 5x^2 + 3x) \\ &\quad = 3x^3y + 5x^2 - 2y + 2x^3y - 5x^2 + 3x \\ &\quad = 3x^3y + 2x^3y + 5x^2 - 5x^2 - 2y + 3x && \text{Use the Commutative Property to rearrange terms.} \\ &\quad = 5x^3y - 2y + 3x && \text{Combine like terms.} \end{aligned}$$

We can add the polynomials in a vertical format by writing like terms in a column and adding the like terms, column by column.

$$\begin{array}{llll} 3x^3y & + 5x^2 & - 2y & \\ 2x^3y & - 5x^2 & & + 3x \\ \hline 5x^3y & & - 2y & + 3x \end{array}$$

**Self Check 1** Add: $(4x^2 + 3x - 5) + (3x^2 - 5x + 7)$.

**Now Try Exercise 21.**

**EXAMPLE 2 Subtracting Polynomials**

Subtract: $(2x^2 + 3y^2) - (x^2 - 2y^2 + 7)$.

**SOLUTION** To subtract the polynomials, we remove parentheses and combine like terms.

$$\begin{aligned} &(2x^2 + 3y^2) - (x^2 - 2y^2 + 7) \\ &\quad = 2x^2 + 3y^2 - x^2 + 2y^2 - 7 \\ &\quad = 2x^2 - x^2 + 3y^2 + 2y^2 - 7 && \text{Use the Commutative Property to rearrange terms.} \\ &\quad = x^2 + 5y^2 - 7 && \text{Combine like terms.} \end{aligned}$$

We can subtract the polynomials in a vertical format by writing like terms in a column and subtracting the like terms, column by column.

$$\begin{array}{r} 2x^2 + 3y^2 \\ \underline{-(x^2 - 2y^2 + 7)} \\ x^2 + 5y^2 - 7 \end{array}$$

$2x^2 - x^2 = (2 - 1)x^2$

$3y^2 - (-2)y^2 = 3y^2 + 2y^2 = (3 + 2)y^2$

$0 - 7 = -7$

**Self Check 2** Subtract: $(4x^2 + 3x - 5) - (3x^2 - 5x + 7)$.

**Now Try Exercise 23.**

We can also use the Distributive Property to remove parentheses enclosing several terms that are multiplied by a constant. For example,

$$\begin{aligned} 4(3x^2 - 2x + 6) &= 4(3x^2) - 4(2x) + 4(6) \\ &= 12x^2 - 8x + 24 \end{aligned}$$

This example suggests that *to add multiples of one polynomial to another, or to subtract multiples of one polynomial from another, we remove parentheses and combine like terms.*

**EXAMPLE 3 Using the Distributive Property and Combining Like Terms**

Simplify: $7x(2y^2 + 13x^2) - 5(xy^2 - 13x^3)$.

**SOLUTION** $7x(2y^2 + 13x^2) - 5(xy^2 - 13x^3)$

$= 14xy^2 + 91x^3 - 5xy^2 + 65x^3$ Use the Distributive Property to remove parentheses.

$= 14xy^2 - 5xy^2 + 91x^3 + 65x^3$ Use the Commutative Property to rearrange terms.

$= 9xy^2 + 156x^3$ Combine like terms.

**Self Check 3** Simplify: $3(2b^2 - 3a^2b) + 2b(b + a^2)$.

**Now Try Exercise 30.**

## 3. Multiply Polynomials

To find the product of $3x^2y^3z$ and $5xyz^2$, we proceed as follows:

$(3x^2y^3z)(5xyz^2) = 3 \cdot x^2 \cdot y^3 \cdot z \cdot 5 \cdot x \cdot y \cdot z^2$

$= 3 \cdot 5 \cdot x^2 \cdot x \cdot y^3 \cdot y \cdot z \cdot z^2$ Use the Commutative Property to rearrange terms.

$= 15x^3y^4z^3$

This illustrates that *to multiply two monomials, we multiply the coefficients and then multiply the variables.*

To find the product of a monomial and a polynomial, we use the Distributive Property.

$$\begin{aligned} 3xy^2(2xy + x^2 - 7yz) &= 3xy^2(2xy) + (3xy^2)(x^2) - (3xy^2)(7yz) \\ &= 6x^2y^3 + 3x^3y^2 - 21xy^3z \end{aligned}$$

This illustrates that *to multiply a polynomial by a monomial, we multiply each term of the polynomial by the monomial.*

To multiply one binomial by another, we use the Distributive Property twice.

**EXAMPLE 4 Multiplying Binomials**

Multiply: **a.** $(x + y)(x + y)$ **b.** $(x - y)(x - y)$ **c.** $(x + y)(x - y)$

**SOLUTION**

**a.**
$$\begin{aligned}(x + y)(x + y) &= (x + y)x + (x + y)y \\ &= x^2 + xy + xy + y^2 \\ &= x^2 + 2xy + y^2\end{aligned}$$

**b.**
$$\begin{aligned}(x - y)(x - y) &= (x - y)x - (x - y)y \\ &= x^2 - xy - xy + y^2 \\ &= x^2 - 2xy + y^2\end{aligned}$$

**c.**
$$\begin{aligned}(x + y)(x - y) &= (x + y)x - (x + y)y \\ &= x^2 + xy - xy - y^2 \\ &= x^2 - y^2\end{aligned}$$

**Self Check 4** Multiply: **a.** $(x + 2)(x + 2)$ **b.** $(x - 3)(x - 3)$
**c.** $(x + 4)(x - 4)$

**Now Try Exercise 45.**

The products in Example 4 are called **special products**. Because they occur so often, it is worthwhile to learn their forms.

**Special Product Formulas**

$$(x + y)^2 = (x + y)(x + y) = x^2 + 2xy + y^2$$
$$(x - y)^2 = (x - y)(x - y) = x^2 - 2xy + y^2$$
$$(x + y)(x - y) = x^2 - y^2$$

**Caution**

Remember that $(x + y)^2$ and $(x - y)^2$ have trinomials for their products and that

$(x + y)^2 \neq x^2 + y^2$ and $(x - y)^2 \neq x^2 - y^2$

For example,

$$\begin{aligned}(3 + 5)^2 &\neq 3^2 + 5^2 \\ 8^2 &\neq 9 + 25 \\ 64 &\neq 34\end{aligned} \qquad \text{and} \qquad \begin{aligned}(3 - 5)^2 &\neq 3^2 - 5^2 \\ (-2)^2 &\neq 9 - 25 \\ 4 &\neq -16\end{aligned}$$

We can use the **FOIL method** to multiply one binomial by another. FOIL is an acronym for **F**irst terms, **O**uter terms, **I**nner terms, and **L**ast terms. To use this method to multiply $3x - 4$ by $2x + 5$, we write

First terms Last terms

$$\begin{aligned}(3x - 4)(2x + 5) &= 3x(2x) + 3x(5) - 4(2x) - 4(5)\\ &= 6x^2 + 15x - 8x - 20\\ &= 6x^2 + 7x - 20\end{aligned}$$

Inner terms

Outer terms

In this example,

- the product of the first terms is $6x^2$,
- the product of the outer terms is $15x$,
- the product of the inner terms is $-8x$, and
- the product of the last terms is $-20$.

The resulting like terms of the product are then combined.

**EXAMPLE 5** **Using the FOIL Method to Multiply Polynomials**

Use the FOIL method to multiply: $(\sqrt{3} + x)(2 - \sqrt{3}x)$.

**SOLUTION**

$$\begin{aligned}(\sqrt{3} + x)(2 - \sqrt{3}x) &= 2\sqrt{3} - \sqrt{3}\sqrt{3}x + 2x - x\sqrt{3}x\\ &= 2\sqrt{3} - 3x + 2x - \sqrt{3}x^2\\ &= 2\sqrt{3} - x - \sqrt{3}x^2\end{aligned}$$

**Self Check 5** Multiply: $(2x + \sqrt{3})(x - \sqrt{3})$.

**Now Try Exercise 63.**

To multiply a polynomial with more than two terms by another polynomial, we multiply each term of one polynomial by each term of the other polynomial and combine like terms whenever possible.

**EXAMPLE 6** **Multiplying Polynomials**

Multiply: **a.** $(x + y)(x^2 - xy + y^2)$ **b.** $(x + 3)^3$

**SOLUTION**

**a.** $$\begin{aligned}(x + y)(x^2 - xy + y^2) &= x^3 - x^2y + xy^2 + yx^2 - xy^2 + y^3\\ &= x^3 + y^3\end{aligned}$$

**b.** $$\begin{aligned}(x + 3)^3 &= (x + 3)(x + 3)^2\\ &= (x + 3)(x^2 + 6x + 9)\\ &= x^3 + 6x^2 + 9x + 3x^2 + 18x + 27\\ &= x^3 + 9x^2 + 27x + 27\end{aligned}$$

**Self Check 6** Multiply: $(x + 2)(2x^2 + 3x - 1)$.

**Now Try Exercise 67.**

We can use a vertical format to multiply two polynomials, such as the polynomials given in Self Check 6. We first write the polynomials as follows and draw a line beneath them. We then multiply each term of the upper polynomial by each term of the lower polynomial and write the results so that like terms appear in each column. Finally, we combine like terms column by column.

$$\begin{array}{r}2x^2 + 3x - 1\\ x + 2\\ \hline 4x^2 + 6x - 2\\ 2x^3 + 3x^2 - x\quad\\ \hline 2x^3 + 7x^2 + 5x - 2\end{array}\qquad\begin{array}{l}\\ \\ \text{Multiply } 2x^2 + 3x - 1 \text{ by } 2.\\ \text{Multiply } 2x^2 + 3x - 1 \text{ by } x.\\ \text{In each column, combine like terms.}\end{array}$$

If $n$ is a whole number, the expressions $a^n + 1$ and $2a^n - 3$ are polynomials and we can multiply them as follows:

$$\begin{aligned}(a^n + 1)(2a^n - 3) &= 2a^{2n} - 3a^n + 2a^n - 3\\ &= 2a^{2n} - a^n - 3 \qquad \text{Combine like terms.}\end{aligned}$$

We can also use the methods previously discussed to multiply expressions that are not polynomials, such as $x^{-2} + y$ and $x^2 - y^{-1}$.

$$\begin{aligned}(x^{-2} + y)(x^2 - y^{-1}) &= x^{-2+2} - x^{-2}y^{-1} + x^2y - y^{1-1}\\ &= x^0 - \frac{1}{x^2y} + x^2y - y^0\\ &= 1 - \frac{1}{x^2y} + x^2y - 1 \qquad x^0 = 1 \text{ and } y^0 = 1.\\ &= x^2y - \frac{1}{x^2y}\end{aligned}$$

## 4. Rationalize Denominators

If the denominator of a fraction is a binomial containing square roots, we can use the product formula $(x + y)(x - y)$ to rationalize the denominator. For example, to rationalize the denominator of

$$\frac{6}{\sqrt{7} + 2} \qquad \text{To rationalize a denominator means to change the denominator into a rational number.}$$

we multiply the numerator and denominator by $\sqrt{7} - 2$ and simplify.

$$\begin{aligned}\frac{6}{\sqrt{7} + 2} &= \frac{6(\sqrt{7} - 2)}{(\sqrt{7} + 2)(\sqrt{7} - 2)} \qquad \frac{\sqrt{7} - 2}{\sqrt{7} - 2} = 1\\ &= \frac{6(\sqrt{7} - 2)}{7 - 4}\\ &= \frac{6(\sqrt{7} - 2)}{3} \qquad \text{Here the denominator is a rational number.}\\ &= 2(\sqrt{7} - 2)\end{aligned}$$

In this example, we multiplied both the numerator and the denominator of the given fraction by $\sqrt{7} - 2$. This binomial is the same as the denominator of the given fraction $\sqrt{7} + 2$, except for the sign between the terms. Such binomials are called **conjugate binomials** or **radical conjugates**.

**Conjugate Binomials** **Conjugate binomials** are binomials that are the same except for the sign between their terms. The conjugate of $a + b$ is $a - b$, and the conjugate of $a - b$ is $a + b$.

**EXAMPLE 7 Rationalizing the Denominator of a Radical Expression**

Rationalize the denominator: $\dfrac{\sqrt{3x}-\sqrt{2}}{\sqrt{3x}+\sqrt{2}}$ $(x > 0)$.

**SOLUTION** We multiply the numerator and the denominator by $\sqrt{3x}-\sqrt{2}$ (the conjugate of $\sqrt{3x}+\sqrt{2}$) and simplify.

$$\begin{aligned}
\frac{\sqrt{3x}-\sqrt{2}}{\sqrt{3x}+\sqrt{2}} &= \frac{\left(\sqrt{3x}-\sqrt{2}\right)\left(\sqrt{3x}-\sqrt{2}\right)}{\left(\sqrt{3x}+\sqrt{2}\right)\left(\sqrt{3x}-\sqrt{2}\right)} && \frac{\sqrt{3x}-\sqrt{2}}{\sqrt{3x}-\sqrt{2}}=1\\
&= \frac{\sqrt{3x}\sqrt{3x}-\sqrt{3x}\sqrt{2}-\sqrt{2}\sqrt{3x}+\sqrt{2}\sqrt{2}}{\left(\sqrt{3x}\right)^2-\left(\sqrt{2}\right)^2}\\
&= \frac{3x-\sqrt{6x}-\sqrt{6x}+2}{3x-2}\\
&= \frac{3x-2\sqrt{6x}+2}{3x-2}
\end{aligned}$$

■

**Self Check 7** Rationalize the denominator: $\dfrac{\sqrt{x}+2}{\sqrt{x}-2}$.

**Now Try Exercise 89.**

In calculus, we often rationalize a numerator.

**EXAMPLE 8 Rationalizing the Numerator of a Radical Expression**

Rationalize the numerator: $\dfrac{\sqrt{x+h}-\sqrt{x}}{h}$.

**SOLUTION** To rid the numerator of radicals, we multiply the numerator and the denominator by the conjugate of the numerator and simplify.

$$\begin{aligned}
\frac{\sqrt{x+h}-\sqrt{x}}{h} &= \frac{\left(\sqrt{x+h}-\sqrt{x}\right)\left(\sqrt{x+h}+\sqrt{x}\right)}{h\left(\sqrt{x+h}+\sqrt{x}\right)} && \frac{\sqrt{x+h}+\sqrt{x}}{\sqrt{x+h}+\sqrt{x}}=1\\
&= \frac{x+h-x}{h\left(\sqrt{x+h}+\sqrt{x}\right)} && \text{Here the numerator has no radicals.}\\
&= \frac{h}{h\left(\sqrt{x+h}+\sqrt{x}\right)}\\
&= \frac{1}{\sqrt{x+h}+\sqrt{x}} && \text{Divide out the common factor of } h.
\end{aligned}$$

■

**Self Check 8** Rationalize the numerator: $\dfrac{\sqrt{4+h}-2}{h}$.

**Now Try Exercise 99.**

## 5. Divide Polynomials

To divide monomials, we write the quotient as a fraction and simplify by using the rules of exponents. For example,

$$\begin{aligned}\frac{6x^2y^3}{-2x^3y} &= -3x^{2-3}y^{3-1}\\ &= -3x^{-1}y^2\\ &= -\frac{3y^2}{x}\end{aligned}$$

To divide a polynomial by a monomial, we write the quotient as a fraction, write the fraction as a sum of separate fractions, and simplify each one. For example, to divide $8x^5y^4 + 12x^2y^5 - 16x^2y^3$ by $4x^3y^4$, we proceed as follows:

$$\begin{aligned}\frac{8x^5y^4 + 12x^2y^5 - 16x^2y^3}{4x^3y^4} &= \frac{8x^5y^4}{4x^3y^4} + \frac{12x^2y^5}{4x^3y^4} + \frac{-16x^2y^3}{4x^3y^4}\\ &= 2x^2 + \frac{3y}{x} - \frac{4}{xy}\end{aligned}$$

To divide two polynomials, we can use long division. To illustrate, we consider the division

$$\frac{2x^2 + 11x - 30}{x + 7}$$

which can be written in long division form as

$$x + 7\overline{)2x^2 + 11x - 30}$$

The binomial $x + 7$ is called the **divisor**, and the trinomial $2x^2 + 11x - 30$ is called the **dividend**. The final answer, called the **quotient**, will appear above the long division symbol.

We begin the division by asking "What monomial, when multiplied by $x$, gives $2x^2$?" Because $x \cdot 2x = 2x^2$, the answer is $2x$. We place $2x$ in the quotient, multiply each term of the divisor by $2x$, subtract, and bring down the –30.

$$\begin{array}{r} 2x \phantom{{}+11x-30} \\ x+7\overline{)2x^2 + 11x - 30} \\ \underline{2x^2 + 14x} \phantom{{}-30} \\ -3x - 30 \end{array}$$

We continue the division by asking "What monomial, when multiplied by $x$, gives $-3x$?" We place the answer, $-3$, in the quotient, multiply each term of the divisor by $-3$, and subtract. This time, there is no number to bring down.

$$\begin{array}{r} 2x - 3 \\ x+7\overline{)2x^2 + 11x - 30} \\ \underline{2x^2 + 14x} \phantom{{}-30} \\ -3x - 30 \\ \underline{-3x - 21} \\ -9 \end{array}$$

Because the degree of the remainder, $-9$, is less than the degree of the divisor, the division process stops, and we can express the result in the form

$$\text{quotient} + \frac{\text{remainder}}{\text{divisor}}$$

Thus,

$$\frac{2x^2 + 11x - 30}{x + 7} = 2x - 3 + \frac{-9}{x + 7}$$

**EXAMPLE 9** **Using Long Division to Divide Polynomials**

Divide $6x^3 - 11$ by $2x + 2$.

**SOLUTION** We set up the division, leaving spaces for the missing powers of $x$ in the dividend.

$$2x + 2\overline{)6x^3 \qquad\qquad - 11}$$

**Comment**

In Example 9, we could write the missing powers of $x$ using coefficients of 0.

$$2x + 2\overline{)6x^3 + 0x^2 + 0x - 11}$$

The division process continues as usual, with the following results:

$$\begin{array}{r}
3x^2 - 3x + 3 \phantom{)} \\
2x + 2\overline{)6x^3 \qquad\qquad - 11} \\
\underline{6x^3 + 6x^2} \qquad\qquad\quad \\
- 6x^2 \qquad\qquad\quad \\
\underline{- 6x^2 - 6x} \qquad\quad \\
+ 6x - 11 \\
\underline{+ 6x + \phantom{1}6} \\
- 17
\end{array}$$

Thus, $\dfrac{6x^3 - 11}{2x + 2} = 3x^2 - 3x + 3 + \dfrac{-17}{2x + 2}$.

**Self Check 9** Divide: $3x + 1\overline{)9x^2 - 1}$.

**Now Try Exercise 113.**

**EXAMPLE 10** **Using Long Division to Divide Polynomials**

Divide $-3x^3 - 3 + x^5 + 4x^2 - x^4$ by $x^2 - 3$.

**SOLUTION** The division process works best when the terms in the divisor and dividend are written with their exponents in descending order.

$$\begin{array}{r}
x^3 - x^2 + 1 \\
x^2 - 3\overline{)x^5 - x^4 - 3x^3 + 4x^2 - 3} \\
\underline{x^5 \qquad - 3x^3} \qquad\qquad\quad \\
- x^4 \qquad\quad + 4x^2 \qquad \\
\underline{- x^4 \qquad\quad + 3x^2} \qquad \\
x^2 - 3 \\
\underline{x^2 - 3} \\
0
\end{array}$$

Thus, $\dfrac{-3x^3 - 3 + x^5 + 4x^2 - x^4}{x^2 - 3} = x^3 - x^2 + 1$.

**Self Check 10** Divide: $x^2 + 1\overline{)3x^2 - x + 1 - 2x^3 + 3x^4}$.

**Now Try Exercise 115.**

**Self Check Answers** **1.** $7x^2 - 2x + 2$ **2.** $x^2 + 8x - 12$ **3.** $8b^2 - 7a^2b$ **4. a.** $x^2 + 4x + 4$ **b.** $x^2 - 6x + 9$ **c.** $x^2 - 16$ **5.** $2x^2 - x\sqrt{3} - 3$ **6.** $2x^3 + 7x^2 + 5x - 2$ **7.** $\dfrac{x + 4\sqrt{x} + 4}{x - 4}$ **8.** $\dfrac{1}{\sqrt{4+h} + 2}$ **9.** $3x - 1$ **10.** $3x^2 - 2x + \dfrac{x+1}{x^2+1}$

## Exercises 0.4

### Getting Ready

***You should be able to complete these vocabulary and concept statements before you proceed to the practice exercises.***

***Fill in the blanks.***

**1.** A ________ is a real number or the product of a real number and one or more ________.

**2.** The ______ of a monomial is the sum of the exponents of its ________.

**3.** A ________ is a polynomial with three terms.

**4.** A ________ is a polynomial with two terms.

**5.** A monomial is a polynomial with _____ term.

**6.** The constant 0 is called the _____ polynomial.

**7.** Terms with the same variables with the same exponents are called _____ terms.

**8.** The _______ of a polynomial is the same as the degree of its term of highest degree.

**9.** To combine like terms, we add their _________ and keep the same ________ and the same exponents.

**10.** The conjugate of $3\sqrt{x} + 2$ is ________.

***Determine whether the given expression is a polynomial. If so, tell whether it is a monomial, a binomial, or a trinomial, and give its degree.***

**11.** $x^2 + 3x + 4$

**12.** $5xy - x^3$

**13.** $x^3 + y^{1/2}$

**14.** $x^{-3} - 5y^{-2}$

**15.** $4x^2 - \sqrt{5}x^3$

**16.** $x^2y^3$

**17.** $\sqrt{15}$

**18.** $\dfrac{5}{x} + \dfrac{x}{5} + 5$

**19.** $0$

**20.** $3y^3 - 4y^2 + 2y + 2$

### Practice

***Perform the operations and simplify.***

**21.** $(x^3 - 3x^2) + (5x^3 - 8x)$

**22.** $(2x^4 - 5x^3) + (7x^3 - x^4 + 2x)$

**23.** $(y^5 + 2y^3 + 7) - (y^5 - 2y^3 - 7)$

**24.** $(3t^7 - 7t^3 + 3) - (7t^7 - 3t^3 + 7)$

**25.** $2(x^2 + 3x - 1) - 3(x^2 + 2x - 4) + 4$

**26.** $5(x^3 - 8x + 3) + 2(3x^2 + 5x) - 7$

**27.** $8(t^2 - 2t + 5) + 4(t^2 - 3t + 2) - 6(2t^2 - 8)$

**28.** $-3(x^3 - x) + 2(x^2 + x) + 3(x^3 - 2x)$

**29.** $y(y^2 - 1) - y^2(y + 2) - y(2y - 2)$

**30.** $-4a^2(a + 1) + 3a(a^2 - 4) - a^2(a + 2)$

**31.** $xy(x - 4y) - y(x^2 + 3xy) + xy(2x + 3y)$

**32.** $3mn(m + 2n) - 6m(3mn + 1) - 2n(4mn - 1)$

**33.** $2x^2y^3(4xy^4)$

**34.** $-15a^3b(-2a^2b^3)$

**35.** $-3m^2n(2mn^2)\left(-\dfrac{mn}{12}\right)$

**36.** $-\dfrac{3r^2s^3}{5}\left(\dfrac{2r^2s}{3}\right)\left(\dfrac{15rs^2}{2}\right)$

**37.** $-4rs(r^2 + s^2)$

**38.** $6u^2v(2uv^2 - y)$

**39.** $6ab^2c(2ac + 3bc^2 - 4ab^2c)$

**40.** $-\dfrac{mn^2}{2}(4mn - 6m^2 - 8)$

**41.** $(a + 2)(a + 2)$

**42.** $(y - 5)(y - 5)$

**43.** $(a - 6)^2$

**44.** $(t + 9)^2$

**45.** $(x + 4)(x - 4)$

**46.** $(z + 7)(z - 7)$

**47.** $(x-3)(x+5)$

**48.** $(z+4)(z-6)$

**49.** $(u+2)(3u-2)$

**50.** $(4x+1)(2x-3)$

**51.** $(5x-1)(2x+3)$

**52.** $(4x-1)(2x-7)$

**53.** $(3a-2b)^2$

**54.** $(4a+5b)(4a-5b)$

**55.** $(3m+4n)(3m-4n)$

**56.** $(4r+3s)^2$

**57.** $(2y-4x)(3y-2x)$

**58.** $(-2x+3y)(3x+y)$

**59.** $(9x-y)(x^2-3y)$

**60.** $(8a^2+b)(a+2b)$

**61.** $(5z+2t)(z^2-t)$

**62.** $(y-2x^2)(x^2+3y)$

**63.** $(\sqrt{5}+3x)(2-\sqrt{5}x)$

**64.** $(\sqrt{2}+x)(3+\sqrt{2}x)$

**65.** $(3x-1)^3$

**66.** $(2x-3)^3$

**67.** $(3x+1)(2x^2+4x-3)$

**68.** $(2x-5)(x^2-3x+2)$

**69.** $(3x+2y)(2x^2-3xy+4y^2)$

**70.** $(4r-3s)(2r^2+4rs-2s^2)$

***Multiply the expressions as you would multiply polynomials.***

**71.** $2y^n(3y^n+y^{-n})$

**72.** $3a^{-n}(2a^n+3a^{n-1})$

**73.** $-5x^{2n}y^n(2x^{2n}y^{-n}+3x^{-2n}y^n)$

**74.** $-2a^{3n}b^{2n}(5a^{-3n}b-ab^{-2n})$

**75.** $(x^n+3)(x^n-4)$

**76.** $(a^n-5)(a^n-3)$

**77.** $(2r^n-7)(3r^n-2)$

**78.** $(4z^n+3)(3z^n+1)$

**79.** $x^{1/2}(x^{1/2}y+xy^{1/2})$

**80.** $ab^{1/2}(a^{1/2}b^{1/2}+b^{1/2})$

**81.** $(a^{1/2}+b^{1/2})(a^{1/2}-b^{1/2})$

**82.** $(x^{3/2}+y^{1/2})^2$

***Rationalize each denominator.***

**83.** $\dfrac{2}{\sqrt{3}-1}$

**84.** $\dfrac{1}{\sqrt{5}+2}$

**85.** $\dfrac{3x}{\sqrt{7}+2}$

**86.** $\dfrac{14y}{\sqrt{2}-3}$

**87.** $\dfrac{x}{x-\sqrt{3}}$

**88.** $\dfrac{y}{2y+\sqrt{7}}$

**89.** $\dfrac{y+\sqrt{2}}{y-\sqrt{2}}$

**90.** $\dfrac{x-\sqrt{3}}{x+\sqrt{3}}$

**91.** $\dfrac{\sqrt{2}-\sqrt{3}}{1-\sqrt{3}}$

**92.** $\dfrac{\sqrt{3}-\sqrt{2}}{1+\sqrt{2}}$

**93.** $\dfrac{\sqrt{x}-\sqrt{y}}{\sqrt{x}+\sqrt{y}}$

**94.** $\dfrac{\sqrt{2x}+y}{\sqrt{2x}-y}$

***Rationalize each numerator.***

**95.** $\dfrac{\sqrt{2}+1}{2}$

**96.** $\dfrac{\sqrt{x}-3}{3}$

**97.** $\dfrac{y-\sqrt{3}}{y+\sqrt{3}}$

**98.** $\dfrac{\sqrt{a}-\sqrt{b}}{\sqrt{a}+\sqrt{b}}$

**99.** $\dfrac{\sqrt{x+3}-\sqrt{x}}{3}$

**100.** $\dfrac{\sqrt{2+h}-\sqrt{2}}{h}$

***Perform each division and write all answers without using negative exponents.***

**101.** $\dfrac{36a^2b^3}{18ab^6}$

**102.** $\dfrac{-45r^2s^2t^3}{27r^6s^2t^8}$

**103.** $\dfrac{16x^6y^4z^9}{-24x^9y^6z^0}$

**104.** $\dfrac{32m^6n^4p^2}{26m^6n^7p^2}$

**105.** $\dfrac{5x^3y^2+15x^3y^4}{10x^2y^3}$

**106.** $\dfrac{9m^4n^9-6m^3n^4}{12m^3n^3}$

**107.** $\dfrac{24x^5y^7-36x^2y^5+12xy}{60x^5y^4}$

**108.** $\dfrac{9a^3b^4+27a^2b^4-18a^2b^3}{18a^2b^7}$

***Perform each division. If there is a nonzero remainder, write the answer in*** quotient $+\frac{\text{remainder}}{\text{divisor}}$ ***form.***

**109.** $x+3\overline{)3x^2+11x+6}$

**110.** $3x+2\overline{)3x^2+11x+6}$

**111.** $2x-5\overline{)2x^2-19x+37}$

**112.** $x-7\overline{)2x^2-19x+35}$

**113.** $\dfrac{2x^3+1}{x-1}$

**114.** $\dfrac{2x^3-9x^2+13x-20}{2x-7}$

**115.** $x^2+x-1\overline{)x^3-2x^2-4x+3}$

**116.** $x^2-3\overline{)x^3-2x^2-4x+5}$

**117.** $\dfrac{x^5-2x^3-3x^2+9}{x^3-2}$

**118.** $\dfrac{x^5 - 2x^3 - 3x^2 + 9}{x^3 - 3}$

**119.** $\dfrac{x^5 - 32}{x - 2}$

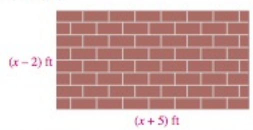
**120.** $\dfrac{x^4 - 1}{x + 1}$

**121.** $11x - 10 + 6x^2\overline{)\,36x^4 - 121x^2 + 120 + 72x^3 - 142x}$

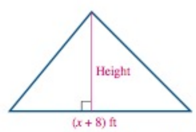
**122.** $x + 6x^2 - 12\overline{)\,-121x^2 + 72x^3 - 142x + 120 + 36x^4}$

## Applications

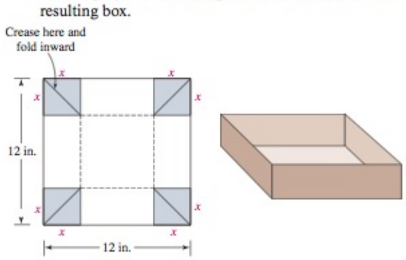
**123. Geometry** Find an expression that represents the area of the brick wall.




**124. Geometry** The area of the triangle shown in the illustration is represented as $(x^2 + 3x - 40)$ square feet. Find an expression that represents its height.




**125. Gift Boxes** The corners of a 12 in.-by-12 in. piece of cardboard are folded inward and glued to make a box. Write a polynomial that represents the volume of the resulting box.




**126. Travel** Complete the following table, which shows the rate (mph), time traveled (hr), and distance traveled (mi) by a family on vacation.

| $r$ | · | $t$ | = | $d$ |
|---|---|---|---|---|
| $3x + 4$ | | | | $3x^2 + 19x + 20$ |

## Discovery and Writing

**127.** Show that a trinomial can be squared by using the formula
$(a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ac.$

**128.** Show that $(a + b + c + d)^2 = a^2 + b^2 + c^2 + d^2 + 2ab + 2ac + 2ad + 2bc + 2bd + 2cd$.

**129.** Explain the FOIL method.

**130.** Explain how to rationalize the numerator of $\frac{\sqrt{x} + 2}{x}$.

**131.** Explain why $(a + b)^2 \neq a^2 + b^2$.

**132.** Explain why $\sqrt{a^2 + b^2} \neq \sqrt{a^2} + \sqrt{b^2}$.

## Review

***Simplify each expression. Assume that all variables represent positive numbers.***

**133.** $9^{3/2}$

**134.** $\left(\dfrac{8}{125}\right)^{-2/3}$

**135.** $\left(\dfrac{625x^4}{16y^8}\right)^{3/4}$

**136.** $\sqrt{80x^4}$

**137.** $\sqrt[3]{16ab^4} - b\sqrt[3]{54ab}$

**138.** $x\sqrt[4]{1{,}280x} + \sqrt[4]{80x^5}$

# 0.5 Factoring Polynomials

In this section, we will learn to

1. Factor out a common monomial.
2. Factor by grouping.
3. Factor the difference of two squares.
4. Factor trinomials.
5. Factor trinomials by grouping.
6. Factor the sum and difference of two cubes.
7. Factor miscellaneous polynomials.

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The television network series *CSI: Crime Scene Investigation* and its spinoffs, *CSI: NY* and *CSI: Miami*, are favorite television shows of many people. Their fans enjoy watching a team of forensic scientists uncover the circumstances that led to an unusual death or crime. These mysteries are captivating because many viewers are interested in the field of forensic science.

In this section, we will investigate a mathematics mystery. We will be given a polynomial and asked to unveil or uncover the two or more polynomials that were multiplied together to obtain the given polynomial. The process that we will use to solve this mystery is called *factoring*.

When two or more polynomials are multiplied together, each one is called a **factor** of the resulting product. For example, the factors of $7(x + 2)(x + 3)$ are

$$7, \quad x + 2, \quad \text{and} \quad x + 3$$

The process of writing a polynomial as the product of several factors is called **factoring**.

In this section, we will discuss factoring where the coefficients of the polynomial and the polynomial factors are integers. If a polynomial cannot be factored by using integers only, we call it a **prime polynomial**.

## 1. Factor Out a Common Monomial

The simplest type of factoring occurs when we see a common monomial factor in each term of the polynomial. In this case, our strategy is to use the Distributive Property and factor out the greatest common factor.

**EXAMPLE 1** **Factoring by Removing a Common Monomial**

Factor: $3xy^2 + 6x$.

**SOLUTION** We note that each term contains a greatest common factor of $3x$:

$$3xy^2 + 6x = \mathbf{3x}(y^2) + \mathbf{3x}(2)$$

We can then use the Distributive Property to factor out the common factor of $3x$:

$$3xy^2 + 6x = \mathbf{3x}(y^2 + 2) \qquad \text{We can check by multiplying: } 3x(y^2 + 2) = 3xy^2 + 6x.$$

**Self Check 1** Factor: $4a^2 - 8ab$.

**Now Try Exercise 11.**



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**EXAMPLE 2** **Factoring by Removing a Common Monomial**

Factor: $x^2y^2z^2 - xyz$.

**SOLUTION** We factor out the greatest common factor of $xyz$:

$$\begin{aligned} x^2y^2z^2 - xyz &= xyz(xyz) - xyz(1) \\ &= xyz(xyz - 1) \quad \text{We can check by multiplying.} \end{aligned}$$

The last term in the expression $x^2y^2z^2 - xyz$ has an understood coefficient of 1. When the $xyz$ is factored out, the 1 must be written.

**Self Check 2** Factor: $a^2b^2c^2 + a^3b^3c^3$.

**Now Try Exercise 13.**

## 2. Factor by Grouping

A strategy that is often helpful when factoring a polynomial with four or more terms is called *grouping*. Terms with common factors are grouped together and then their greatest common factors are factored out using the Distributive Property.

**EXAMPLE 3** **Factoring by Grouping**

Factor: $ax + bx + a + b$.

**SOLUTION** Although there is no factor common to all four terms, we can factor $x$ out of the first two terms and write the expression as

$$ax + bx + a + b = x(a + b) + (a + b)$$

We can now factor out the common factor of $a + b$.

$$\begin{aligned} ax + bx + a + b &= x(a + b) + (a + b) \\ &= x(a + b) + 1(a + b) \\ &= (a + b)(x + 1) \quad \text{We can check by multiplying.} \end{aligned}$$

**Self Check 3** Factor: $x^2 + xy + 2x + 2y$.

**Now Try Exercise 17.**

## 3. Factor the Difference of Two Squares

A binomial that is the difference of the squares of two quantities factors easily. The strategy used is to write the polynomial as the product of two factors. The first factor is the sum of the quantities and the other factor is the difference of the quantities.

**EXAMPLE 4** **Factoring the Difference of Two Squares**

Factor: $49x^2 - 4$.

**SOLUTION** We observe that each term is a perfect square:

$$49x^2 - 4 = (7x)^2 - 2^2$$

The difference of the squares of two quantities is the product of two factors. One is the sum of the quantities, and the other is the difference of the quantities. Thus, $49x^2 - 4$ factors as

$$\begin{aligned} 49x^2 - 4 &= (7x)^2 - 2^2 \\ &= (7x + 2)(7x - 2) \quad \text{We can check by multiplying.} \end{aligned}$$

**Self Check 4** Factor: $9a^2 - 16b^2$.

**Now Try Exercise 21.**

Example 4 suggests a formula for factoring the difference of two squares.

**Factoring the Difference of Two Squares** $x^2 - y^2 = (x + y)(x - y)$

**EXAMPLE 5 Factoring the Difference of Two Squares Twice in One Problem**

Factor: $16m^4 - n^4$.

**SOLUTION** The binomial $16m^4 - n^4$ can be factored as the difference of two squares:

$$\begin{aligned} 16m^4 - n^4 &= (4m^2)^2 - (n^2)^2 \\ &= (4m^2 + n^2)(4m^2 - n^2) \end{aligned}$$

The first factor is the sum of two squares and is prime. The second factor is a difference of two squares and can be factored:

$$\begin{aligned} 16m^4 - n^4 &= (4m^2 + n^2)[(2m)^2 - n^2] \\ &= (4m^2 + n^2)(2m + n)(2m - n) \quad \text{We can check by multiplying.} \end{aligned}$$

**Caution**

If you are limited to integer coefficients, the sum of two squares **cannot** be factored. For example, $x^2 + y^2$ is a prime polynomial.

**Self Check 5** Factor: $a^4 - 81b^4$.

**Now Try Exercise 23.**

**EXAMPLE 6 Removing a Common Factor and Factoring the Difference of Two Squares**

Factor: $18t^2 - 32$.

**SOLUTION** We begin by factoring out the common monomial factor of 2.

$$18t^2 - 32 = 2(9t^2 - 16)$$

Since $9t^2 - 16$ is the difference of two squares, it can be factored.

$$\begin{aligned} 18t^2 - 32 &= 2(9t^2 - 16) \\ &= 2(3t + 4)(3t - 4) \quad \text{We can check by multiplying.} \end{aligned}$$

**Self Check 6** Factor: $-3x^2 + 12$.

**Now Try Exercise 61.**

## 4. Factor Trinomials

Trinomials that are squares of binomials can be factored by using the following formulas.

**Factoring Trinomial Squares**
(1) $x^2 + 2xy + y^2 = (x + y)(x + y) = (x + y)^2$
(2) $x^2 - 2xy + y^2 = (x - y)(x - y) = (x - y)^2$

For example, to factor $a^2 - 6a + 9$, we note that it can be written in the form

$$a^2 - 2(3a) + 3^2 \quad x = a \text{ and } y = 3$$

which matches the left side of Equation 2 above. Thus,

$$\begin{aligned} a^2 - 6a + 9 &= a^2 - 2(3a) + 3^2 \\ &= (a - 3)(a - 3) \\ &= (a - 3)^2 \quad \text{We can check by multiplying.} \end{aligned}$$

Factoring trinomials that are not squares of binomials often requires some guesswork. If a trinomial with no common factors is factorable, it will factor into the product of two binomials.

**EXAMPLE 7 Factoring a Trinomial with Leading Coefficient of 1**

Factor: $x^2 + 3x - 10$.

**SOLUTION** To factor $x^2 + 3x - 10$, we must find two binomials $x + a$ and $x + b$ such that

$$x^2 + 3x - 10 = (x + a)(x + b)$$

where the product of $a$ and $b$ is $-10$ and the sum of $a$ and $b$ is 3.

$$ab = -10 \quad \text{and} \quad a + b = 3$$

To find such numbers, we list the possible factorizations of $-10$:

$$10(-1) \qquad 5(-2) \qquad -10(1) \qquad -5(2)$$

Only in the factorization $5(-2)$ do the factors have a sum of 3. Thus, $a = 5$ and $b = -2$, and

$$x^2 + 3x - 10 = (x + a)(x + b)$$

(3) $$x^2 + 3x - 10 = (x + 5)(x - 2) \quad \text{We can check by multiplying.}$$

Because of the Commutative Property of Multiplication, the order of the factors in Equation 3 is not important. Equation 3 can also be written as

$$x^2 + 3x - 10 = (x - 2)(x + 5)$$

**Self Check 7** Factor: $p^2 - 5p - 6$.

**Now Try Exercise 29.**

**EXAMPLE 8 Factoring a Trinomial with Leading Coefficient not 1**

Factor: $2x^2 - x - 6$.

**SOLUTION** Since the first term is $2x^2$, the first terms of the binomial factors must be $2x$ and $x$:

$$2x^2 - x - 6 = (2x \quad)(x \quad)$$

The product of the last terms must be $-6$, and the sum of the products of the outer terms and the inner terms must be $-x$. Since the only factorization of $-6$ that will cause this to happen is $3(-2)$, we have

$$2x^2 - x - 6 = (2x + 3)(x - 2)$$ We can check by multiplying.

**Self Check 8** Factor: $6x^2 - x - 2$.

**Now Try Exercise 33.**

It is not easy to give specific rules for factoring trinomials, because some guesswork is often necessary. However, the following hints are helpful.

**Strategy for Factoring a General Trinomial with Integer Coefficients**

1. Write the trinomial in descending powers of one variable.
2. Factor out any greatest common factor, including $-1$ if that is necessary to make the coefficient of the first term positive.
3. When the sign of the first term of a trinomial is $+$ and the sign of the third term is $+$, the sign between the terms of each binomial factor is the same as the sign of the middle term of the trinomial.
   When the sign of the first term is $+$ and the sign of the third term is $-$, one of the signs between the terms of the binomial factors is $+$ and the other is $-$.
4. Try various combinations of first terms and last terms until you find one that works. If no possibilities work, the trinomial is prime.
5. Check the factorization by multiplication.

**EXAMPLE 9** **Factoring a Trinomial Completely**

Factor: $10xy + 24y^2 - 6x^2$.

**SOLUTION** We write the trinomial in descending powers of $x$ and then factor out the common factor of $-2$.

$$\begin{aligned}10xy + 24y^2 - 6x^2 &= -6x^2 + 10xy + 24y^2\\ &= -2(3x^2 - 5xy - 12y^2)\end{aligned}$$

Since the sign of the third term of $3x^2 - 5xy - 12y^2$ is $-$, the signs between the binomial factors will be opposite. Since the first term is $3x^2$, the first terms of the binomial factors must be $3x$ and $x$:

$$-2(3x^2 - 5xy - 12y^2) = -2(3x \quad)(x \quad)$$

The product of the last terms must be $-12y^2$, and the sum of the outer terms and the inner terms must be $-5xy$. Of the many factorizations of $-12y^2$, only $4y(-3y)$ leads to a middle term of $-5xy$. So we have

$$\begin{aligned}10xy + 24y^2 - 6x^2 &= -6x^2 + 10xy + 24y^2\\ &= -2(3x^2 - 5xy - 12y^2)\\ &= -2(3x + 4y)(x - 3y)\end{aligned}$$ We can check by multiplying.

**Self Check 9** Factor: $-6x^2 - 15xy - 6y^2$.

**Now Try Exercise 69.**

## 5. Factor Trinomials by Grouping

Another way of factoring trinomials involves factoring by grouping. This method can be used to factor trinomials of the form $ax^2 + bx + c$. For example, to factor $6x^2 + 5x - 6$, we note that $a = 6$, $b = 5$, and $c = -6$, and proceed as follows:

1. Find the product $ac$: $6(-6) = -36$. This number is called the **key number**.
2. Find two factors of the key number $(-36)$ whose sum is $b = 5$. Two such numbers are 9 and $-4$.

   $9(-4) = -36 \qquad \text{and} \qquad 9 + (-4) = 5$

3. Use the factors 9 and $-4$ as coefficients of two terms to be placed between $6x^2$ and $-6$.

   $6x^2 + 5x - 6 = 6x^2 + 9x - 4x - 6$

4. Factor by grouping:

$$\begin{aligned} 6x^2 + 9x - 4x - 6 &= 3x(2x + 3) - 2(2x + 3) \\ &= (2x + 3)(3x - 2) \qquad \text{Factor out } 2x + 3. \end{aligned}$$

**EXAMPLE 10** **Factoring a Trinomial by Grouping**

Factor: $15x^2 + x - 2$.

**SOLUTION** Since $a = 15$ and $c = -2$ in the trinomial, $ac = -30$. We now find factors of $-30$ whose sum is $b = 1$. Such factors are 6 and $-5$. We use these factors as coefficients of two terms to be placed between $15x^2$ and $-2$.

$15x^2 + 6x - 5x - 2$

Finally, we factor by grouping.

$3x(5x + 2) - (5x + 2) = (5x + 2)(3x - 1)$

**Self Check 10** Factor: $15a^2 + 17a - 4$.

**Now Try Exercise 39.**

We can often factor polynomials with variable exponents. For example, if $n$ is a natural number,

$a^{2n} - 5a^n - 6 = (a^n + 1)(a^n - 6)$

because

$$\begin{aligned} (a^n + 1)(a^n - 6) &= a^{2n} - 6a^n + a^n - 6 \\ &= a^{2n} - 5a^n - 6 \qquad \text{Combine like terms.} \end{aligned}$$

## 6. Factor the Sum and Difference of Two Cubes

Two other types of factoring involve binomials that are the sum or the difference of two cubes. Like the difference of two squares, they can be factored by using a formula.

**Factoring the Sum and Difference of Two Cubes**

$$x^3 + y^3 = (x + y)(x^2 - xy + y^2)$$
$$x^3 - y^3 = (x - y)(x^2 + xy + y^2)$$

**EXAMPLE 11 Factoring the Sum of Two Cubes**

Factor: $27x^6 + 64y^3$.

**SOLUTION** We can write this expression as the sum of two cubes and factor it as follows:

$$\begin{aligned} 27x^6 + 64y^3 &= (3x^2)^3 + (4y)^3 \\ &= (3x^2 + 4y)[(3x^2)^2 - (3x^2)(4y) + (4y)^2] \\ &= (3x^2 + 4y)(9x^4 - 12x^2y + 16y^2) \quad \text{We can check by multiplying.} \end{aligned}$$

**Self Check 11** Factor: $8a^3 + 1{,}000b^6$.

**Now Try Exercise 41.**

**EXAMPLE 12 Factoring the Difference of Two Cubes**

Factor: $x^3 - 8$.

**SOLUTION** This binomial can be written as $x^3 - 2^3$, which is the difference of two cubes. Substituting into the formula for the difference of two cubes gives

$$\begin{aligned} x^3 - 2^3 &= (x - 2)(x^2 + 2x + 2^2) \\ &= (x - 2)(x^2 + 2x + 4) \quad \text{We can check by multiplying.} \end{aligned}$$

**Self Check 12** Factor: $p^3 - 64$.

**Now Try Exercise 43.**

## 7. Factor Miscellaneous Polynomials

**EXAMPLE 13 Factoring a Miscellaneous Polynomial**

Factor: $x^2 - y^2 + 6x + 9$.

**SOLUTION** Here we will factor a trinomial and a difference of two squares.

$$\begin{aligned} x^2 - y^2 + 6x + 9 &= x^2 + 6x + 9 - y^2 && \text{Use the Commutative Property to rearrange the terms.} \\ &= (x + 3)^2 - y^2 && \text{Factor } x^2 + 6x + 9. \\ &= (x + 3 + y)(x + 3 - y) && \text{Factor the difference of two squares.} \end{aligned}$$

We could try to factor this expression in another way.

$$x^2 - y^2 + 6x + 9 = (x + y)(x - y) + 3(2x + 3) \quad \text{Factor } x^2 - y^2 \text{ and } 6x + 9.$$

However, we are unable to finish the factorization. If grouping in one way doesn't work, try various other ways.

**Self Check 13** Factor: $a^2 + 8a - b^2 + 16$.

**Now Try Exercise 97.**

**EXAMPLE 14 Factoring a Miscellaneous Trinomial**

Factor: $z^4 - 3z^2 + 1$.

**SOLUTION** This trinomial cannot be factored as the product of two binomials, because no combination will give a middle term of $-3z^2$. However, if the middle term were $-2z^2$, the trinomial would be a perfect square, and the factorization would be easy:

$$\begin{aligned} z^4 - 2z^2 + 1 &= (z^2 - 1)(z^2 - 1) \\ &= (z^2 - 1)^2 \end{aligned}$$

We can change the middle term in $z^4 - 3z^2 + 1$ to $-2z^2$ by adding $z^2$ to it. However, to make sure that adding $z^2$ does not change the value of the trinomial, we must also subtract $z^2$. We can then proceed as follows.

$$\begin{aligned} z^4 - 3z^2 + 1 &= z^4 - 3z^2 + z^2 + 1 - z^2 && \text{Add and subtract } z^2. \\ &= z^4 - 2z^2 + 1 - z^2 && \text{Combine } -3z^2 \text{ and } z^2. \\ &= (z^2 - 1)^2 - z^2 && \text{Factor } z^4 - 2z^2 + 1. \\ &= (z^2 - 1 + z)(z^2 - 1 - z) && \text{Factor the difference of two squares.} \end{aligned}$$

In this type of problem, we will always try to add and subtract a perfect square in hopes of making a perfect-square trinomial that will lead to factoring a difference of two squares.

**Self Check 14** Factor: $x^4 + 3x^2 + 4$.

**Now Try Exercise 103.**

It is helpful to identify the problem type when we must factor polynomials that are given in random order.

**Factoring Strategy**

1. Factor out all common monomial factors.
2. If an expression has two terms, check whether the problem type is
   a. The difference of two squares:
   $$x^2 - y^2 = (x + y)(x - y)$$
   b. The sum of two cubes:
   $$x^3 + y^3 = (x + y)(x^2 - xy + y^2)$$
   c. The difference of two cubes:
   $$x^3 - y^3 = (x - y)(x^2 + xy + y^2)$$
3. If an expression has three terms, try to factor it as a *trinomial*.
4. If an expression has four or more terms, try factoring by *grouping*.
5. Continue until each individual factor is prime.
6. Check the results by multiplying.

**Self Check Answers**
**1.** $4a(a - 2b)$ **2.** $a^2b^2c^2(1 + abc)$ **3.** $(x + y)(x + 2)$
**4.** $(3a + 4b)(3a - 4b)$ **5.** $(a^2 + 9b^2)(a + 3b)(a - 3b)$
**6.** $-3(x + 2)(x - 2)$ **7.** $(p - 6)(p + 1)$ **8.** $(3x - 2)(2x + 1)$
**9.** $-3(x + 2y)(2x + y)$ **10.** $(3a + 4)(5a - 1)$
**11.** $8(a + 5b^2)(a^2 - 5ab^2 + 25b^4)$ **12.** $(p - 4)(p^2 + 4p + 16)$
**13.** $(a + 4 + b)(a + 4 - b)$ **14.** $(x^2 + 2 + x)(x^2 + 2 - x)$

# Exercises 0.5

## Getting Ready

***You should be able to complete these vocabulary and concept statements before you proceed to the practice exercises.***

***Fill in the blanks.***

1. When polynomials are multiplied together, each polynomial is a ______ of the product.
2. If a polynomial cannot be factored using ______ coefficients, it is called a ______ polynomial.

***Complete each factoring formula.***

3. $ax + bx =$ ________
4. $x^2 - y^2 =$ ________
5. $x^2 + 2xy + y^2 =$ ________
6. $x^2 - 2xy + y^2 =$ ________
7. $x^3 + y^3 =$ ________
8. $x^3 - y^3 =$ ________

## Practice

***In each expression, factor out the greatest common monomial.***

9. $3x - 6$
10. $5y - 15$
11. $8x^2 + 4x^3$
12. $9y^3 + 6y^2$
13. $7x^2y^2 + 14x^3y^2$
14. $25y^2z - 15yz^2$

***In each expression, factor by grouping.***

15. $a(x + y) + b(x + y)$
16. $b(x - y) + a(x - y)$
17. $4a + b - 12a^2 - 3ab$
18. $x^2 + 4x + xy + 4y$

***In each expression, factor the difference of two squares.***

19. $4x^2 - 9$
20. $36z^2 - 49$
21. $4 - 9r^2$
22. $16 - 49x^2$
23. $81x^4 - 1$
24. $81 - x^4$
25. $(x + z)^2 - 25$
26. $(x - y)^2 - 9$

***In each expression, factor the trinomial.***

27. $x^2 + 8x + 16$
28. $a^2 - 12a + 36$
29. $b^2 - 10b + 25$
30. $y^2 + 14y + 49$
31. $m^2 + 4mn + 4n^2$
32. $r^2 - 8rs + 16s^2$
33. $12x^2 - xy - 6y^2$
34. $8x^2 - 10xy - 3y^2$

***In each expression, factor the trinomial by grouping.***

35. $x^2 + 10x + 21$
36. $x^2 + 7x + 10$
37. $x^2 - 4x - 12$
38. $x^2 - 2x - 63$
39. $6p^2 + 7p - 3$
40. $4q^2 - 19q + 12$

***In each expression, factor the sum of two cubes.***

41. $t^3 + 343$
42. $r^3 + 8s^3$

***In each expression, factor the difference of two cubes.***

43. $8z^3 - 27$
44. $125a^3 - 64$

***Factor each expression completely. If an expression is prime, so indicate.***

45. $3a^2bc + 6ab^2c + 9abc^2$
46. $5x^3y^3z^3 + 25x^2y^2z^2 - 125xyz$
47. $3x^3 + 3x^2 - x - 1$
48. $4x + 6xy - 9y - 6$
49. $2txy + 2ctx - 3ty - 3ct$
50. $2ax + 4ay - bx - 2by$
51. $ax + bx + ay + by + az + bz$
52. $6x^2y^3 + 18xy + 3x^2y^2 + 9x$
53. $x^2 - (y - z)^2$
54. $z^2 - (y + 3)^2$
55. $(x - y)^2 - (x + y)^2$
56. $(2a + 3)^2 - (2a - 3)^2$
57. $x^4 - y^4$
58. $z^4 - 81$
59. $3x^2 - 12$
60. $3x^3y - 3xy$
61. $18xy^2 - 8x$
62. $27x^2 - 12$
63. $x^2 - 2x + 15$
64. $x^2 + x + 2$
65. $-15 + 2a + 24a^2$
66. $-32 - 68x + 9x^2$
67. $6x^2 + 29xy + 35y^2$
68. $10x^2 - 17xy + 6y^2$
69. $12p^2 - 58pq - 70q^2$
70. $3x^2 - 6xy - 9y^2$
71. $-6m^2 + 47mn - 35n^2$
72. $-14r^2 - 11rs + 15s^2$
73. $-6x^3 + 23x^2 + 35x$
74. $-y^3 - y^2 + 90y$

**75.** $6x^4 - 11x^3 - 35x^2$

**76.** $12x + 17x^2 - 7x^3$

**77.** $x^4 + 2x^2 - 15$

**78.** $x^4 - x^2 - 6$

**79.** $a^{2n} - 2a^n - 3$

**80.** $a^{2n} + 6a^n + 8$

**81.** $6x^{2n} - 7x^n + 2$

**82.** $9x^{2n} + 9x^n + 2$

**83.** $4x^{2n} - 9y^{2n}$

**84.** $8x^{2n} - 2x^n - 3$

**85.** $10y^{2n} - 11y^n - 6$

**86.** $16y^{4n} - 25y^{2n}$

**87.** $2x^3 + 2{,}000$

**88.** $3y^3 + 648$

**89.** $(x + y)^3 - 64$

**90.** $(x - y)^3 + 27$

**91.** $64a^6 - y^6$

**92.** $a^6 + b^6$

**93.** $a^3 - b^3 + a - b$

**94.** $(a^2 - y^2) - 5(a + y)$

**95.** $64x^6 + y^6$

**96.** $z^2 + 6z + 9 - 225y^2$

**97.** $x^2 - 6x + 9 - 144y^2$

**98.** $x^2 + 2x - 9y^2 + 1$

**99.** $(a + b)^2 - 3(a + b) - 10$

**100.** $2(a + b)^2 - 5(a + b) - 3$

**101.** $x^6 + 7x^3 - 8$

**102.** $x^6 - 13x^4 + 36x^2$

**103.** $x^4 + x^2 + 1$

**104.** $x^4 + 3x^2 + 4$

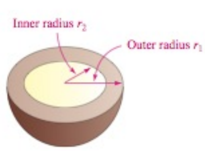
**105.** $x^4 + 7x^2 + 16$

**106.** $y^4 + 2y^2 + 9$

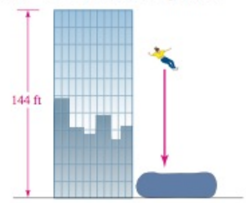
**107.** $4a^4 + 1 + 3a^2$

**108.** $x^4 + 25 + 6x^2$

**109. Candy** To find the amount of chocolate used in the outer coating of one of the malted-milk balls shown, we can find the volume $V$ of the chocolate shell using the formula $V = \frac{4}{3}\pi r_1^3 - \frac{4}{3}\pi r_2^3$. Factor the expression on the right side of the formula.




**110. Movie Stunts** The formula that gives the distance a stuntwoman is above the ground $t$ seconds after she falls over the side of a 144-foot tall building is $f = 144 - 16t^2$. Factor the right side.




## Discovery and Writing

**111.** Explain how to factor the difference of two squares.

**112.** Explain how to factor the difference of two cubes.

**113.** Explain how to factor $a^2 - b^2 + a + b$.

**114.** Explain how to factor $x^2 + 2x + 1$.

***Factor the indicated monomial from the given expression.***

**115.** $3x + 2; 2$

**116.** $5x - 3; 5$

**117.** $x^2 + 2x + 4; 2$

**118.** $3x^2 - 2x - 5; 3$

**119.** $a + b; a$

**120.** $a - b; b$

**121.** $x + x^{1/2}; x^{1/2}$

**122.** $x^{3/2} - x^{1/2}; x^{1/2}$

**123.** $2x + \sqrt{2}y; \sqrt{2}$

**124.** $\sqrt{3}a - 3b; \sqrt{3}$

**125.** $ab^{3/2} - a^{3/2}b; ab$

**126.** $ab^2 + b; b^{-1}$

***Factor each expression by grouping three terms and two terms.***

**127.** $x^2 + x - 6 + xy - 2y$

**128.** $2x^2 + 5x + 2 - xy - 2y$

**129.** $a^4 + 2a^3 + a^2 + a + 1$

**130.** $a^4 + a^3 - 2a^2 + a - 1$

## Review

**131.** Which natural number is neither prime nor composite?

**132.** Graph the interval $[-2,3)$.

**133.** Simplify: $(x^3x^2)^4$.

**134.** Simplify: $\dfrac{(a^3)^3(a^2)^4}{(a^2a^3)^3}$.

**135.** $\left(\dfrac{3x^4x^3}{6x^{-2}x^4}\right)^0$

**136.** Simplify: $\sqrt{20x^5}$.

**137.** Simplify: $\sqrt{20x} - \sqrt{125x}$.

**138.** Rationalize the denominator: $\dfrac{3}{\sqrt[3]{3}}$.

# 0.6 Rational Expressions

In this section, we will learn to

1. Define rational expressions.
2. Simplify rational expressions.
3. Multiple and divide rational expressions.
4. Add and subtract rational expressions.
5. Simplify complex fractions.

Abercrombie and Fitch (A&F) is a very successful American clothing company founded in 1892 by David Abercrombie and Ezra Fitch. Today the stores are popular shopping destinations for university students wanting to keep up with the latest styles and trends.

Photo by Tim Boyle/Getty Images

Suppose that a clothing manufacturer finds that the cost in dollars of producing $x$ fleece vintage shirts is given by the algebraic expression $13x + 1{,}000$. The average cost of producing each shirt could be obtained by dividing the production cost, $13x + 1{,}000$, by the number of shirts produced, $x$. The algebraic fraction

$$\frac{13x + 1{,}000}{x}$$

represents the average cost per shirt. We see that the average cost of producing 200 shirts would be $18.

$$\frac{13(\mathbf{200}) + 1{,}000}{\mathbf{200}} = 18$$

An understanding of algebraic fractions is important in solving many real-life problems.

## 1. Define Rational Expressions

If $x$ and $y$ are real numbers, the quotient $\frac{x}{y}$ $(y \neq 0)$ is called a **fraction**. The number $x$ is called the **numerator**, and the number $y$ is called the **denominator**.

**Algebraic fractions** are quotients of algebraic expressions. If the expressions are polynomials, the fraction is called a **rational expression**. The first two of the following algebraic fractions are rational expressions. The third is not, because the numerator and denominator are not polynomials.

$$\frac{5y^2 + 2y}{y^2 - 3y - 7} \qquad \frac{8ab^2 - 16c^3}{2x + 3} \qquad \frac{x^{1/2} + 4x}{x^{3/2} - x^{1/2}}$$

**Caution**

Remember that the denominator of a fraction **cannot** be zero.

We summarize some of the properties of fractions as follows:

**Properties of Fractions** If $a$, $b$, $c$, and $d$ are real numbers and no denominators are 0, then

**Equality of Fractions**

$$\frac{a}{b} = \frac{c}{d} \quad \text{if and only if} \quad ad = bc$$

**Fundamental Property of Fractions**

$$\frac{ax}{bx} = \frac{a}{b}$$

**Multiplication and Division of Fractions**

$$\frac{a}{b} \cdot \frac{c}{d} = \frac{ac}{bd} \quad \text{and} \quad \frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{c} = \frac{ad}{bc}$$

**Addition and Subtraction of Fractions**

$$\frac{a}{b} + \frac{c}{b} = \frac{a+c}{b} \quad \text{and} \quad \frac{a}{b} - \frac{c}{b} = \frac{a-c}{b}$$

The first two examples illustrate each of the previous properties of fractions.

**EXAMPLE 1 Illustrating the Properties of Fractions**

Assume that no denominators are 0.

**a.** $\frac{2a}{3} = \frac{4a}{6}$ Because $2a(6) = 3(4a)$

**b.** $\frac{6xy}{10xy} = \frac{3(2xy)}{5(2xy)}$ Factor the numerator and denominator and divide out the common factors.

$= \frac{3}{5}$

**Self Check 1** **a.** Is $\frac{3y}{5} = \frac{15z}{25}$? **b.** Simplify: $\frac{15a^2b}{25ab^2}$.

**Now Try Exercise 9.**

**EXAMPLE 2 Illustrating the Properties of Fractions**

Assume that no denominators are 0.

**a.** $\frac{2r}{7s} \cdot \frac{3r}{5s} = \frac{2r \cdot 3r}{7s \cdot 5s}$

$= \frac{6r^2}{35s^2}$

**b.** $\frac{3mn}{4pq} \div \frac{2pq}{7mn} = \frac{3mn}{4pq} \cdot \frac{7mn}{2pq}$

$= \frac{21m^2n^2}{8p^2q^2}$

**c.** $\frac{2ab}{5xy} + \frac{ab}{5xy} = \frac{2ab + ab}{5xy}$

$= \frac{3ab}{5xy}$

**d.** $\frac{6uv^2}{7w^2} - \frac{3uv^2}{7w^2} = \frac{6uv^2 - 3uv^2}{7w^2}$

$= \frac{3uv^2}{7w^2}$

**Self Check 2** Perform each operation:

**a.** $\frac{3a}{5b} \cdot \frac{2a}{7b}$ **b.** $\frac{2ab}{3rs} \div \frac{2rs}{4ab}$ **c.** $\frac{5pq}{3t} + \frac{3pq}{3t}$ **d.** $\frac{5mn^2}{3w} - \frac{mn^2}{3w}$

**Now Try Exercise 19.**

To add or subtract rational expressions with unlike denominators, we write each expression as an equivalent expression with a common denominator. We can then add or subtract the expressions. For example,

$$\begin{aligned}\frac{3x}{5} + \frac{2x}{7} &= \frac{3x(7)}{5(7)} + \frac{2x(5)}{7(5)} \\ &= \frac{21x}{35} + \frac{10x}{35} \\ &= \frac{21x + 10x}{35} \\ &= \frac{31x}{35}\end{aligned}$$

$$\begin{aligned}\frac{4a^2}{15} - \frac{3a^2}{10} &= \frac{4a^2(2)}{15(2)} - \frac{3a^2(3)}{10(3)} \\ &= \frac{8a^2}{30} - \frac{9a^2}{30} \\ &= \frac{8a^2 - 9a^2}{30} \\ &= \frac{-a^2}{30} \\ &= -\frac{a^2}{30}\end{aligned}$$

A rational expression is **in lowest terms** if all factors common to the numerator and the denominator have been removed. To **simplify a rational expression** means to write it in lowest terms.

## 2. Simplify Rational Expressions

To simplify rational expressions, we use the Fundamental Property of Fractions. This enables us to divide out all factors that are common to the numerator and the denominator.

**EXAMPLE 3** **Simplifying a Rational Expression**

Simplify: $\frac{x^2 - 9}{x^2 - 3x}$ $(x \neq 0, 3)$.

**SOLUTION** We factor the difference of two squares in the numerator, factor out $x$ in the denominator, and divide out the common factor of $x - 3$.

$$\begin{aligned}\frac{x^2 - 9}{x^2 - 3x} &= \frac{(x + 3)\cancel{(x - 3)}}{x\cancel{(x - 3)}} \qquad \frac{x-3}{x-3} = 1 \\ &= \frac{x + 3}{x}\end{aligned}$$

**Self Check 3** Simplify: $\frac{a^2 - 4a}{a^2 - a - 12}$ $(a \neq 4, -3)$.

**Now Try Exercise 23.**

We will encounter the following properties of fractions in the next examples.

**Properties of Fractions** If $a$ and $b$ represent real numbers and there are no divisions by 0, then

- $\frac{a}{1} = a$
- $\frac{a}{a} = 1$
- $\frac{a}{b} = \frac{-a}{-b} = -\frac{a}{-b} = -\frac{-a}{b}$
- $-\frac{a}{b} = \frac{a}{-b} = \frac{-a}{b} = -\frac{-a}{-b}$

**EXAMPLE 4 Simplifying a Rational Expression**

Simplify: $\frac{x^2 - 2xy + y^2}{y - x}$ $(x \neq y)$.

**SOLUTION** We factor the trinomial in the numerator, factor $-1$ from the denominator, and divide out the common factor of $x - y$.

$$\begin{aligned}\frac{x^2 - 2xy + y^2}{y - x} &= \frac{(x - y)\cancel{(x - y)}}{-1\cancel{(x - y)}} \qquad \frac{x-y}{x-y} = 1\\ &= \frac{x - y}{-1}\\ &= -\frac{x - y}{1}\\ &= -(x - y)\end{aligned}$$

**Self Check 4** Simplify: $\frac{a^2 - ab - 2b^2}{2b - a}$ $(2b - a \neq 0)$.

**Now Try Exercise 25.**

**EXAMPLE 5 Simplifying a Rational Expression**

Simplify: $\frac{x^2 - 3x + 2}{x^2 - x - 2}$ $(x \neq 2, -1)$.

**SOLUTION** We factor the numerator and denominator and divide out the common factor of $x - 2$.

$$\begin{aligned}\frac{x^2 - 3x + 2}{x^2 - x - 2} &= \frac{(x - 1)\cancel{(x - 2)}}{(x + 1)\cancel{(x - 2)}} \qquad \frac{x-2}{x-2} = 1\\ &= \frac{x - 1}{x + 1}\end{aligned}$$

**Self Check 5** Simplify: $\frac{a^2 + 3a - 4}{a^2 + 2a - 3}$ $(a \neq 1, -3)$.

**Now Try Exercise 27.**

## 3. Multiply and Divide Rational Expressions

**EXAMPLE 6 Multiplying Rational Expressions**

Multiply: $\frac{x^2 - x - 2}{x^2 - 1} \cdot \frac{x^2 + 2x - 3}{x - 2}$ $(x \neq 1, -1, 2)$.

**SOLUTION** To multiply the rational expressions, we multiply the numerators, multiply the denominators, and divide out the common factors.

$$\begin{aligned}\frac{x^2-x-2}{x^2-1}\cdot\frac{x^2+2x-3}{x-2} &= \frac{(x^2-x-2)(x^2+2x-3)}{(x^2-1)(x-2)}\\ &= \frac{\cancel{(x-2)}\cancel{(x+1)}\cancel{(x-1)}(x+3)}{\cancel{(x+1)}\cancel{(x-1)}\cancel{(x-2)}} \qquad \frac{x-2}{x-2}=1,\ \frac{x+1}{x+1}=1,\ \frac{x-1}{x-1}=1\\ &= x+3\end{aligned}$$

**Self Check 6** Simplify: $\frac{x^2-9}{x^2-x}\cdot\frac{x-1}{x^2-3x}$ $(x \neq 0, 1, 3)$.

**Now Try Exercise 33.**

**EXAMPLE 7 Dividing Rational Expressions**

Divide: $\frac{x^2-2x-3}{x^2-4} \div \frac{x^2+2x-15}{x^2+3x-10}$ $(x \neq 2, -2, 3, -5)$.

**SOLUTION** To divide the rational expressions, we multiply by the reciprocal of the second rational expression. We then simplify by factoring the numerator and denominator and dividing out the common factors.

$$\begin{aligned}\frac{x^2-2x-3}{x^2-4}\div\frac{x^2+2x-15}{x^2+3x-10} &= \frac{x^2-2x-3}{x^2-4}\cdot\frac{x^2+3x-10}{x^2+2x-15}\\ &= \frac{(x^2-2x-3)(x^2+3x-10)}{(x^2-4)(x^2+2x-15)}\\ &= \frac{\cancel{(x-3)}(x+1)\cancel{(x-2)}\cancel{(x+5)}}{(x+2)\cancel{(x-2)}\cancel{(x+5)}\cancel{(x-3)}} \qquad \frac{x-3}{x-3}=1,\ \frac{x-2}{x-2}=1,\ \frac{x+5}{x+5}=1\\ &= \frac{x+1}{x+2}\end{aligned}$$

**Self Check 7** Simplify: $\frac{a^2-a}{a+2} \div \frac{a^2-2a}{a^2-4}$ $(a \neq 0, 2, -2)$.

**Now Try Exercise 39.**

**EXAMPLE 8 Using Multiplication and Division to Simplify a Rational Expression**

Simplify: $\frac{2x^2-5x-3}{3x-1}\cdot\frac{3x^2+2x-1}{x^2-2x-3}\div\frac{2x^2+x}{3x}$ $\left(x \neq \frac{1}{3}, -1, 3, 0, -\frac{1}{2}\right)$.

**SOLUTION** We can change the division to a multiplication, factor, and simplify.

$$\begin{aligned}&\frac{2x^2-5x-3}{3x-1}\cdot\frac{3x^2+2x-1}{x^2-2x-3}\div\frac{2x^2+x}{3x}\\ &= \frac{2x^2-5x-3}{3x-1}\cdot\frac{3x^2+2x-1}{x^2-2x-3}\cdot\frac{3x}{2x^2+x}\\ &= \frac{(2x^2-5x-3)(3x^2+2x-1)(3x)}{(3x-1)(x^2-2x-3)(2x^2+x)}\\ &= \frac{\cancel{(x-3)}\cancel{(2x+1)}\cancel{(3x-1)}\cancel{(x+1)}3\cancel{x}}{\cancel{(3x-1)}\cancel{(x+1)}\cancel{(x-3)}\cancel{x}\cancel{(2x+1)}} \qquad \frac{x-3}{x-3}=1,\ \frac{2x+1}{2x+1}=1,\ \frac{3x-1}{3x-1}=1,\ \frac{x+1}{x+1}=1,\ \frac{x}{x}=1\\ &= 3\end{aligned}$$

**Self Check 8** Simplify: $\frac{x^2-25}{x-2} \div \frac{x^2-5x}{x^2-2x} \cdot \frac{x^2+2x}{x^2+5x}$ $(x \neq 0, 2, 5, -5)$.

**Now Try Exercise 45.**

## 4. Add and Subtract Rational Expressions

To add (or subtract) rational expressions with like denominators, we add (or subtract) the numerators and keep the common denominator.

**EXAMPLE 9** **Adding Rational Expression with Like Deonominators**

Add: $\frac{2x+5}{x+5} + \frac{3x+20}{x+5}$ $(x \neq -5)$.

**SOLUTION**

$$\frac{2x+5}{x+5} + \frac{3x+20}{x+5} = \frac{5x+25}{x+5} \quad \text{Add the numerators and keep the common denominator.}$$

$$= \frac{5\cancel{(x+5)}}{1\cancel{(x+5)}} \quad \text{Factor out 5 and divide out the common factor of } x+5.$$

$$= 5$$

**Self Check 9** Add: $\frac{3x-2}{x-2} + \frac{x-6}{x-2}$ $(x \neq 2)$.

**Now Try Exercise 49.**

To add (or subtract) rational expressions with unlike denominators, we must find a common denominator, called the **least** (or lowest) **common denominator (LCD)**. Suppose the unlike denominators of three rational expressions are 12, 20, and 35. To find the LCD, we first find the prime factorization of each number.

$$12 = 4 \cdot 3 \qquad 20 = 4 \cdot 5 \qquad 35 = 5 \cdot 7$$
$$= 2^2 \cdot 3 \qquad = 2^2 \cdot 5$$

Because the LCD is the smallest number that can be divided by 12, 20, and 35, it must contain factors of $2^2$, 3, 5, and 7. Thus, the

$$\text{LCD} = 2^2 \cdot 3 \cdot 5 \cdot 7 = 420$$

That is, 420 is the smallest number that can be divided without remainder by 12, 20, and 35.

When finding an LCD, we always factor each denominator and then create the LCD by using each factor the greatest number of times that it appears in any one denominator. The product of these factors is the LCD.

This rule also applies if the unlike denominators of the rational expressions contain variables. Suppose the unlike denominators are $x^2(x-5)$ and $x(x-5)^3$. To find the LCD, we use $x^2$ and $(x-5)^3$. Thus, the LCD is the product $x^2(x-5)^3$.

**Comment**

Remember to find the *least* common denominator, use each factor the *greatest* number of times that it occurs.

**EXAMPLE 10** **Adding Rational Expressions with Unlike Denominators**

Add: $\frac{1}{x^2-4} + \frac{2}{x^2-4x+4}$ $(x \neq 2, -2)$.

**SOLUTION** We factor each denominator and find the LCD.

$$x^2 - 4 = (x+2)(x-2)$$
$$x^2 - 4x + 4 = (x-2)(x-2) = (x-2)^2$$

The LCD is $(x+2)(x-2)^2$. We then write each rational expression with its denominator in factored form, convert each rational expression into an equivalent expression with a denominator of $(x+2)(x-2)^2$, add the expressions, and simplify.

$$\begin{aligned}
\frac{1}{x^2-4} + \frac{2}{x^2-4x+4} &= \frac{1}{(x+2)(x-2)} + \frac{2}{(x-2)(x-2)} \\
&= \frac{1(x-2)}{(x+2)(x-2)(x-2)} + \frac{2(x+2)}{(x-2)(x-2)(x+2)} \qquad \frac{x-2}{x-2}=1, \frac{x+2}{x+2}=1 \\
&= \frac{1(x-2)+2(x+2)}{(x+2)(x-2)(x-2)} \\
&= \frac{x-2+2x+4}{(x+2)(x-2)(x-2)} \\
&= \frac{3x+2}{(x+2)(x-2)^2}
\end{aligned}$$

**Comment**

Always attempt to simplify the final result. In this case, the final fraction is already in lowest terms.

**Self Check 10** Add: $\frac{3}{x^2-6x+9} + \frac{1}{x^2-9}$ $(x \neq 3, -3)$.

**Now Try Exercise 59.**

**EXAMPLE 11 Combining and Simplifying Rational Expressions with Unlike Denominators**

Simplify: $\frac{x-2}{x^2-1} - \frac{x+3}{x^2+3x+2} + \frac{3}{x^2+x-2}$ $(x \neq 1, -1, -2)$.

**SOLUTION** We factor the denominators to find the LCD.

$$x^2 - 1 = (x+1)(x-1)$$
$$x^2 + 3x + 2 = (x+2)(x+1)$$
$$x^2 + x - 2 = (x+2)(x-1)$$

The LCD is $(x+1)(x+2)(x-1)$. We now write each rational expression as an equivalent expression with this LCD, and proceed as follows:

$$\begin{aligned}
&\frac{x-2}{x^2-1} - \frac{x+3}{x^2+3x+2} + \frac{3}{x^2+x-2} \\
&= \frac{x-2}{(x+1)(x-1)} - \frac{x+3}{(x+1)(x+2)} + \frac{3}{(x-1)(x+2)} \\
&= \frac{(x-2)(x+2)}{(x+1)(x-1)(x+2)} - \frac{(x+3)(x-1)}{(x+1)(x+2)(x-1)} + \frac{3(x+1)}{(x-1)(x+2)(x+1)} \qquad \frac{x+2}{x+2}=1, \frac{x-1}{x-1}=1, \frac{x+1}{x+1}=1 \\
&= \frac{(x^2-4)-(x^2+2x-3)+(3x+3)}{(x+1)(x+2)(x-1)} \\
&= \frac{x^2-4-x^2-2x+3+3x+3}{(x+1)(x+2)(x-1)} \\
&= \frac{x+2}{(x+1)(x+2)(x-1)} \\
&= \frac{1}{(x+1)(x-1)} \qquad \text{Divide out the common factor of } x+2. \ \frac{x+2}{x+2}=1
\end{aligned}$$

**Self Check 11** Simplify: $\frac{4y}{y^2-1} - \frac{2}{y+1} + 2$ $(y \neq 1, -1)$.

**Now Try Exercise 61.**

## 5. Simplify Complex Fractions

A **complex fraction** is a fraction that has a fraction in its numerator or a fraction in its denominator. There are two methods generally used to simplify complex fractions. These are stated here for you.

**Strategies for Simplifying Complex Fractions**

**Method 1: Multiply the Complex Fraction by 1**

- Determine the LCD of all fractions in the complex fraction.
- Multiply both numerator and denominator of the complex fraction by the LCD. Note that when we multiply by $\frac{LCD}{LCD}$ we are multiplying by 1.

**Method 2: Simplify the Numerator and Denominator and then Divide**

- Simplify the numerator and denominator so that both are single fractions.
- Perform the division by multiplying the numerator by the reciprocal of the denominator.

**EXAMPLE 12** **Simplifying Complex Fractions**

Simplify: $\dfrac{\frac{1}{x}+\frac{1}{y}}{\frac{x}{y}}$ $(x, y \neq 0)$.

**Method 1:** We note that the LCD of the three fractions in the complex fraction is $xy$. So we multiply the numerator and denominator of the complex fraction by $xy$ and simplify:

$$\frac{\frac{1}{x}+\frac{1}{y}}{\frac{x}{y}} = \frac{xy\left(\frac{1}{x}+\frac{1}{y}\right)}{xy\left(\frac{x}{y}\right)} = \frac{\frac{xy}{x}+\frac{xy}{y}}{\frac{xyx}{y}} = \frac{y+x}{x^2}$$

**Method 2:** We combine the fractions in the numerator of the complex fraction to obtain a single fraction over a single fraction.

$$\frac{\frac{1}{x}+\frac{1}{y}}{\frac{x}{y}} = \frac{\frac{1(y)}{x(y)}+\frac{1(x)}{y(x)}}{\frac{x}{y}} = \frac{\frac{y+x}{xy}}{\frac{x}{y}}$$

Then we use the fact that any fraction indicates a division:

$$\frac{\frac{y+x}{xy}}{\frac{x}{y}} = \frac{y+x}{xy} \div \frac{x}{y} = \frac{y+x}{xy}\cdot\frac{y}{x} = \frac{(y+x)y}{xyx} = \frac{y+x}{x^2}$$

**Self Check 12** Simplify: $\dfrac{\frac{1}{x}-\frac{1}{y}}{\frac{1}{x}+\frac{1}{y}}$ $(x, y \neq 0)$.

**Now Try Exercise 81.**

**Self Check Answers** **1. a.** no **b.** $\frac{3a}{5b}$ **2. a.** $\frac{6a^2}{35b^2}$ **b.** $\frac{4a^2b^2}{3r^2s^2}$ **c.** $\frac{8pq}{3t}$ **d.** $\frac{4mn^2}{3w}$ **3.** $\frac{a}{a+3}$ **4.** $-(a+b)$ **5.** $\frac{a+4}{a+3}$ **6.** $\frac{x+3}{x^2}$ **7.** $a-1$ **8.** $x+2$ **9.** 4 **10.** $\frac{4x+6}{(x+3)(x-3)^2}$ **11.** $\frac{2y}{y-1}$ **12.** $\frac{y-x}{y+x}$

## Exercises 0.6

### Getting Ready

***You should be able to complete these vocabulary and concept statements before you proceed to the practice exercises.***

***Fill in the blanks.***

1. In the fraction $\frac{a}{b}$, $a$ is called the ________.
2. In the fraction $\frac{a}{b}$, $b$ is called the ________.
3. $\frac{a}{b} = \frac{c}{d}$ if and only if ________.
4. The denominator of a fraction can never be _____.

***Complete each formula.***

5. $\frac{a}{b} \cdot \frac{c}{d} =$ ____
6. $\frac{a}{b} \div \frac{c}{d} =$ ____
7. $\frac{a}{b} + \frac{c}{b} =$ _____
8. $\frac{a}{b} - \frac{c}{b} =$ _____

***Determine whether the fractions are equal. Assume that no denominators are 0.***

9. $\frac{8x}{3y}, \frac{16x}{6y}$
10. $\frac{3x^2}{4y^2}, \frac{12y^2}{16x^2}$
11. $\frac{25xyz}{12ab^2c}, \frac{50a^2bc}{24xyz}$
12. $\frac{15rs^2}{4rs^2}, \frac{37.5a^3}{10a^3}$

### Practice

***Simplify each rational expression. Assume that no denominators are 0.***

13. $\frac{7a^2b}{21ab^2}$
14. $\frac{35p^3q^2}{49p^4q}$

***Perform the operations and simplify, whenever possible. Assume that no denominators are 0.***

15. $\frac{4x}{7} \cdot \frac{2}{5a}$
16. $\frac{-5y}{2z} \cdot \frac{4}{y^2}$
17. $\frac{8m}{5n} \div \frac{3m}{10n}$
18. $\frac{15p}{8q} \div \frac{-5p}{16q^2}$
19. $\frac{3z}{5c} + \frac{2z}{5c}$
20. $\frac{7a}{4b} - \frac{3a}{4b}$
21. $\frac{15x^2y}{7a^2b^3} - \frac{x^2y}{7a^2b^3}$
22. $\frac{8rst^2}{15m^4t^2} + \frac{7rst^2}{15m^4t^2}$

***Simplify each fraction. Assume that no denominators are 0.***

23. $\frac{2x-4}{x^2-4}$
24. $\frac{x^2-16}{x^2-8x+16}$
25. $\frac{4-x^2}{x^2-5x+6}$
26. $\frac{25-x^2}{x^2+10x+25}$
27. $\frac{6x^3+x^2-12x}{4x^3+4x^2-3x}$
28. $\frac{6x^4-5x^3-6x^2}{2x^3-7x^2-15x}$
29. $\frac{x^3-8}{x^2+ax-2x-2a}$
30. $\frac{xy+2x+3y+6}{x^3+27}$

***Perform the operations and simplify, whenever possible. Assume that no denominators are 0.***

31. $\frac{x^2-1}{x} \cdot \frac{x^2}{x^2+2x+1}$

32. $\dfrac{y^2-2y+1}{y}\cdot\dfrac{y+2}{y^2+y-2}$

33. $\dfrac{3x^2+7x+2}{x^2+2x}\cdot\dfrac{x^2-x}{3x^2+x}$

34. $\dfrac{x^2+x}{2x^2+3x}\cdot\dfrac{2x^2+x-3}{x^2-1}$

35. $\dfrac{x^2+x}{x-1}\cdot\dfrac{x^2-1}{x+2}$

36. $\dfrac{x^2+5x+6}{x^2+6x+9}\cdot\dfrac{x+2}{x^2-4}$

37. $\dfrac{2x^2+32}{8}\div\dfrac{x^2+16}{2}$

38. $\dfrac{x^2+x-6}{x^2-6x+9}\div\dfrac{x^2-4}{x^2-9}$

39. $\dfrac{z^2+z-20}{z^2-4}\div\dfrac{z^2-25}{z-5}$

40. $\dfrac{ax+bx+a+b}{a^2+2ab+b^2}\div\dfrac{x^2-1}{x^2-2x+1}$

41. $\dfrac{3x^2+5x-2}{x^3+2x^2}\div\dfrac{6x^2+13x-5}{2x^3+5x^2}$

42. $\dfrac{x^2+13x+12}{8x^2-6x-5}\div\dfrac{2x^2-x-3}{8x^2-14x+5}$

43. $\dfrac{x^2+7x+12}{x^3-x^2-6x}\cdot\dfrac{x^2-3x-10}{x^2+2x-3}\cdot\dfrac{x^3-4x^2+3x}{x^2-x-20}$

44. $\dfrac{x(x-2)-3}{x(x+7)-3(x-1)}\cdot\dfrac{x(x+1)-2}{x(x-7)+3(x+1)}$

45. $\dfrac{x^2-2x-3}{21x^2-50x-16}\cdot\dfrac{3x-8}{x-3}\div\dfrac{x^2+6x+5}{7x^2-33x-10}$

46. $\dfrac{x^3+27}{x^2-4}\div\left(\dfrac{x^2+4x+3}{x^2+2x}\div\dfrac{x^2+x-6}{x^2-3x+9}\right)$

47. $\dfrac{3}{x+3}+\dfrac{x+2}{x+3}$

48. $\dfrac{3}{x+1}+\dfrac{x+2}{x+1}$

49. $\dfrac{4x}{x-1}-\dfrac{4}{x-1}$

50. $\dfrac{6x}{x-2}-\dfrac{3}{x-2}$

51. $\dfrac{2}{5-x}+\dfrac{1}{x-5}$

52. $\dfrac{3}{x-6}-\dfrac{2}{6-x}$

53. $\dfrac{3}{x+1}+\dfrac{2}{x-1}$

54. $\dfrac{3}{x+4}+\dfrac{x}{x-4}$

55. $\dfrac{a+3}{a^2+7a+12}+\dfrac{a}{a^2-16}$

56. $\dfrac{a}{a^2+a-2}+\dfrac{2}{a^2-5a+4}$

57. $\dfrac{x}{x^2-4}-\dfrac{1}{x+2}$

58. $\dfrac{b^2}{b^2-4}-\dfrac{4}{b^2+2b}$

59. $\dfrac{3x-2}{x^2+2x+1}-\dfrac{x}{x^2-1}$

60. $\dfrac{2t}{t^2-25}-\dfrac{t+1}{t^2+5t}$

61. $\dfrac{2}{y^2-1}+3+\dfrac{1}{y+1}$

62. $2+\dfrac{4}{t^2-4}-\dfrac{1}{t-2}$

63. $\dfrac{1}{x-2}+\dfrac{3}{x+2}-\dfrac{3x-2}{x^2-4}$

64. $\dfrac{x}{x-3}-\dfrac{5}{x+3}+\dfrac{3(3x-1)}{x^2-9}$

65. $\left(\dfrac{1}{x-2}+\dfrac{1}{x-3}\right)\cdot\dfrac{x-3}{2x}$

66. $\left(\dfrac{1}{x+1}-\dfrac{1}{x-2}\right)\div\dfrac{1}{x-2}$

67. $\dfrac{3x}{x-4}-\dfrac{x}{x+4}-\dfrac{3x+1}{16-x^2}$

68. $\dfrac{7x}{x-5}+\dfrac{3x}{5-x}+\dfrac{3x-1}{x^2-25}$

69. $\dfrac{1}{x^2+3x+2}-\dfrac{2}{x^2+4x+3}+\dfrac{1}{x^2+5x+6}$

70. $\dfrac{-2}{x-y}+\dfrac{2}{x-z}-\dfrac{2z-2y}{(y-x)(z-x)}$

71. $\dfrac{3x-2}{x^2+x-20}-\dfrac{4x^2+2}{x^2-25}+\dfrac{3x^2-25}{x^2-16}$

72. $\dfrac{3x+2}{8x^2-10x-3}+\dfrac{x+4}{6x^2-11x+3}-\dfrac{1}{4x+1}$

***Simplify each complex fraction. Assume that no denominators are 0.***

73. $\dfrac{\frac{3a}{b}}{\frac{6ac}{b^2}}$

74. $\dfrac{\frac{3t^2}{9x}}{\frac{t}{18x}}$

**75.** $\dfrac{\frac{3a^2b}{ab}}{27}$

**76.** $\dfrac{\frac{3u^2v}{4t}}{3uv}$

**77.** $\dfrac{\frac{x-y}{ab}}{\frac{y-x}{ab}}$

**78.** $\dfrac{\frac{x^2-5x+6}{2x^2y}}{\frac{x^2-9}{2x^2y}}$

**79.** $\dfrac{\frac{1}{x}+\frac{1}{y}}{xy}$

**80.** $\dfrac{xy}{\frac{11}{x}+\frac{11}{y}}$

**81.** $\dfrac{\frac{1}{x}+\frac{1}{y}}{\frac{1}{x}-\frac{1}{y}}$

**82.** $\dfrac{\frac{1}{x}-\frac{1}{y}}{\frac{1}{x}+\frac{1}{y}}$

**83.** $\dfrac{\frac{3a}{b}-\frac{4a^2}{x}}{\frac{1}{b}+\frac{1}{ax}}$

**84.** $\dfrac{1-\frac{x}{y}}{\frac{x^2}{y^2}-1}$

**85.** $\dfrac{x+1-\frac{6}{x}}{x+5+\frac{6}{x}}$

**86.** $\dfrac{2z}{1-\frac{3}{z}}$

**87.** $\dfrac{3xy}{1-\frac{1}{xy}}$

**88.** $\dfrac{x-3+\frac{1}{x}}{-\frac{1}{x}-x+3}$

**89.** $\dfrac{3x}{x+\frac{1}{x}}$

**90.** $\dfrac{2x^2+4}{2+\frac{4x}{5}}$

**91.** $\dfrac{\frac{x}{x+2}-\frac{2}{x-1}}{\frac{3}{x+2}+\frac{x}{x-1}}$

**92.** $\dfrac{\frac{2x}{x-3}+\frac{1}{x-2}}{\frac{3}{x-3}-\frac{x}{x-2}}$

***Write each expression without using negative exponents, and simplify the resulting complex fraction. Assume that no denominators are 0.***

**93.** $\dfrac{1}{1+x^{-1}}$

**94.** $\dfrac{y^{-1}}{x^{-1}+y^{-1}}$

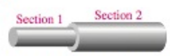
**95.** $\dfrac{3(x+2)^{-1}+2(x-1)^{-1}}{(x+2)^{-1}}$

**96.** $\dfrac{2x(x-3)^{-1}-3(x+2)^{-1}}{(x-3)^{-1}(x+2)^{-1}}$

## Applications

**97. Engineering** The stiffness $k$ of the shaft shown in the illustration is given by the following formula where $k_1$ and $k_2$ are the individual stiffnesses of each section. Simplify the complex fraction on the right side of the formula.

$$k = \frac{1}{\frac{1}{k_1}+\frac{1}{k_2}}$$




**98. Electronics** The combined resistance $R$ of three resistors with resistances of $R_1$, $R_2$, and $R_3$ is given by the following formula. Simplify the complex fraction on the right side of the formula.

$$R = \frac{1}{\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}}$$

## Discovery and Writing

***Simplify each complex fraction. Assume that no denominators are 0.***

**99.** $\dfrac{x}{1+\frac{1}{3x^{-1}}}$

**100.** $\dfrac{ab}{2+\frac{3}{2a^{-1}}}$

**101.** $\dfrac{1}{1+\frac{1}{1+\frac{1}{x}}}$

**102.** $\dfrac{y}{2+\frac{2}{2+\frac{2}{y}}}$

**103.** Explain why the formula $\frac{a}{b}+\frac{c}{d}=\frac{ad+bc}{bd}$ is valid.

**104.** Explain why the formula $\frac{a}{b}\div\frac{c}{d}=\frac{a}{b}\cdot\frac{d}{c}$ is valid.

**105.** Explain the Commutative Property of Addition and explain why it is useful.

**106.** Explain the Distributive Property and explain why it is useful.

## Review

***Write each expression without using absolute value symbols.***

**107.** $|-6|$

**108.** $|5-x|$, given that $x<0$

***Simplify each expression.***

**109.** $\left(\dfrac{x^3y^{-2}}{x^{-1}y}\right)^{-3}$

**110.** $(27x^6)^{2/3}$

**111.** $\sqrt{20}-\sqrt{45}$

**112.** $2(x^2+4)-3(2x^2+5)$

# CHAPTER REVIEW

## SECTION 0.1 Sets of Real Numbers

| Definitions and Concepts | Examples |
|---|---|
| **Natural numbers:** The numbers that we count with. | 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, ... |
| **Whole numbers:** The natural numbers and 0. | 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, ... |
| **Integers:** The whole numbers and the negatives of the natural numbers. | ..., −5, −4, −3, −2, −1, 0, 1, 2, 3, 4, 5, ... |
| **Rational numbers:** $\{x \mid x$ can be written in the form $\frac{a}{b}$ $(b \neq 0)$, where $a$ and $b$ are integers.$\}$ | $2 = \frac{2}{1}, -5 = -\frac{5}{1}, \frac{2}{3}, -\frac{7}{3}, 0.25 = \frac{1}{4}$ |
| All decimals that either terminate or repeat. | $\frac{3}{4} = 0.25, \frac{13}{5} = 2.6, \frac{2}{3} = 0.666\ldots = 0.\overline{6}, \frac{7}{11} = 0.\overline{63}$ |
| **Irrational numbers:** Nonrational real numbers. All decimals that neither terminate nor repeat. | $\sqrt{2}, -\sqrt{26}, \pi, 0.232232223\ldots$ |
| **Real numbers:** Any number that can be expressed as a decimal. | $-6, -\frac{9}{13}, 0, \pi, \sqrt{31}, 10.73$ |
| **Prime numbers:** A natural number greater than 1 that is divisible only by itself and 1. | 2, 3, 5, 7, 11, 13, 17, ... |
| **Composite numbers:** A natural number greater than 1 that is not prime. | 4, 6, 8, 9, 10, 12, 14, 15, ... |
| **Even integers:** The integers that are exactly divisible by 2. | ..., −6, −4, −2, 0, 2, 4, 6, ... |
| **Odd integers:** The integers that are not exactly divisible by 2. | ..., −5, −3, −1, 1, 3, 5, ... |
| **Associative Properties:** of addition $(a + b) + c = a + (b + c)$ | $5 + (4 + 7) = (5 + 4) + 7$ |
| of multiplication $(ab)c = a(bc)$ | $(5 \cdot 4) \cdot 7 = 5(4 \cdot 7)$ |
| **Commutative Properties:** of addition $a + b = b + a$ | $4 + 7 = 7 + 4$    $1.7 + 2.5 = 2.5 + 1.7$ |
| of multiplication $ab = ba$ | $4 \cdot 7 = 7 \cdot 4$    $1.7(2.5) = 2.5(1.7)$ |
| **Distributive Property:** $a(b + c) = ab + ac$ | $3(x + 6) = 3x + 3 \cdot 6$    $0.2(y - 10) = 0.2y - 0.2(10)$ |
| **Double Negative Rule:** $-(-a) = a$ | $-(-5) = 5$    $-(-a) = a$ |
| **Open intervals** have no endpoints. | $(-3, 2)$  |
| **Closed intervals** have two endpoints. | $[-3, 2]$  |
| **Half-open intervals** have one endpoint. | $(-3, 2]$  |

| Definitions and Concepts | Examples |
| --- | --- |
| **Absolute value:** | |
| If $x \geq 0$, then $\lvert x \rvert = x$. | $\lvert 7 \rvert = 7 \quad \lvert 3.5 \rvert = 3.5 \quad \left\lvert \frac{7}{2} \right\rvert = \frac{7}{2} \quad \lvert 0 \rvert = 0$ |
| If $x < 0$, then $\lvert x \rvert = -x$. | $\lvert -7 \rvert = 7 \quad \lvert -3.5 \rvert = 3.5 \quad \left\lvert -\frac{7}{2} \right\rvert = \frac{7}{2}$ |
| **Distance:** The **distance** between points $a$ and $b$ on a number line is $d = \lvert b - a \rvert$. | The distance on the number line between points with coordinates of $-3$ and 2 is $d = \lvert 2 - (-3) \rvert = \lvert 5 \rvert = 5$. |

## EXERCISES

***Consider the set $\{-6, -3, 0, \frac{1}{2}, 3, \pi, \sqrt{5}, 6, 8\}$. List the numbers in this set that are***

1. natural numbers.
2. whole numbers.
3. integers.
4. rational numbers.
5. irrational numbers.
6. real numbers.

***Consider the set $\{-6, -3, 0, \frac{1}{2}, 3, \pi, \sqrt{5}, 6, 8\}$. List the numbers in this set that are***

7. prime numbers.
8. composite numbers.
9. even integers.
10. odd integers.

***Determine which property of real numbers justifies each statement.***

11. $(a + b) + 2 = a + (b + 2)$
12. $a + 7 = 7 + a$
13. $4(2x) = (4 \cdot 2)x$
14. $3(a + b) = 3a + 3b$
15. $(5a)7 = 7(5a)$
16. $(2x + y) + z = (y + 2x) + z$
17. $-(-6) = 6$

***Graph each subset of the real numbers:***

18. the prime numbers between 10 and 20
19. the even integers from 6 to 14

***Graph each interval on the number line.***

20. $-3 < x \leq 5$
21. $x \geq 0$ or $x < -1$
22. $(-2, 4]$
23. $(-\infty, 2) \cap (-5, \infty)$
24. $(-\infty, -4) \cup [6, \infty)$

***Write each expression without absolute value symbols.***

25. $\lvert 6 \rvert$
26. $\lvert -25 \rvert$
27. $\lvert 1 - \sqrt{2} \rvert$
28. $\lvert \sqrt{3} - 1 \rvert$
29. On a number line, find the distance between points with coordinates of $-5$ and 7.

## SECTION 0.2 Integer Exponents and Scientific Notation

| Definitions and Concepts | Examples |
| --- | --- |
| **Natural-number exponents:** $x^n = \overbrace{x \cdot x \cdot x \cdot \cdots \cdot x}^{n \text{ factors of } x}$ | $x^5 = x \cdot x \cdot x \cdot x \cdot x$ |
| **Rules of exponents:** If there are no divisions by 0, • $x^m x^n = x^{m+n}$ • $(x^m)^n = x^{mn}$ • $(xy)^n = x^n y^n$ • $\left(\frac{x}{y}\right)^n = \frac{x^n}{y^n}$ • $x^0 = 1 \ (x \neq 0)$ • $x^{-n} = \frac{1}{x^n}$ • $\frac{x^m}{x^n} = x^{m-n}$ • $\left(\frac{x}{y}\right)^{-n} = \left(\frac{y}{x}\right)^n$ | $x^2x^5 = x^{2+5} = x^7$ $\quad (x^2)^7 = x^{2 \cdot 7} = x^{14}$ <br> $(xy)^5 = x^5y^5$ $\quad \left(\frac{x}{y}\right)^5 = \frac{x^5}{y^5}$ <br> $6^0 = 1$ $\quad 6^{-2} = \frac{1}{6^2} = \frac{1}{36}$ <br> $\frac{x^6}{x^4} = x^{6-4} = x^2$ $\quad \left(\frac{x}{6}\right)^{-3} = \left(\frac{6}{x}\right)^3 = \frac{6^3}{x^3} = \frac{216}{x^3}$ |
| **Scientific notation:** A number is written in **scientific notation** when it is written in the form $N \times 10^n$, where $1 \leq \lvert N \rvert < 10$. | Write each number in scientific notation. $386{,}000 = 3.86 \times 10^5$ $\quad 0.0025 = 2.5 \times 10^{-3}$ <br> Write each number in standard form. $7.3 \times 10^3 = 7{,}300$ $\quad 5.25 \times 10^{-4} = 0.000525$ |

## EXERCISES

*Write each expression without using exponents.*

**30.** $-5a^3$

**31.** $(-5a)^2$

*Write each expression using exponents.*

**32.** $3ttt$

**33.** $(-2b)(3b)$

*Simplify each expression.*

**34.** $n^2n^4$

**35.** $(p^3)^2$

**36.** $(x^3y^2)^4$

**37.** $\left(\frac{a^4}{b^2}\right)^3$

**38.** $(m^{-3}n^0)^2$

**39.** $\left(\frac{p^{-2}q^2}{2}\right)^3$

**40.** $\frac{a^5}{a^8}$

**41.** $\left(\frac{a^2}{b^3}\right)^{-2}$

**42.** $\left(\frac{3x^2y^{-2}}{x^2y^2}\right)^{-2}$

**43.** $\left(\frac{a^{-3}b^2}{ab^{-3}}\right)^{-2}$

**44.** $\left(\frac{-3x^3y}{xy^3}\right)^{-2}$

**45.** $\left(-\frac{2m^{-2}n^0}{4m^2n^{-1}}\right)^{-3}$

**46.** If $x = -3$ and $y = 3$, evaluate $-x^2 - xy^2$

*Write each number in scientific notation.*

**47.** 6,750

**48.** 0.00023

*Write each number in standard form.*

**49.** $4.8 \times 10^2$

**50.** $0.25 \times 10^{-3}$

**51.** Use scientific notation to simplify $\frac{(45{,}000)(350{,}000)}{0.000105}$.

## SECTION 0.3 Rational Exponents and Radicals

| Definitions and Concepts | Examples |
|---|---|
| **Summary of $a^{1/n}$ definitions:**<br>• If $a \geq 0$, then $a^{1/n}$ is the nonnegative number $b$ such that $b^n = a$.<br>• If $a < 0$ and $n$ is odd, then $a^{1/n}$ is the real number $b$ such that $b^n = a$.<br>• If $a < 0$ and $n$ is even, then $a^{1/n}$ is not a real number. | $16^{1/2} = 4$ because $4^2 = 16$<br>$(-27)^{1/3} = -3$ because $(-3)^3 = -27$<br>$(-16)^{1/2}$ is not a real number because no real number squared is $-16$. |
| **Rule for rational exponents:**<br>If $m$ and $n$ are positive integers, $\frac{m}{n}$ is in lowest terms, and $a^{1/n}$ is a real number, then<br>$a^{m/n} = (a^{1/n})^m = (a^m)^{1/n}$ | $8^{2/3} = (8^{1/3})^2 = 2^2 = 4$ or $(8^2)^{1/3} = 64^{1/3} = 4$ |
| **Definition of $\sqrt[n]{a}$:**<br>$\sqrt[n]{a} = a^{1/n}$ | $\sqrt[3]{125} = 125^{1/3} = 5$ |
| **Properties of radicals:**<br>If all radicals are real numbers and there are no divisions by 0, then<br>• $\sqrt[n]{ab} = \sqrt[n]{a}\sqrt[n]{b}$<br>• $\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}$<br>• $\sqrt[m]{\sqrt[n]{a}} = \sqrt[n]{\sqrt[m]{a}} = \sqrt[mn]{a}$ | $\sqrt[5]{32x^{10}} = \sqrt[5]{32}\sqrt[5]{x^{10}} = 2x^2$<br>$\sqrt[4]{\frac{x^{12}}{625}} = \frac{\sqrt[4]{x^{12}}}{\sqrt[4]{625}} = \frac{x^{12/4}}{5} = \frac{x^3}{5}$<br>$\sqrt[3]{\sqrt{64}} = \sqrt[3]{8} = 2$ $\sqrt{\sqrt[3]{64}} = \sqrt{4} = 2$ $\sqrt[2\cdot 3]{64} = \sqrt[6]{64} = 2$ |

## EXERCISES

*Simplify each expression, if possible.*

**52.** $121^{1/2}$

**53.** $\left(\frac{27}{125}\right)^{1/3}$

**54.** $(32x^5)^{1/5}$

**55.** $(81a^4)^{1/4}$

**56.** $(-1{,}000x^6)^{1/3}$

**57.** $(-25x^2)^{1/2}$

**58.** $(x^{12}y^2)^{1/2}$

**59.** $\left(\frac{x^{12}}{y^4}\right)^{-1/2}$

**60.** $\left(\frac{-c^{2/3}c^{5/3}}{c^{-2/3}}\right)^{1/3}$

**61.** $\left(\frac{a^{-1/4}a^{3/4}}{a^{9/2}}\right)^{-1/2}$

*Simplify each expression.*

**62.** $64^{2/3}$

**63.** $32^{-3/5}$

**64.** $\left(\frac{16}{81}\right)^{3/4}$

**65.** $\left(\frac{32}{243}\right)^{2/5}$

**66.** $\left(\frac{8}{27}\right)^{-2/3}$

**67.** $\left(\frac{16}{625}\right)^{-3/4}$

**68.** $(-216x^3)^{2/3}$

**69.** $\frac{p^{a/2}p^{a/3}}{p^{a/6}}$

*Simplify each expression.*

**70.** $\sqrt{36}$

**71.** $-\sqrt{49}$

**72.** $\sqrt{\frac{9}{25}}$

**73.** $\sqrt[3]{\frac{27}{125}}$

**74.** $\sqrt{x^2y^4}$

**75.** $\sqrt[3]{x^3}$

**76.** $\sqrt[4]{\frac{m^8n^4}{p^{16}}}$

**77.** $\sqrt[5]{\frac{a^{15}b^{10}}{c^5}}$

*Simplify and combine terms.*

**78.** $\sqrt{50} + \sqrt{8}$

**79.** $\sqrt{12} + \sqrt{3} - \sqrt{27}$

**80.** $\sqrt[3]{24x^4} - \sqrt[3]{3x^4}$

*Rationalize each denominator.*

**81.** $\frac{\sqrt{7}}{\sqrt{5}}$

**82.** $\frac{8}{\sqrt{8}}$

**83.** $\frac{1}{\sqrt[3]{2}}$

**84.** $\frac{2}{\sqrt[3]{25}}$

***Rationalize each numerator.***

**85.** $\dfrac{\sqrt{2}}{5}$ **86.** $\dfrac{\sqrt{5}}{5}$ **87.** $\dfrac{\sqrt{2x}}{3}$ **88.** $\dfrac{3\sqrt[3]{7x}}{2}$

## SECTION 0.4 Polynomials

| Definitions and Concepts | Examples |
|---|---|
| **Monomial:** A polynomial with one term. | $2, 3x, 4x^2y, -x^3y^2z$ |
| **Binomial:** A polynomial with two terms. | $2t + 3, 3r^2 - 6r, 4m + 5n$ |
| **Trinomial:** A polynomial with three terms. | $3p^2 - 7p + 8, 3m^2 + 2n - p$ |
| The **degree of a monomial** is the sum of the exponents on its variables. | The degree of $4x^2y^3$ is $2 + 3 = 5$ |
| The **degree of a polynomial** is the degree of the term in the polynomial with highest degree. | The degree of the first term of $3p^2q^3 - 6p^3q^4 + 9pq^2$ is $2 + 3 = 5$, the degree of the second term is $3 + 4 = 7$, and the degree of the third term is $1 + 2 = 3$. The degree of the polynomial is the largest of these. It is 7. |
| **Multiplying a monomial times a polynomial:** $a(b + c + d + \cdots) = ab + ac + ad + \cdots$ | $3(x + 2) = 3x + 6$<br>$2x(3x^2 - 2y + 3) = 6x^3 - 4xy + 6x$ |
| **Addition and subtraction of polynomials:** To add or subtract polynomials, remove parentheses and combine like terms. | Add: $(3x^2 + 5x) + (2x^2 - 2x)$.<br>$(3x^2 + 5x) + (2x^2 - 2x) = 3x^2 + 5x + 2x^2 - 2x$<br>$= 3x^2 + 2x^2 + 5x - 2x$<br>$= 5x^2 + 3x$<br>Subtract: $(4a^2 - 5b) - (3a^2 - 7b)$.<br>$(4a^2 - 5b) - (3a^2 - 7b) = 4a^2 - 5b - 3a^2 + 7b$<br>$= 4a^2 - 3a^2 - 5b + 7b$<br>$= a^2 + 2b$ |
| **Special products:**<br>$(x + y)^2 = x^2 + 2xy + y^2$<br>$(x - y)^2 = x^2 - 2xy + y^2$<br>$(x + y)(x - y) = x^2 - y^2$ | $(2m + 3)^2 = 4m^2 + 12m + 9$<br>$(4t - 3s)^2 = 16t^2 - 24ts + 9s^2$<br>$(2m + n)(2m - n) = 4m^2 - 2mn + 2mn - n^2 = 4m^2 - n^2$ |
| **Multiplying a binomial times a binomial:** Use the FOIL method. | $(2a + b)(a - b) = 2a(a) + 2a(-b) + ba + b(-b)$<br>$= 2a^2 - 2ab + ab - b^2$<br>$= 2a^2 - ab - b^2$ |

| Definitions and Concepts | Examples |
| --- | --- |
| The **conjugate** of $a + b$ is $a - b$. To rationalize the denominator of a radical expression, multiply both the numerator and denominator of the rational expression by the conjugate of the denominator. | Rationalize the denominator: $\frac{x}{\sqrt{x}+2}$. $\frac{x}{\sqrt{x}+2} = \frac{x(\sqrt{x}-2)}{(\sqrt{x}+2)(\sqrt{x}-2)}$ Multiply the numerator and denominator by the conjugate of $\sqrt{x}+2$. $= \frac{x\sqrt{x}-2x}{x-4}$ Simplify. |
| **Division of polynomials:** To divide polynomials, use long division. | Divide: $2x + 3\overline{)6x^3 + 7x^2 - x + 3}$. $$\begin{array}{r} 3x^2 - x + 1 \\ 2x+3\overline{)6x^3 + 7x^2 - x + 3} \\ \underline{6x^3 + 9x^2} \quad\quad\quad \\ -2x^2 - x \quad\; \\ \underline{-2x^2 - 3x} \quad\; \\ +2x + 3 \\ \underline{+2x + 3} \\ 0 \end{array}$$ |

## EXERCISES

***Give the degree of each polynomial and tell whether the polynomial is a monomial, a binomial, or a trinomial.***

**89.** $x^3 - 8$

**90.** $8x - 8x^2 - 8$

**91.** $\sqrt{3}x^2$

**92.** $4x^4 - 12x^2 + 1$

***Perform the operations and simplify.***

**93.** $2(x + 3) + 3(x - 4)$

**94.** $3x^2(x - 1) - 2x(x + 3) - x^2(x + 2)$

**95.** $(3x + 2)(3x + 2)$

**96.** $(3x + y)(2x - 3y)$

**97.** $(4a + 2b)(2a - 3b)$

**98.** $(z + 3)(3z^2 + z - 1)$

**99.** $(a^n + 2)(a^n - 1)$

**100.** $\left(\sqrt{2} + x\right)^2$

**101.** $\left(\sqrt{2} + 1\right)\left(\sqrt{3} + 1\right)$

**102.** $\left(\sqrt[3]{3} - 2\right)\left(\sqrt[3]{9} + 2\sqrt[3]{3} + 4\right)$

***Rationalize each denominator.***

**103.** $\frac{2}{\sqrt{3} - 1}$

**104.** $\frac{-2}{\sqrt{3} - \sqrt{2}}$

**105.** $\frac{2x}{\sqrt{x} - 2}$

**106.** $\frac{\sqrt{x} - \sqrt{y}}{\sqrt{x} + \sqrt{y}}$

***Rationalize each numerator.***

**107.** $\frac{\sqrt{x} + 2}{5}$

**108.** $\frac{1 - \sqrt{a}}{a}$

***Perform each division.***

**109.** $\frac{3x^2y^2}{6x^3y}$

**110.** $\frac{4a^2b^3 + 6ab^4}{2b^2}$

**111.** $2x + 3\overline{)2x^3 + 7x^2 + 8x + 3}$

**112.** $x^2 - 1\overline{)x^5 + x^3 - 2x - 3x^2 - 3}$

## SECTION 0.5 Factoring Polynomials

| Definitions and Concepts | Examples |
|---|---|
| **Factoring out a common monomial:** $ab + ac = a(b + c)$ | $3p^3 - 6p^2q + 9p = 3p(p^2 - 2pq + 3)$ |
| **Factoring the difference of two squares:** $x^2 - y^2 = (x + y)(x - y)$ | $4x^2 - 9 = (2x + 3)(2x - 3)$ |
| **Factoring trinomials:** • Trinomial squares $x^2 + 2xy + y^2 = (x + y)^2$ | $9a^2 + 12ab + 4b^2 = (3a + 2b)(3a + 2b) = (3a + 2b)^2$ |
| $x^2 - 2xy + y^2 = (x - y)^2$ | $r^2 - 4rs + 4s^2 = (r - 2s)(r - 2s) = (r - 2s)^2$ |
| • To factor general trinomials use trial and error or grouping. | $6x^2 - 5x - 6 = (2x - 3)(3x + 2)$ |
| **Factoring the sum and difference of two cubes:** $x^3 + y^3 = (x + y)(x^2 - xy + y^2)$ | $r^3 + 8 = (r + 2)(r^2 - 2r + 4)$ |
| $x^3 - y^3 = (x - y)(x^2 + xy + y^2)$ | $27a^3 - 8b^3 = (3a - 2b)(9a^2 + 6ab + 4b^2)$ |

### EXERCISES

***Factor each expression completely, if possible.***

**113.** $3t^3 - 3t$
**114.** $5r^3 - 5$
**115.** $6x^2 + 7x - 24$
**116.** $3a^2 + ax - 3a - x$
**117.** $8x^3 - 125$
**118.** $6x^2 - 20x - 16$
**119.** $x^2 + 6x + 9 - t^2$
**120.** $3x^2 - 1 + 5x$
**121.** $8z^3 + 343$
**122.** $1 + 14b + 49b^2$
**123.** $121z^2 + 4 - 44z$
**124.** $64y^3 - 1{,}000$
**125.** $2xy - 4zx - wy + 2zw$
**126.** $x^8 + x^4 + 1$

## SECTION 0.6 Algebraic Fractions

| Definitions and Concepts | Examples |
|---|---|
| **Properties of fractions:** If there are no divisions by 0, then • $\frac{a}{b} = \frac{c}{d}$ if and only if $ad = bc$. | $\frac{3x}{4} = \frac{6x}{8}$ because $(3x)8$ and $4(6x)$ both equal $24x$. |
| • $\frac{a}{b} = \frac{ax}{bx}$ | $\frac{6x^2}{8x^3} = \frac{3 \cdot 2 \cdot x \cdot x}{2 \cdot 4 \cdot x \cdot x \cdot x} = \frac{3}{4x}$ |
| • $\frac{a}{b} \cdot \frac{c}{d} = \frac{ac}{bd}$ | $\frac{3p}{2q} \cdot \frac{2p}{6q} = \frac{3p \cdot 2p}{2q \cdot 6q} = \frac{3p \cdot 2p}{2q \cdot 3 \cdot 2q} = \frac{p^2}{2q^2}$ |
| • $\frac{a}{b} \div \frac{c}{d} = \frac{ad}{bc}$ | $\frac{2t}{3s} \div \frac{2t}{6s} = \frac{2t}{3s} \cdot \frac{6s}{2t} = \frac{2t \cdot 6s}{3s \cdot 2t} = \frac{2t \cdot 2 \cdot 3s}{3s2t} = 2$ |

| Definitions and Concepts | Examples |
| --- | --- |
| • $\frac{a}{b}+\frac{c}{b}=\frac{a+c}{b}$ • $\frac{a}{b}-\frac{c}{b}=\frac{a-c}{b}$ | $\frac{x}{4}+\frac{y}{4}=\frac{x+y}{4}$ $\frac{3p}{2q}-\frac{p}{2q}=\frac{3p-p}{2q}=\frac{2p}{2q}=\frac{p}{q}$ |
| • $a\cdot 1=a$ • $\frac{a}{1}=a$ • $\frac{a}{a}=1$ | $7\cdot 1=1$ $\frac{7}{1}=7$ $\frac{7}{7}=1$ |
| • $\frac{a}{b}=\frac{-a}{-b}=-\frac{a}{-b}=-\frac{-a}{b}$ | $\frac{7}{2}=\frac{-7}{-2}=-\frac{7}{-2}=-\frac{-7}{2}$ |
| • $-\frac{a}{b}=\frac{a}{-b}=\frac{-a}{b}=-\frac{-a}{-b}$ | $-\frac{7}{2}=\frac{7}{-2}=\frac{-7}{2}=-\frac{-7}{-2}$ |
| **Simplifying rational expressions:** To simplify a rational expression, factor the numerator and denominator, if possible, and divide out factors that are common to the numerator and denominator. | To simplify $\frac{2-x}{2x-4}$, factor $-1$ from the numerator and 2 from the denominator to get $\frac{2-x}{2x-4}=\frac{-1(-2+x)}{2(x-2)}=\frac{-1\cancel{(x-2)}}{2\cancel{(x-2)}}=\frac{-1}{2}=-\frac{1}{2}$ |
| **Adding or subtracting rational expressions:** To add or subtract rational expressions with unlike denominators, find the LCD of the expressions, write each expression with a denominator that is the LCD, add or subtract the expressions, and simplify the result, if possible. | $\frac{2x}{x+2}-\frac{2x}{x-3}=\frac{2x(x-3)}{(x+2)(x-3)}-\frac{2x(x+2)}{(x-3)(x+2)}$ $=\frac{2x(x-3)-2x(x+2)}{(x+2)(x-3)}$ $=\frac{2x^2-6x-2x^2-4x}{(x+2)(x-3)}$ $=\frac{-10x}{(x+2)(x-3)}$ |
| **Complex fractions:** To simplify complex fractions, multiply the numerator and denominator of the complex fraction by the LCD of all the fractions. | $\frac{1-\frac{y}{2}}{\frac{1}{y}+\frac{1}{2}}=\frac{2y\left(1-\frac{y}{2}\right)}{2y\left(\frac{1}{y}+\frac{1}{2}\right)}=\frac{2y-y^2}{2+y}=\frac{y(2-y)}{2+y}$ |

## EXERCISES

***Simplify each rational expression.***

**127.** $\frac{2-x}{x^2-4x+4}$

**128.** $\frac{a^2-9}{a^2-6a+9}$

***Perform each operation and simplify. Assume that no denominators are 0.***

**129.** $\frac{x^2-4x+4}{x+2}\cdot\frac{x^2+5x+6}{x-2}$

**130.** $\frac{2y^2-11y+15}{y^2-6y+8}\cdot\frac{y^2-2y-8}{y^2-y-6}$

**131.** $\frac{2t^2+t-3}{3t^2-7t+4}\div\frac{10t+15}{3t^2-t-4}$

**132.** $\frac{p^2+7p+12}{p^3+8p^2+4p}\div\frac{p^2-9}{p^2}$

**133.** $\frac{x^2+x-6}{x^2-x-6}\cdot\frac{x^2-x-6}{x^2+x-2}\div\frac{x^2-4}{x^2-5x+6}$

**134.** $\left(\frac{2x+6}{x+5}\div\frac{2x^2-2x-4}{x^2-25}\right)\frac{x^2-x-2}{x^2-2x-15}$

**135.** $\frac{2}{x-4}+\frac{3x}{x+5}$

**136.** $\frac{5x}{x-2}-\frac{3x+7}{x+2}+\frac{2x+1}{x+2}$

**137.** $\frac{x}{x-1}+\frac{x}{x-2}+\frac{x}{x-3}$

**138.** $\frac{x}{x+1}-\frac{3x+7}{x+2}+\frac{2x+1}{x+2}$

**139.** $\dfrac{3(x+1)}{x} - \dfrac{5(x^2+3)}{x^2} + \dfrac{x}{x+1}$

**140.** $\dfrac{3x}{x+1} + \dfrac{x^2+4x+3}{x^2+3x+2} - \dfrac{x^2+x-6}{x^2-4}$

***Simplify each complex fraction. Assume that no denominators are 0.***

**141.** $\dfrac{\frac{5x}{2}}{\frac{3x^2}{8}}$

**142.** $\dfrac{\frac{3x}{y}}{\frac{6x}{y^2}}$

**143.** $\dfrac{\frac{1}{x}+\frac{1}{y}}{x-y}$

**144.** $\dfrac{x^{-1}+y^{-1}}{y^{-1}-x^{-1}}$

## CHAPTER TEST

***Consider the set*** $\{-7, -\frac{2}{3}, 0, 1, 3, \sqrt{10}, 4\}$.

**1.** List the numbers in the set that are odd integers.

**2.** List the numbers in the set that are prime numbers.

***Determine which property justifies each statement.***

**3.** $(a+b)+c=(b+a)+c$

**4.** $a(b+c)=ab+ac$

***Graph each interval on a number line.***

**5.** $-4 < x \le 2$

**6.** $(-\infty, -3) \cup [6, \infty)$

***Write each expression without using absolute value symbols.***

**7.** $|-17|$

**8.** $|x-7|$, when $x < 0$

***Find the distance on a number line between points with the following coordinates.***

**9.** $-4$ and $12$

**10.** $-20$ and $-12$

***Simplify each expression. Assume that all variables represent positive numbers, and write all answers without using negative exponents.***

**11.** $x^4x^5x^2$

**12.** $\dfrac{r^2r^3s}{r^4s^2}$

**13.** $\dfrac{(a^{-1}a^2)^{-2}}{a^{-3}}$

**14.** $\left(\dfrac{x^0x^2}{x^{-2}}\right)^6$

***Write each number in scientific notation.***

**15.** 450,000

**16.** 0.000345

***Write each number in standard notation.***

**17.** $3.7 \times 10^3$

**18.** $1.2 \times 10^{-3}$

***Simplify each expression. Assume that all variables represent positive numbers, and write all answers without using negative exponents.***

**19.** $(25a^4)^{1/2}$

**20.** $\left(\dfrac{36}{81}\right)^{3/2}$

**21.** $\left(\dfrac{8t^6}{27s^9}\right)^{-2/3}$

**22.** $\sqrt[3]{27a^6}$

**23.** $\sqrt{12}+\sqrt{27}$

**24.** $2\sqrt[3]{3x^4} - 3x\sqrt[3]{24x}$

**25.** Rationalize the denominator: $\dfrac{x}{\sqrt{x}-2}$.

**26.** Rationalize the numerator: $\dfrac{\sqrt{x}-\sqrt{y}}{\sqrt{x}+\sqrt{y}}$.

***Perform each operation.***

**27.** $(a^2+3)-(2a^2-4)$

**28.** $(3a^3b^2)(-2a^3b^4)$

**29.** $(3x-4)(2x+7)$

**30.** $(a^n+2)(a^n-3)$

**31.** $(x^2+4)(x^2-4)$

**32.** $(x^2-x+2)(2x-3)$

**33.** $x-3\overline{)6x^2+x-23}$

**34.** $2x-1\overline{)2x^3+3x^2-1}$

***Factor each polynomial.***

**35.** $3x+6y$

**36.** $x^2-100$

**37.** $10t^2-19tw+6w^2$

**38.** $3a^3 - 648$

**39.** $x^4 - x^2 - 12$

**40.** $6x^4 + 11x^2 - 10$

***Perform each operation and simplify if possible. Assume that no denominators are 0.***

**41.** $\dfrac{x}{x+2} + \dfrac{2}{x+2}$

**42.** $\dfrac{x}{x+1} - \dfrac{x}{x-1}$

**43.** $\dfrac{x^2+x-20}{x^2-16} \cdot \dfrac{x^2-25}{x-5}$

**44.** $\dfrac{x+2}{x^2+2x+1} \div \dfrac{x^2-4}{x+1}$

***Simplify each complex fraction. Assume that no denominators are 0.***

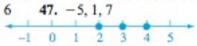
**45.** $\dfrac{\frac{1}{a}+\frac{1}{b}}{\frac{1}{b}}$

**46.** $\dfrac{x^{-1}}{x^{-1}+y^{-1}}$

# Answers to Selected Exercises

## Section 0.1

**1.** set **3.** union **5.** decimal **7.** 2 **9.** composite
**11.** decimals **13.** negative **5.** $x + (y + z)$
**17.** $5m + 5 \cdot 2$ **19.** interval **21.** two **23.** positive
**25.** true **27.** false **29.** true **31.** {a, b, c, d, e, f, g}
**33.** {a, c, e} **35.** terminates **37.** repeats
**39.** 1, 2, 6, 7 **41.** $-5, -4, 0, 1, 2, 6, 7$ **43.** $\sqrt{2}$
**45.** 6 **47.** $-5, 1, 7$

**49.** 


**51.** 

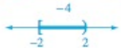

**53.** 


**55.** 

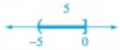

**57.** $(2, \infty)$ 


**59.** $(0, 5)$ 

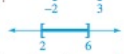

**61.** $(-4, \infty)$ 


**63.** $[-2, 2)$ 


**65.** $(-\infty, 5]$ 


**67.** $(-5, 0]$ 

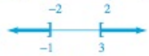

**69.** $[-2, 3]$ 


**71.** $[2, 6]$ 


**73.** $(-5, \infty) \cap (-\infty, 4)$ 


**75.** $[-8, \infty) \cap (-\infty, -3]$ 


**77.** $(-\infty, -2) \cup (2, \infty)$ 

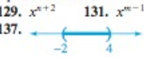

**79.** $(-\infty, -1] \cup [3, \infty)$ 


**81.** 13 **83.** 0 **85.** $-8$ **87.** $-32$ **89.** $5 - \pi$
**91.** 0 **93.** $x + 1$ **95.** $-(x - 4)$ **97.** 5 **99.** 5
**101.** natural numbers **103.** integers

## Section 0.2

**1.** factor **3.** 3, $2x$ **5.** scientific, integer **7.** $x^{m+n}$
**9.** $x^n y^n$ **11.** 1 **13.** 169 **15.** $-25$
**17.** $4 \cdot x \cdot x \cdot x$ **19.** $(-5x)(-5x)(-5x)(-5x)$
**21.** $-8 \cdot x \cdot x \cdot x \cdot x$ **23.** $7x^3$ **25.** $x^2$ **27.** $-27t^3$
**29.** $x^3y^2$ **31.** 10.648 **33.** $-0.0625$ **35.** $x^5$ **37.** $z^6$
**39.** $y^{21}$ **41.** $z^{26}$ **43.** $a^{14}$ **45.** $27x^3$ **47.** $x^6y^3$
**49.** $\frac{a^6}{b^3}$ **51.** 1 **53.** 1 **55.** $\frac{1}{z^4}$ **57.** $\frac{1}{y^5}$ **59.** $x^2$

**61.** $x^4$ **63.** $a^4$ **65.** $x$ **67.** $\frac{m^9}{n^6}$ **69.** $\frac{1}{a^9}$ **71.** $\frac{a^{12}}{b^4}$
**73.** $\frac{1}{r^4}$ **75.** $\frac{x^{32}}{y^{16}}$ **77.** $\frac{9x^{10}}{25y^2}$ **79.** $\frac{4y^{12}}{x^{20}}$ **81.** $\frac{64z^7}{25y^6}$
**83.** $-50$ **85.** 4 **87.** $-8$ **89.** 216 **91.** $-12$
**93.** 20 **95.** $\frac{3}{64}$ **97.** $3.72 \times 10^5$ **99.** $-1.77 \times 10^8$
**101.** $7 \times 10^{-3}$ **103.** $-6.93 \times 10^{-7}$ **105.** $1 \times 10^{12}$
**107.** 937,000 **109.** 0.0000221 **111.** 3.2
**113.** $-0.0032$ **115.** $1.17 \times 10^4$ **117.** $7 \times 10^4$
**119.** $5.3 \times 10^{19}$ **121.** $1.986 \times 10^4$ meters per min
**123.** $1.67248 \times 10^{-15}$ g **125.** $9.3 \times 10^7$ mi, $1.48 \times 10^8$ mi
**127.** polar radius: $6.356750 \times 10^3$ km;
equatorial radius: $6.378135 \times 10^3$ km
**129.** $x^{n+2}$ **131.** $x^{m-1}$ **133.** $x^{m+4}$

**137.** 


**139.** $5 - \pi$

## Section 0.3

**1.** 0 **3.** not **5.** $a^{1/n}$ **7.** $\sqrt[n]{ab}$ **9.** $\neq$ **11.** 3
**13.** $\frac{1}{5}$ **15.** $-3$ **17.** 10 **19.** $-\frac{3}{2}$
**21.** not a real number **23.** $4|a|$ **25.** $2|a|$
**27.** $-2a$ **29.** $-6b^2$ **31.** $\frac{4a^2}{5|b|}$ **33.** $-\frac{10x^2}{3y}$
**35.** 8 **37.** $-64$ **39.** $-100$ **41.** $\frac{1}{8}$ **43.** $\frac{1}{512}$
**45.** $-\frac{1}{27}$ **47.** $\frac{32}{243}$ **49.** $\frac{16}{9}$ **51.** $10s^2$ **53.** $\frac{1}{2y^2z}$
**55.** $x^6y^3$ **57.** $\frac{1}{r^6s^{12}}$ **59.** $\frac{4a^4}{25b^6}$ **61.** $\frac{100s^8}{9r^4}$ **63.** $a$
**65.** 7 **67.** 5 **69.** $-5$ **71.** $-\frac{1}{5}$ **73.** $6|x|$
**75.** $3y^2$ **77.** 2y **79.** $\frac{|x|y^2}{|z^3|}$ **81.** $\sqrt{2}$ **83.** $17x\sqrt{2}$
**85.** $2y^2\sqrt{3y}$ **87.** $12\sqrt[3]{3}$ **89.** $6z\sqrt[4]{3z}$ **91.** $6x\sqrt{2y}$
**93.** 0 **95.** $\sqrt{3}$ **97.** $\frac{2\sqrt{x}}{x}$ **99.** $\sqrt[3]{4}$ **101.** $\sqrt[3]{5a^2}$
**103.** $\frac{2b\sqrt[4]{27a^2}}{3a}$ **105.** $\frac{u\sqrt[3]{6uv^2}}{3v}$ **107.** $\frac{1}{2\sqrt{5}}$ **109.** $\frac{1}{\sqrt[3]{3}}$
**111.** $\frac{b}{32a\sqrt[5]{2b^2}}$ **113.** $\frac{2\sqrt{3}}{9}$ **115.** $-\frac{\sqrt{2x}}{8}$ **117.** $\sqrt{3}$
**119.** $\sqrt[5]{4x^3}$ **121.** $\sqrt[6]{32}$ **123.** $\frac{\sqrt[4]{12}}{2}$ **129.** $(-2, 5]$
**131.** $-5$ **133.** $6.17 \times 10^8$

## Section 0.4

**1.** monomial, variables **3.** trinomial **5.** one **7.** like
**9.** coefficients, variables **11.** yes, trinomial, 2nd degree

**13.** no **15.** yes, binomial, 3rd degree **17.** yes, monomial, 0th degree **19.** yes, monomial, no defined degree
**21.** $6x^3 - 3x^2 - 8x$ **23.** $4y^3 + 14$ **25.** $-x^2 + 14$
**27.** $-28t + 96$ **29.** $-4y^2 + y$ **31.** $2x^2y - 4xy^2$
**33.** $8x^3y^7$ **35.** $\frac{m^4n^4}{2}$ **37.** $-4r^3s - 4rs^3$
**39.** $12a^2b^2c^2 + 18ab^3c^3 - 24a^2b^4c^2$ **41.** $a^2 + 4a + 4$
**43.** $a^2 - 12a + 36$ **45.** $x^2 - 16$ **47.** $x^2 + 2x - 15$
**49.** $3u^2 + 4u - 4$ **51.** $10x^2 + 13x - 3$
**53.** $9a^2 - 12ab + 4b^2$ **55.** $9m^2 - 16n^2$
**57.** $6y^2 - 16xy + 8x^2$ **59.** $9x^3 - x^2y - 27xy + 3y^2$
**61.** $5z^3 - 5tz + 2tz^2 - 2t^2$ **63.** $-3\sqrt{5}x^2 + x + 2\sqrt{5}$
**65.** $27x^3 - 27x^2 + 9x - 1$ **67.** $6x^3 + 14x^2 - 5x - 3$
**69.** $6x^3 - 5x^2y + 6xy^2 + 8y^3$ **71.** $6y^{2n} + 2$
**73.** $-10x^{4n} - 15y^{2n}$ **75.** $x^{2n} - x^n - 12$
**77.** $6r^{2n} - 25r^n + 14$ **79.** $xy + x^{3/2}y^{1/2}$
**81.** $a - b$ **83.** $\sqrt{3} + 1$ **85.** $x\left(\sqrt{7} - 2\right)$
**87.** $\frac{x\left(x + \sqrt{3}\right)}{x^2 - 3}$ **89.** $\frac{\left(y + \sqrt{2}\right)^2}{y^2 - 2}$
**91.** $\frac{\sqrt{3} + 3 - \sqrt{2} - \sqrt{6}}{2}$ **93.** $\frac{x - 2\sqrt{xy} + y}{x - y}$
**95.** $\frac{1}{2\left(\sqrt{2} - 1\right)}$ **97.** $\frac{y^2 - 3}{y^2 + 2y\sqrt{3} + 3}$
**99.** $\frac{1}{\sqrt{x + 3} + \sqrt{x}}$ **101.** $\frac{2a}{b^3}$ **103.** $-\frac{2z^9}{3x^3y^2}$
**105.** $\frac{x}{2y} + \frac{3xy}{2}$ **107.** $\frac{2y^3}{5} - \frac{3y}{5x^3} + \frac{1}{5x^4y^3}$ **109.** $3x + 2$
**111.** $x - 7 + \frac{2}{2x - 5}$ **113.** $2x^2 + 2x + 2 + \frac{3}{x - 1}$
**115.** $x - 3$ **117.** $x^2 - 2 + \frac{-x^2 + 5}{x^3 - 2}$
**119.** $x^4 + 2x^3 + 4x^2 + 8x + 16$ **121.** $6x^2 + x - 12$
**123.** $(x^2 + 3x - 10)$ ft$^2$ **125.** $(4x^3 - 48x^2 + 144x)$ in.$^3$
**133.** 27 **135.** $\frac{125x^3}{8y^6}$ **137.** $-b\sqrt[3]{2ab}$

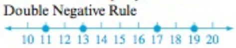
## Section 0.5

**1.** factor **3.** $x(a + b)$ **5.** $(x + y)^2$
**7.** $(x + y)(x^2 - xy + y^2)$ **9.** $3(x - 2)$ **11.** $4x^2(2 + x)$
**13.** $7x^2y^2(1 + 2x)$ **15.** $(x + y)(a + b)$
**17.** $(4a + b)(1 - 3a)$ **19.** $(2x + 3)(2x - 3)$
**21.** $(2 + 3r)(2 - 3r)$ **23.** $(9x^2 + 1)(3x + 1)(3x - 1)$
**25.** $(x + z + 5)(x + z - 5)$ **27.** $(x + 4)^2$
**29.** $(b - 5)^2$ **31.** $(m + 2n)^2$
**33.** $(4x - 3y)(3x + 2y)$ **35.** $(x + 7)(x + 3)$
**37.** $(x - 6)(x + 2)$ **39.** $(2p + 3)(3p - 1)$
**41.** $(t + 7)(t^2 - 7t + 49)$ **43.** $(2z - 3)(4z^2 + 6z + 9)$
**45.** $3abc(a + 2b + 3c)$ **47.** $(x + 1)(3x^2 - 1)$
**49.** $t(y + c)(2x - 3)$ **51.** $(a + b)(x + y + z)$
**53.** $(x + y - z)(x - y + z)$ **55.** $-4xy$
**57.** $(x^2 + y^2)(x + y)(x - y)$ **59.** $3(x + 2)(x - 2)$
**61.** $2x(3y + 2)(3y - 2)$ **63.** prime
**65.** $(6a + 5)(4a - 3)$ **67.** $(3x + 7y)(2x + 5y)$
**69.** $2(6p - 35q)(p + q)$ **71.** $-(6m - 5n)(m - 7n)$
**73.** $-x(6x + 7)(x - 5)$ **75.** $x^2(2x - 7)(3x + 5)$
**77.** $(x^2 + 5)(x^2 - 3)$ **79.** $(a^n - 3)(a^n + 1)$
**81.** $(3x^n - 2)(2x^n - 1)$ **83.** $(2x^n + 3y^n)(2x^n - 3y^n)$
**85.** $(5y^n + 2)(2y^n - 3)$ **87.** $2(x + 10)(x^2 - 10x + 100)$
**89.** $(x + y - 4)(x^2 + 2xy + y^2 + 4x + 4y + 16)$
**91.** $(2a + y)(2a - y)(4a^2 - 2ay + y^2)(4a^2 + 2ay + y^2)$
**93.** $(a - b)(a^2 + ab + b^2 + 1)$
**95.** $(4x^2 + y^2)(16x^4 - 4x^2y^2 + y^4)$
**97.** $(x - 3 + 12y)(x - 3 - 12y)$
**99.** $(a + b - 5)(a + b + 2)$
**101.** $(x + 2)(x^2 - 2x + 4)(x - 1)(x^2 + x + 1)$
**103.** $(x^2 + 1 + x)(x^2 + 1 - x)$
**105.** $(x^2 + x + 4)(x^2 - x + 4)$
**107.** $(2a^2 + a + 1)(2a^2 - a + 1)$
**109.** $\frac{4}{3}\pi(r_1 - r_2)(r_1^2 + r_1r_2 + r_2^2)$ **115.** $2\left(\frac{3}{2}x + 1\right)$
**117.** $2\left(\frac{1}{2}x^2 + x + 2\right)$ **119.** $a\left(1 + \frac{b}{a}\right)$ **121.** $x^{1/2}(x^{1/2} + 1)$
**123.** $\sqrt{2}\left(\sqrt{2}x + y\right)$ **125.** $ab(b^{1/2} - a^{1/2})$
**127.** $(x - 2)(x + 3 + y)$ **129.** $(a + 1)(a^3 + a^2 + 1)$
**131.** 1 **133.** $x^{20}$ **135.** 1 **137.** $-3\sqrt{5x}$

## Section 0.6

**1.** numerator **3.** $ad = bc$ **5.** $\frac{ac}{bd}$ **7.** $\frac{a + c}{b}$
**9.** equal **11.** not equal **13.** $\frac{a}{3b}$ **15.** $\frac{8x}{35a}$
**17.** $\frac{16}{3}$ **19.** $\frac{z}{c}$ **21.** $\frac{2x^2y}{a^2b^3}$ **23.** $\frac{2}{x + 2}$
**25.** $-\frac{x + 2}{x - 3}$ **27.** $\frac{3x - 4}{2x - 1}$ **29.** $\frac{x^2 + 2x + 4}{x + a}$
**31.** $\frac{x(x - 1)}{x + 1}$ **33.** $\frac{x - 1}{x}$ **35.** $\frac{x(x + 1)^2}{x + 2}$
**37.** $\frac{1}{2}$ **39.** $\frac{z - 4}{(z + 2)(z - 2)}$ **41.** 1 **43.** 1
**45.** $\frac{x - 5}{x + 5}$ **47.** $\frac{x + 5}{x + 3}$ **49.** 4 **51.** $\frac{-1}{x - 5}$
**53.** $\frac{5x - 1}{(x + 1)(x - 1)}$ **55.** $\frac{2(a - 2)}{(a + 4)(a - 4)}$
**57.** $\frac{2}{(x + 2)(x - 2)}$ **59.** $\frac{2(x^2 - 3x + 1)}{(x + 1)^2(x - 1)}$
**61.** $\frac{3y - 2}{y - 1}$ **63.** $\frac{1}{x + 2}$ **65.** $\frac{2x - 5}{2x(x - 2)}$
**67.** $\frac{2x^2 + 19x + 1}{(x + 4)(x - 4)}$ **69.** 0
**71.** $\frac{-x^4 + 3x^3 - 43x^2 - 58x + 697}{(x + 5)(x - 5)(x + 4)(x - 4)}$ **73.** $\frac{b}{2c}$
**75.** $81a$ **77.** $-1$ **79.** $\frac{y + x}{x^2y^2}$ **81.** $\frac{y + x}{y - x}$
**83.** $\frac{a^2(3x - 4ab)}{ax + b}$ **85.** $\frac{x - 2}{x + 2}$ **87.** $\frac{3x^2y^2}{xy - 1}$
**89.** $\frac{3x^2}{x^2 + 1}$ **91.** $\frac{x^2 - 3x - 4}{x^2 + 5x - 3}$ **93.** $\frac{x}{x + 1}$
**95.** $\frac{5x + 1}{x - 1}$ **97.** $\frac{k_1k_2}{k_2 + k_1}$ **99.** $\frac{3x}{3 + x}$
**101.** $\frac{x + 1}{2x + 1}$ **107.** 6 **109.** $\frac{y^9}{x^{12}}$ **111.** $-\sqrt{5}$

## Chapter Review

**1.** 3, 6, 8 **2.** 0, 3, 6, 8 **3.** $-6, -3, 0, 3, 6, 8$
**4.** $-6, -3, 0, \frac{1}{2}, 3, 6, 8$ **5.** $\pi, \sqrt{5}$
**6.** $-6, -3, 0, \frac{1}{2}, 3, \pi, \sqrt{5}, 6, 8$ **7.** 3 **8.** 6, 8
**9.** $-6, 0, 6, 8$ **10.** $-3, 3$
**11.** Associative Property of Addition
**12.** Commutative Property of Addition
**13.** Associative Property of Multiplication
**14.** Distributive Property
**15.** Commutative Property of Multiplication
**16.** Commutative Property of Addition
**17.** Double Negative Rule
**18.** 

**19.** 

**20.** 

**21.** 

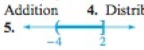
**22.** 

**23.** 

**24.** 

**25.** 6 **26.** 25 **27.** $\sqrt{2} - 1$ **28.** $\sqrt{3} - 1$
**29.** 12 **30.** $-5aaa$ **31.** $(-5a)(-5a)$ **32.** $3r^3$
**33.** $-6b^2$ **34.** $n^6$ **35.** $p^6$ **36.** $x^{12}y^8$
**37.** $\frac{a^{12}}{b^6}$ **38.** $\frac{1}{m^6}$ **39.** $\frac{q^6}{8p^6}$ **40.** $\frac{1}{a^3}$ **41.** $\frac{b^6}{a^4}$
**42.** $\frac{y^8}{9}$ **43.** $\frac{a^8}{b^{10}}$ **44.** $\frac{y^4}{9x^4}$ **45.** $-\frac{8m^{12}}{n^3}$
**46.** 18 **47.** $6.75 \times 10^3$ **48.** $2.3 \times 10^{-4}$
**49.** 480 **50.** 0.00025 **51.** $1.5 \times 10^{14}$ **52.** 11
**53.** $\frac{3}{5}$ **54.** $2x$ **55.** $3|a|$ **56.** $-10x^2$
**57.** not a real number **58.** $x^6|y|$ **59.** $\frac{y^2}{x^6}$
**60.** $-c$ **61.** $a^2$ **62.** 16 **63.** $\frac{1}{8}$ **64.** $\frac{8}{27}$ **65.** $\frac{4}{9}$
**66.** $\frac{9}{4}$ **67.** $\frac{125}{8}$ **68.** $36x^2$ **69.** $p^{2a/3}$ **70.** 6
**71.** $-7$ **72.** $\frac{3}{5}$ **73.** $\frac{3}{5}$ **74.** $|x|y^2$ **75.** $x$
**76.** $\frac{m^2|n|}{p^4}$ **77.** $\frac{a^3b^2}{c}$ **78.** $7\sqrt{2}$ **79.** 0 **80.** $x\sqrt[3]{3x}$
**81.** $\frac{\sqrt{35}}{5}$ **82.** $2\sqrt{2}$ **83.** $\frac{\sqrt[3]{4}}{2}$ **84.** $\frac{2\sqrt[3]{5}}{5}$
**85.** $\frac{2}{5\sqrt{2}}$ **86.** $\frac{1}{\sqrt{5}}$ **87.** $\frac{2x}{3\sqrt{2x}}$ **88.** $\frac{21x}{2\sqrt[3]{49x^2}}$
**89.** 3rd degree, binomial **90.** 2nd degree, trinomial
**91.** 2nd degree, monomial **92.** 4th degree, trinomial
**93.** $5x - 6$ **94.** $2x^3 - 7x^2 - 6x$ **95.** $9x^2 + 12x + 4$
**96.** $6x^2 - 7xy - 3y^2$ **97.** $8a^2 - 8ab - 6b^2$
**98.** $3z^3 + 10z^2 + 2z - 3$ **99.** $a^{2n} + a^n - 2$
**100.** $2 + 2x\sqrt{2} + x^2$ **101.** $\sqrt{6} + \sqrt{2} + \sqrt{3} + 1$
**102.** $-5$ **103.** $\sqrt{3} + 1$ **104.** $-2\left(\sqrt{3} + \sqrt{2}\right)$
**105.** $\frac{2x\left(\sqrt{x} + 2\right)}{x - 4}$ **106.** $\frac{x - 2\sqrt{xy} + y}{x - y}$
**107.** $\frac{x - 4}{5\left(\sqrt{x} - 2\right)}$ **108.** $\frac{1 - a}{a\left(1 + \sqrt{a}\right)}$ **109.** $\frac{y}{2x}$
**110.** $2a^2b + 3ab^2$ **111.** $x^2 + 2x + 1$
**112.** $x^3 + 2x - 3 - \frac{6}{x^2 - 1}$ **113.** $3t(t + 1)(t - 1)$
**114.** $5(r - 1)(r^2 + r + 1)$ **115.** $(3x + 8)(2x - 3)$
**116.** $(3a + x)(a - 1)$ **117.** $(2x - 5)(4x^2 + 10x + 25)$
**118.** $2(3x + 2)(x - 4)$ **119.** $(x + 3 + t)(x + 3 - t)$
**120.** prime **121.** $(2z + 7)(4z^2 - 14z + 49)$
**122.** $(7b + 1)^2$ **123.** $(11z - 2)^2$
**124.** $8(2y - 5)(4y^2 + 10y + 25)$ **125.** $(y - 2z)(2x - w)$
**126.** $(x^2 + 1 + x)(x^2 + 1 - x)(x^4 + 1 - x^2)$
**127.** $\frac{-1}{x - 2}$ **128.** $\frac{a + 3}{a - 3}$ **129.** $(x - 2)(x + 3)$
**130.** $\frac{2y - 5}{y - 2}$ **131.** $\frac{t + 1}{5}$ **132.** $\frac{p(p + 4)}{(p^2 + 8p + 4)(p - 3)}$
**133.** $\frac{(x - 2)(x + 3)(x - 3)}{(x - 1)(x + 2)^2}$ **134.** 1
**135.** $\frac{3x^2 - 10x + 10}{(x - 4)(x + 5)}$ **136.** $\frac{2(2x^2 + 3x + 6)}{(x + 2)(x - 2)}$
**137.** $\frac{3x^3 - 12x^2 + 11x}{(x - 1)(x - 2)(x - 3)}$ **138.** $\frac{-5x - 6}{(x + 1)(x + 2)}$
**139.** $\frac{-x^3 + x^2 - 12x - 15}{x^2(x + 1)}$ **140.** $\frac{3x}{x + 1}$
**141.** $\frac{20}{3x}$ **142.** $\frac{y}{2}$ **143.** $\frac{y + x}{xy(x - y)}$ **144.** $\frac{y + x}{x - y}$

## Chapter Test

**1.** $-7, 1, 3$ **2.** 3 **3.** Commutative Property of Addition **4.** Distributive Property
**5.** 

**6.** 

**7.** 17 **8.** $-(x - 7)$ **9.** 16 **10.** 8 **11.** $x^{11}$
**12.** $\frac{r}{s}$ **13.** $a$ **14.** $x^{24}$ **15.** $4.5 \times 10^5$
**16.** $3.45 \times 10^{-4}$ **17.** 3,700 **18.** 0.0012
**19.** $5a^2$ **20.** $\frac{216}{729}$ **21.** $\frac{9s^6}{4t^4}$ **22.** $3a^2$
**23.** $5\sqrt{3}$ **24.** $-4x\sqrt[3]{3x}$ **25.** $\frac{x\left(\sqrt{x} + 2\right)}{x - 4}$
**26.** $\frac{x - y}{x + 2\sqrt{xy} + y}$ **27.** $-a^2 + 7$ **28.** $-6a^6b^6$
**29.** $6x^2 + 13x - 28$ **30.** $a^{2n} - a^n - 6$ **31.** $x^4 - 16$
**32.** $2x^3 - 5x^2 + 7x - 6$ **33.** $6x + 19 + \frac{34}{x - 3}$
**34.** $x^2 + 2x + 1$ **35.** $3(x + 2y)$ **36.** $(x + 10)(x - 10)$
**37.** $(5t - 2w)(2t - 3w)$ **38.** $3(a - 6)(a^2 + 6a + 36)$
**39.** $(x + 2)(x - 2)(x^2 + 3)$ **40.** $(3x^2 - 2)(2x^2 + 5)$
**41.** 1 **42.** $\frac{-2x}{(x + 1)(x - 1)}$ **43.** $\frac{(x + 5)^2}{x + 4}$
**44.** $\frac{1}{(x + 1)(x - 2)}$ **45.** $\frac{b + a}{a}$ **46.** $\frac{y}{y + x}$