Mini Lecture 1.1

Introduction to Algebra: Variables and Mathematical Models

Learning Objectives:

1. Evaluate algebraic expressions.
2. Translate English phrases into algebraic expressions.
3. Determine whether a number is a solution of an equation.
4. Translate English sentences into algebraic equations.
5. Evaluate formulas.

Examples:

1. Evaluate each expression for $x = 5$.

   a. $4(x-3)$ b. $\dfrac{6x-15}{3x}$

2. Evaluate each expression for $x = 3$ and $y = 6$.

   a. $5(x+y)$ b. $\dfrac{2x+3y}{2y}$

3. Write each English phrase as an algebraic expression. Let x represent the number.
   a. the difference of a number and six
   b. eight more than four times a number
   c. four less than the quotient of a number and twelve

4. Determine whether the given number is a solution of the equation.
   a. $x - 8 = 12$; 20 b. $4x - 7 = 9$; 3 c. $3(y - 5) = 6$; 7

5. Write each English sentence as an equation. Let x represent the number.
   a. The product of a number and seven is twenty-one.
   b. The difference of twice a number and three is equal to twenty-seven.
   c. Six less than three times a number is the same as the number increased by twelve.

Teaching Notes:

- It may be helpful to draw students' attention to the word "evaluate". Help them see the letters v – a – l – u . This will help them remember that evaluate means to find the value of an expression.
- Students often make mistakes with the phrase "less than" so they should be cautioned about the order of the subtraction.
- Translating from English to algebra is an important skill that will be used often .

Answers: 1a. 8 b. 1 2a. 45 b. 2 3a. $x-6$ b. $4x+8$ c. $\dfrac{x}{12}-4$ 4a. yes b. not a solution c. yes 5a. $7x = 21$ b. $2x - 3 = 27$ c. $3x - 6 = x + 12$

Mini Lecture 1.2

Fractions in Algebra

Learning Objectives:

1. Convert between mixed numbers and improper fractions.
2. Write the prime factorization of a composite number.
3. Reduce or simplify fractions.
4. Multiply fractions.
5. Divide fractions.
6. Add and subtract fractions with identical denominators.
7. Add and subtract fractions with unlike denominators.
8. Solve problems involving fractions in algebra.

Examples:

1. Convert each mixed number to an improper fraction.

a. $3\frac{7}{10}$ b. $8\frac{3}{7}$ c. $5\frac{2}{3}$ d. $9\frac{1}{4}$

2. Convert each improper fraction to a mixed number.

a. $\frac{13}{8}$ b. $\frac{12}{11}$ c. $\frac{25}{3}$ d. $\frac{37}{7}$

3. Give the prime factorization of each of the following composite numbers.

a. 24 b. 48 c. 90 d. 108

4. What makes a number a prime?

5. Reduce the following fractions to lowest terms by factoring each numerator and denominator and dividing out common factors.

a. $\frac{10}{12}$ b. $\frac{32}{48}$ c. $\frac{24}{50}$ d. $\frac{77}{98}$

6. Perform the indicated operation. Always reduce answer, if possible.

a. $\frac{3}{4} + \frac{1}{6}$ b. $8\frac{1}{8} + 3\frac{1}{3}$ c. $\frac{7}{10} - \frac{3}{8}$

d. $10\frac{11}{12} - 4\frac{1}{4}$ e. $\left(\frac{7}{9}\right)\left(\frac{18}{19}\right)$ f. $\left(6\frac{2}{3}\right)\left(2\frac{1}{4}\right)$

g. $\frac{7}{8} \div \frac{3}{4}$ h. $5\frac{3}{8} \div 2\frac{1}{4}$

Teaching Notes:

- When teaching factorization, it is often helpful to review divisibility rules.
- To add or subtract fractions, you must have a LCD.
- To divide fractions, multiply by the reciprocal of the divisor.
- To multiply or divide mixed numbers, change to improper fractions first.

Answers: 1. a. 37/10 b. 59/7 c. 17/3 d. 37/4 2. a. 1 5/8 b. 1 1/11 c. 8 1/3 d. 5 2/7
3. a. $2 \cdot 2 \cdot 2 \cdot 3$ b. $2 \cdot 2 \cdot 2 \cdot 2 \cdot 3$ c. $2 \cdot 3 \cdot 3 \cdot 5$ d. $2 \cdot 2 \cdot 3 \cdot 3 \cdot 3$
4. a number whose only factors are 1 and itself 5. a. 5/6 b. 2/3 c. 12/25 d. 11/14 6. a. 11/12
b. $\frac{275}{24}$ or 11 11/24 c. 13/40 d. $\frac{20}{3}$ or 6 2/3 e. 14/19 f. 15 g. 7/6 or 1 1/6 h. 43/18 or 2 7/18

Mini Lecture 1.3

The Real Numbers

Learning Objectives:

1. Define the sets of numbers that make up the set of real numbers.
2. Graph numbers on a number line.
3. Express rational numbers as decimals.
4. Classify numbers as belonging to one or more sets of real numbers.
5. Understand and use inequality symbols.
6. Find the absolute value of a real number.

Examples:

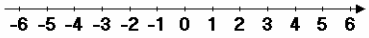
1. Answer the following questions about each number:

   Is it a natural number? Is it rational?
   Is it a whole number? Is it irrational?
   Is it an integer? Is it a real number?

   a. 18 b. –3.5 c. $\sqrt{5}$ d. 0 e. $-\frac{3}{4}$ f. π g. –5 h. 0.45

2. Graph each number on the number line.

   a. 5.5 b. $-\frac{16}{4}$ c. $2\frac{1}{4}$ d. –3.2



3. Express each rational number as a decimal.

   a. $\frac{7}{8}$ b. $\frac{9}{11}$ c. $\frac{5}{3}$ d. $\frac{1}{4}$

4. Use > or < to compare the numbers.

   a. 18 □ –20 b. –16 □ –13 c. –4.3 □ –6.2

   d. $\frac{4}{7}$ □ $\frac{8}{11}$ e. $-\frac{3}{5}$ □ $\frac{2}{3}$

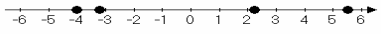
5. Give the absolute value.

   a. $|8|$ b. $|-5|$ c. $|-3.2|$ d. $|22|$

Teaching Notes:

- Make sure the students have minimal understanding of square roots.
- Absolute value is ALWAYS POSITIVE because it measures distance from zero.
- A number cannot be rational and irrational.
- To change a rational number to a decimal, divide the numerator by the denominator.

Answers: 1. a. natural, whole, integer, rational , real b. rational, real c. irrational, real d. whole, integer, rational, real e. rational, real f. irrational, real g., integer, rational, real h. rational, real 2. See below 3. a. 0.875 b. 0.81 c. 0.6 d. 0.25 4. a. > b. < c. > d. < e. < 5. a. 8 b. 5 c. 3.2 d. 22



Mini Lecture 1.4

Basic Rules of Algebra

Learning Objectives:

1. Understand and use the vocabulary of algebraic expressions.
2. Use commutative properties.
3. Use associative properties.
4. Use distributive properties.
5. Combine like terms.
6. Simplify algebraic expressions.

Examples:

1. Fill in the blanks.

Algebraic Expression	# of terms	coefficients	like terms
a. $6y - 3x - 4y + 8$	_____	_____	_____
b. $5x^2 + 2y - 2x^2 + 9 - 3y$	_____	_____	_____
c. $6x^2 - 9y + 4x + 8 - y + 5$	_____	_____	_____

2. Name the property being illustrated and then simplify if possible.
   a. $6(x + 2) = 6x + 12$ _____
   b. $(9 \cdot 12)5 = 9(12 \cdot 5)$ _____
   c. $(x + 4) + 8 = x + (4 + 8)$ _____
   d. $(2)(3.14)(5) = 2(5)(3.14)$ _____

3. Simplify.
   a. $6x - x + 2x =$ _____ b. $3a - 8 + 2a + 10 =$ _____
   c. $6(x + 3) - 5 =$ _____ d. $2(x - 4) - (x - 2) =$ _____
   e. $5(y - 2) + 3(4 - y) =$ _____

Teaching Notes:

- A coefficient is the number factor of a term.
- Like terms have the very same variables raised to the same exponents.
- When applying the commutative property, only the order changes.
- The commutative property holds for addition and multiplication only.
- When applying the associative property the grouping changes.
- The associative property holds for addition and multiplication only.
- When combining like terms, add or subtract the coefficients, the variable part remains the same.
- Always use parentheses when substituting a value for a variable.

Answers: 1 a. 4; 6, –3, –4, 8; $6y$ and $-4y$ b. 5; 5, 2, –2, 9, –3; $5x^2$ and $-2x^2$; $2y$ and $-3y$
c. 6; 6, –9, 4, 8, –1, 5; $9y$ and $-y$; 8 and 5 2. a. distributive b. associative of multiplication
c. associative of addition d. commutative of multiplication 3. a. $7x$ b. $5a + 2$ c. $6x + 13$
d. $x - 6$ e. $2y + 2$

Mini Lecture 1.5

Addition of Real Numbers

Learning Objectives:

1. Add numbers with a number line.
2. Find sums using identity and inverse properties.
3. Add numbers without a number line.
4. Use addition rules to simplify algebraic expressions.
5. Solve applied problems using a series of additions.

Examples:

1. Find the sum using a number line.

   a. $3 + -5$ b. $-4 + -6$ c. $-1 + 2$ d. $5 + 4$

2. Add without using a number line.

   a. $-7 + -11$ b. $-0.4 + -3.2$ c. $-\frac{4}{5} + -\frac{3}{10}$ d. $-15 + 4$

   e. $7.1 + 8.5$ f. $-8 + 25$ g. $-6.4 + 6.1$ h. $\frac{5}{8} + -\frac{3}{4}$

3. Simplify the following.

   a. $-30x + 5x$ b. $-2y + 5x + 8x + 3y$ c. $-2(3x + 5y) + 6(x + 2y)$

4. Write a sum of signed numbers that represents the following situation. Then, add to find the overall change.

   If the stock you purchased last week rose 2 points, then fell 4, rose 1, fell 2, and rose 1, what was the overall change for the week?

Teaching Notes:

- When adding numbers with like signs, add and take the sign.
- When adding unlike signs, subtract the smaller absolute value from the larger absolute value, and the answer will have the sign of the number with the larger absolute value.

Answers: 1. a. -2 b. -10 c. 1 d. 9 2. a. -18 b. -3.6 c. $-\frac{11}{10}$ or $-1\frac{1}{10}$ d. -11 e. 15.6 f. 17 g. -0.3 h. $-\frac{1}{8}$ 3. a. $-25x$ b. $13x + y$ c. $2y$

4. $2 + (-4) + 1 + (-2) + 1 = -2$; fell 2 points

Mini Lecture 1.6

Subtraction of Real Numbers

Learning Objectives:

1. Subtract real numbers.
2. Simplify a series of additions and subtractions.
3. Use the definition of subtraction to identify terms.
4. Use the subtraction definition to simplify algebraic expressions.
5. Solve problems involving subtraction.

Examples:

1. Subtract by changing each subtraction to addition of the opposite first.

   a. $6 - 12$ b. $-15 - 15$ c. $13 - 21$ d. $\frac{2}{5} - \frac{5}{6}$

   e. $4.2 - 6.8$ f. $25 - (-25)$ g. $-51 - (-13)$ h. $14 - (-13)$

2. Simplify.

   a. $-16 - 14 - (-10)$ b. $-20.3 - (-40.1) - 18$

   c. $15 - (-3) - 10 - 18$ d. $-11 - 21 - 31 - 41$

3. Identify the number of terms in each expression; then name the terms.

   a. $4x - 6y + 12 - 3y$ b. $16 - 2x - 15$

   c. $15a - 2ab + 3b - 6a + 18$ d. $5y - x + 3y - 14xy$

4. Simplify each algebraic expression.

   a. $8x + 7 - x$ b. $-11y - 14 + 2y - 10$

   c. $15a - 10 - 12a + 12$ d. $25 - (-3x) - 15 - (-2x)$

5. Applications.

   a. The temperature at dawn was –7 degrees but fortunately the sun came out and by 4:00 p.m. the temperature had reached 38 degrees. What was the difference in the temperature at dawn and 4:00 p.m.?

   b. Express 214 feet below sea level as a negative integer. Express 10,510 above sea level as a positive integer. What is the difference between the two elevations?

Teaching Notes:

- Say the problem to yourself. When you hear the word "minus", immediately make a "change-change". That means to "change" the subtraction to addition and "change" the sign of the number that follows to its opposite.
- Remember, the sign in front of a term goes with the term.
- The symbol "–"can have different meanings:
  1. subtract or "minus" only when it is between 2 terms
  2. the opposite of
  3. negative

Answers: 1. a. –6 b. –30 c. –8 d. $-\frac{13}{30}$ e. –2.6 f. 50 g. –38 h. 27 2. a. –20 b. 1.8 c. –10 d. –104 3. a. 4 terms; $4x$,$-6y$, 12, $-3y$ b. 3 terms; 16, $-2x$, –15 c. 5 terms; $15a$, $-2ab$, $3b$, $-6a$, 18 d. 4 terms; $5y$, $-x$, $3y$, $-14xy$ 4. a. $7x + 7$ b. $-9y - 24$ c. $3a + 2$ d. $5x + 10$ 5. a. 45 degrees b. –214 feet. 10,500 feet; 10, 724 feet

Mini Lecture 1.7

Multiplication and Division of Real Numbers

Learning Objectives:

1. Multiply real numbers.
2. Multiply more than two real numbers.
3. Find multiplicative inverses.
4. Use the definition of division.
5. Divide real numbers.
6. Simplify algebraic expressions involving multiplication.
7. Determine whether a number is a solution of an equation.
8. Use mathematical models involving multiplication and division.

Examples:

1. Multiply.

   a. $(3)(-4)$ b. $(-6)(-5)$ c. $(-8)(0)$ d. $(-3.2)(-1.1)$ e. $\left(-\frac{3}{4}\right)\left(\frac{2}{9}\right)$

   f. $(-5)(2)(-1)$ g. $(-2)(2)(-3)(-3)$

2. Find the multiplicative inverse of each number.

   a. -8 b. $\frac{2}{5}$ c. -7 d. $\frac{1}{4}$

3. Use the definition of division to find each quotient.

   a. $-49 \div 7$ b. $\frac{-24}{-4}$

4. Divide or state that the expression is undefined.

   a. $\frac{-18}{0}$ b. $-\frac{4}{5} \div \frac{20}{25}$ c. $-32.4 \div 8$ d. $0 \div -8$

5. Simplify.

   a. $-3(2x)$ b. $9x + x$ c. $-12a + 4a$ d. $-(5x - 3)$

   e. $-2(3y + 4)$ f. $2(3x + 4) - (4x - 6)$

Teaching Notes:

- The product of an even number of negative numbers is positive.
- The product of an odd number of negative numbers is negative.
- Any product using zero as a factor will equal zero.
- The quotient of two real numbers with different signs is negative.
- The quotient of two real numbers with same signs is positive.
- Division of a non-zero number by zero is undefined.
- Any non-zero number divided into 0 is 0.

Answers: 1. a. -12 b. 30 c. 0 d. 3.52 e. $-\frac{1}{6}$ f. 10 g. -36 2. a. $-\frac{1}{8}$ b. $\frac{5}{2}$ c. $-\frac{1}{7}$ d. $\frac{4}{1}$
3. a. -7 b. 6 4. a. undefined b. -1 c. -4.05 d. 0 5.a. $-6x$ b. $10x$ c. $-8a$ d. $-5x + 3$
e. $-6y - 8$ f. $2x + 14$

Mini Lecture 1.8

Exponents and Order of Operations

Learning Objectives:

1. Evaluate exponential expressions.
2. Simplify algebraic expressions with exponents.
3. Use order of operation agreement.
4. Evaluate mathematical models.

Examples:

1. Identify the base and the exponent, then evaluate.
   a. 3^4 b. $(-4)^3$ c. -8^2 d. $(-8)^2$

2. Evaluate.
   a. 13^2 b. 2^5 c. $(-3)^3$ d. 5^2

3. Simplify if possible.
   a. $6x^2 - x^2$ b. $5y^3 + 2y - 3y^3$ c. $6a^2 + 2a - 4a^2 - 6a$
   d. $10p^3 - 8p^2$

4. Simplify by using the order of operations.
   a. $30 \div 2 \cdot 3 - 52$ b. $14 - (33 \div 11) + 4$
   c. $(5 + 2)^2$ d. $10 - 7(32 \div 8) + 5 \cdot 3$
   e. $\left(\frac{1}{4}\right) + \left(\frac{1}{3}\right)^2$ f. $15 - 3[8 - (-12 \div 2^2) - 4^2]$
   g. $\frac{16 + 4^2 \div 8}{-2 - (-5)}$ h. $22 + 5(x + 7) - 3x - 10$

5. 5. Evaluate each expression for the given value.
   a. $-a - a^2$ if $a = -3$ b. $-a - a^2$ if $a = 3$ c. $4x^2 - x + 3x$ if $x = -1$

6. Use the formula for perimeter of a rectangle, $P = 2w + 2l$ to find the perimeter of a rectangle if the length is 28 cm and the width is 15 cm.

Teaching Notes:

- If the negative sign is part of the base, it will be inside the parentheses.
- **NEVER** multiply the base and the exponent together.
- The exponent tells how many times to write the base as a factor.
- Always use parentheses when substituting a value for a variable.
- The Order of Operations must be followed on every problem.

Answers: 1. a. 81 b. –64 c. –64 d. 64 2. a. 169 b. 32 c. –27 d. 25 3. a. $5x^2$ b. $2y^3 + 2y$ c. $2a^2 - 4a$ d. $10p^3 - 8p^2$ 4. a. –7 b. 15 c. 49 d. –3 e. $\frac{13}{36}$ f. 30 g. 6 h. 2x + 47 5. a. –6 b. –12 c. 2 6. 86 cm