Mini Lecture 2.1

The Addition Property of Equality

Learning Objectives:

1. Identify linear equations in one variable.
2. Use the addition property of equality to solve equations.
3. Solve applied problems using formulas.

Examples:

1. Identify the linear equations in one variable.
   a. $x + 7 = 10$
   b. $x^2 - 2 = 7$
   c. $\frac{3}{x} = 5$
   d. $|x + 1| = 6$
2. Solve the following equations using the addition property of equality. Be sure to check your proposed solution.
   a. $x + 2 = 17$

   b. $-12 = x - 9$

   c. $x - \frac{1}{2} = 4$

   d. $3x - 2x = 8$

   e. $5x + 1 = 4(x - 2)$

   f. $x + 3.5 = 4.8$

   g. $2x + 5 = x - 2$

   h. $3x + 5 = 2x + 5$

   3. If Sue is 2 years older than John then we will use S to represent Sue's age and J to represent John's age. Use the equation $S = J + 2$ to find John's age if Sue is 41.

Teaching Notes:

- Solving an equation is the process of finding the number (or numbers) that make the equation a true statement. These numbers are called the solutions, or roots, or the equation.
- To apply the addition property of equality, one must add the same number or expression to both sides of the equation.
- Equivalent equations are equations that have the same solution.

Answers: 1. a. linear b. not linear c. not linear d. not linear 2. a. 15 b. -3 c. $4\frac{1}{2}$ or $\frac{9}{2}$ d. 8 e. -9 f. 1.3 g. -7 h. 0 3. 39

Mini Lecture 2.2

The Multiplication Property of Equality

Learning Objectives:

1. Use multiplication property of equality to solve equations.
2. Solve equations in the form $-x = c$.
3. Use addition and multiplication properties to solve equations.

Examples:

1. Multiply both sides of the equation by the reciprocal of the coefficient of the variable to solve for the variable.
   a. $6x = 18$ b. $-2x = -14$ c. $15y = -10$ d. $24 = -3x$

2. Divide both sides of the equation by the coefficient of the variable to solve for the variable.
   a. $6x = 18$ b. $-2x = -14$ c. $15y = -10$ d. $24 = -3x$

Both of the above methods of isolating the variable are effective for solving equations.

3. Solve each equation by multiplying or dividing.
   a. $18y = -108$ b. $\frac{3}{5}x = 12$ c. $124 = \frac{x}{3}$ d. $-7x = -63$

4. Multiply or divide both sides of each equation by –1 to get a positive x.
   a. $-x = -7$ b. $82 = -x$ c. $-a = -\frac{3}{7}$ d. $14 = -x$

5. Solve each equation.
   a. $3x - 5 = 13$ b. $18 - 6x = 14 - 2x$ c. $23 = 2a - 7$
   d. $-6y - 21 = 21$ e. $33 - x = 3x - 11$ f. $\frac{2}{3}x - 6 = 12$

Teaching Notes:

- Remind students that reciprocals always have the same sign.
- When students see $-x$ they must realize the coefficient is –1.

Answers: 1. a. $x = 3$ b. $x = 7$ c. $y = -\frac{2}{3}$ d. $x = -8$ 2. a. $x = 3$ b. $x = 7$ c. $y = -\frac{2}{3}$ d. $x = -8$

3. a. $y = -6$ b. $x = 20$ c. $x = 372$ d. $x = 9$ 4. a. $x = 7$ b. $x = -82$ c. $a = \frac{3}{7}$ d. $x = -14$

5. a. $x = 6$ b. $x = 1$ c. $a = 15$ d. $y = -7$ e. $x = 11$ f. $x = 27$

Mini Lecture 2.3

Solving Linear Equations

Learning Objectives:

1. Solve linear equations.
2. Solve linear equations containing fractions.
3. Identify equations with no solution or infinitely many solutions.
4. Solve applied problems using formulas.

Examples:

1. $3x + 2x + 8 = -7 + x + 11$

2. $6x = 3(x + 9)$

3. $5(2x - 1) - 15 = 3(4x + 2) + 4$

4. $\frac{x}{5} = \frac{2x}{3} + \frac{7}{15}$

5. $2x + 9 = 2(x + 4)$

6. $4(x + 2) + 5 = 5(x + 1) + 8$

7. Use the formula $P = 4s$ to find the length of a side of a square whose perimeter is 32 in.

Teaching Notes:

- Simplify the algebraic expression on each side of the equal sign.
- Collect variable terms on one side of the equal sign and all constant terms on the other side of the equal sign.
- Isolate the variable and solve.
- Check your solution in the original expression.

Answers: 1. –1 2. 9 3. –15 4. –1 5. inconsistent, no solution 6. 0 7. 8 inches

Mini Lecture 2.4

Formulas and Percents

Learning Objectives:

1. Solve a formula for a variable.
2. Express decimals as percents and percents as decimals.
3. Solve applied problems involving percent.

Examples:

1. Solve the formula for the indicated variable by isolating the variable.

   a. $A = \frac{B_1 + B_2}{2}$ for B_1

   b. $P = a + b + c$ for c

   c. $A = \pi r^2 h$ for h

   d. $4p + H = M$ for p

   e $Ax + By = C$ for A

   f. $y = mx + b$ for b

2. Express each percent as a decimal.

   a. 42% b. 4% c. 0.8% d. 56% e. 310% f. $\frac{3}{4}\%$

3. Express each decimal as a percent.

   a. 0.47 b. 0.33 c. 0.05 d. 6.21 e. 110 f. 0.004

4. Translate each question into an equation using the percent formula, $A = PB$, then solve the equation.

   a. What is 15 percent of 60?

   b. 62% of what number is 31?

   c. What percent of 132 is 33?

   d. 60 is what percent of 500?

Teaching Notes:

- Many students have trouble solving formulas for a letter and need to be reminded the same steps are used when solving for a letter in a formula as are used when solving any equation for a variable.
- When changing a decimal to a percent, move the decimal point two places to the right and use the % symbol.
- When changing a percent to a decimal, move the decimal point two places to the left and drop the % symbol.
- When translating English into a mathematical equation, the word "is" translates to equals and the word "of" means multiply.

Answers: 1. a. $B_1 = 2A - B_2$ b. $c = P - a - b$ c. $h = \frac{A}{\pi r^2}$ d. $p = \frac{M - H}{4}$ e. $A = \frac{C - By}{x}$ f. $b = y - mx$ 2. a. 0.42 b. 0.04 c. 0.008 d. 0.56 e. 3.1 f. 0.0075 3. a. 47% b. 33% c. 5% d. 621% e. 11000% f. 0.4% 4. a. $x = 0.15(60)$; 9 b. $0.62x = 31$; 50 c. $x \cdot 132 = 33$; 25% d. $60 = x \cdot 500$; 12%

Mini Lecture 2.5

An Introduction to Problem Solving

Learning Objectives:

1. Translate English phrases into algebraic expressions.
2. Solve algebraic word problems using linear equations.

Examples:

1. Translate each English phrase into an algebraic expression. Let "x" represent the unknown.
   a. Three times a number decreased by 11.
   b. The product of seven and a number increased by 2.
   c. Eight more than a number.

2. Translate each sentence into an algebraic equation and then solve the equation.
   a. Twice a number less five is eleven.
   b. Five times the sum of a number and eight is 30.

3. Identify all unknowns, set up an equation, and then solve.
   a. Bill earns five dollars more per hour than Joe. Together their pay for one hour totals $21. How much does each man earn per hour?
   b. Two consecutive even integers equal 42. Find the integers.

Teaching Notes for solving algebraic equations:

- Make sure to familiarize all students with basic mathematical terms and the proper way to translate to algebraic terms.
- First, read the problem carefully and assign a variable for one of the unknown quantities.
- Write expressions if necessary for any other unknown quantities in terms of same variable.
- Write an equation for the stated problem.
- Solve the equation and answer the question.
- Check the solution in the original stated problem.

Answers: 1. a. $3x - 11$
b. $7x + 2$
c. $x + 8$

2. a. $2x - 5 = 11$
$x = 8$
b. $5(x + 8) = 30$
$x = -2$

3. a. x = Joe
$x + 5$ = Bill
$x + (x + 5) = 21$
x = \$8 (Joe)
$x + 5$ = \$13 (Bill)

b. $x = 1^{st}$ even integer
$x + 2 = 2^{nd}$ even integer
$x + (x + 2) = 42$
$x = 20$
$x + 2 = 22$

Mini Lecture 2.6

Solving Linear Inequalities

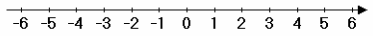
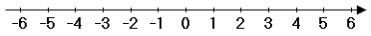
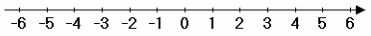
Learning Objectives:

1. Graph inequality solutions on a number line.
2. Use set builder notation and interval notation.
3. Solve linear inequalities in one variable.

Examples:

1. Graph each inequality on the number line.

   a. $x \geq -4$ b. $x < 3$ c. $-1 \leq x < 5$

2. Solve each inequality. Write answers in set builder notation and interval notation.

   a. $4x - 3 \leq 5$ b. $6 - x \geq 3$ c. $6x - 12 < 8x - 14$

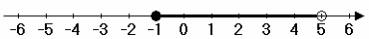
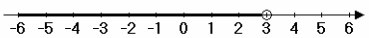
3. Solve each inequality and give the solution in set builder notation: Graph solution on a number line.

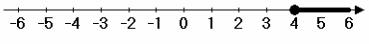
   a. $\frac{1}{5}x > -3$ b. $4(6 - 2x) \geq 12 - 4x$ c. $12x - 3 \geq 4(3x + 2)$

   d. $5(x - 3) \geq 5x - 15$ e. $20 < 3x + 5$ f. $2(x - 5) > 5x + 3$

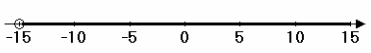
Teaching Notes:

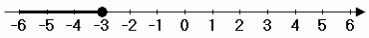
- When graphing the solution of an inequality:
  Use a solid dot when the end point is included in the solution. ($\geq$ or $\leq$)
- When graphing the solution of an inequality:
  Use an open dot when the end point is not included in the solution. ($>$ or $<$)
- When an inequality is multiplied or divided by a negative value, the inequality symbol must be reversed.

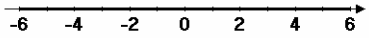
Answers: 1. a.  b. 

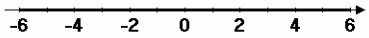
c. 

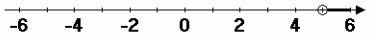
2. a. $\{x \mid x \leq 2\}$ $(-\infty, 2]$ b. $\{x \mid x \leq 3\}$ $(-\infty, 3]$ c. $\{x \mid x > 1\}$ $(1, \infty)$

3. a. $x > -15$ $\{x \mid x > -15\}$ 

b. $x \leq 3$ $\{x \mid x \leq 3\}$ 

c. No Solution { } or Ø 

d. All Real Numbers $\{x \mid x$ is a real number$\}$ 

e. $x > 5$ $\{x \mid x > 5\}$ 

f. $x > -\frac{13}{3}$ $\left\{x \mid x > -\frac{13}{3}\right\}$