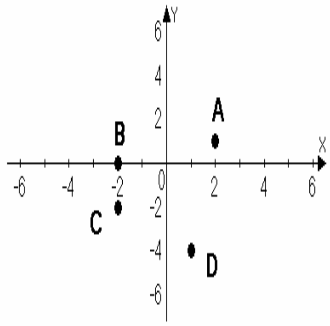
Mini Lecture 4.1
Graphing Equations in Two Variables

Learning Objectives:

1. Plot ordered pairs in the rectangular coordinate system.
2. Find coordinates of points in the rectangular coordinate system.
3. Determine whether an ordered pair is a solution of an equation.
4. Find solutions of an equation in two variables.
5. Use point plotting to graph linear equations.
6. Use graphs of linear equations to solve problems.

Examples:

1. Plot the given points in a rectangular coordinate system. Indicate in which quadrant each point lies.
   a. (–2, –4)
   b. (3, 1)
   c. (–2, 3)
   d. (5, –2)
2. Give the ordered pairs that correspond to the points labeled.



3. Determine if the ordered pair is a solution for the given equation.
   a. $2x + 3y = 10$ (2, 2) b. $3x - y = 5$ (–1, 2)

4. Find five solutions for $y = 2x - 1$ by completing the table of values.
   a.

x	***y* = 2*x* – 1**	**(*x*, *y*)**
–2		
–1		
0		
1		
2		

   b. Plot the ordered pairs to graph the line $y = 2x - 1$.

5. Find five solutions for $y = -x + 1$ by completing the table of values.
   a.

x	***y* = –*x* + 1**	**(*x*, *y*)**
–2		
–1		
0		
1		
2		

   b. Plot the ordered pairs to graph the quadratic equation $y = -x + 1$.

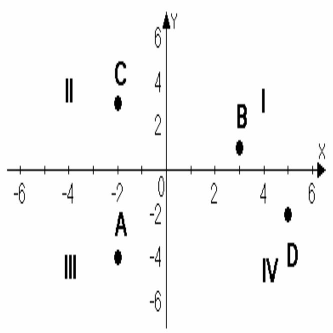
6. a. Your cell phone contract has a base charge of $10 per month and a $.03 per minute charge for nation-wide calling. Create a table of values for $y = .03x + 10$. Use 0, 60, 120, 180, 240 for x.

b. Plot the ordered pairs to graph the above equation.

Teaching Notes:

- The basics for plotting ordered pairs comes from section 1.3.
- When graphing linear equations $(y = mx + b)$ on a coordinate plane, the ordered pairs will form a line when connected.
- Quadratic equations $y = ax^2 + bx + c$ when graphed on a coordinate plane form a parabola.

Answers: 1.

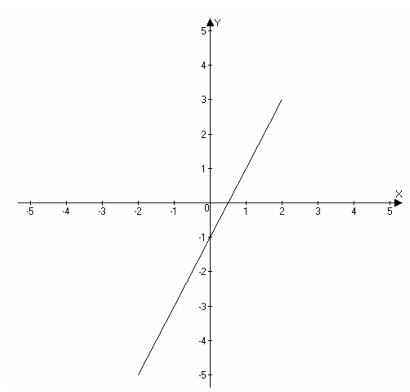


2.a. (2, 1) b. (–2, 0) c. (–1, –2) d. (1, –4)

3.a. yes b. no

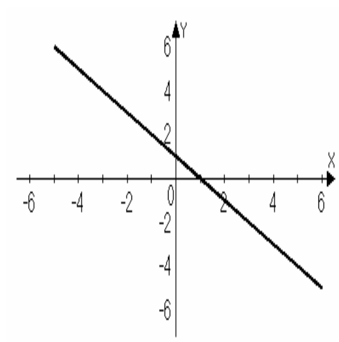
4. a. (–2, –5) (–1, –3) (0, –1) (1, 1) (2, 3)

b.



5. a. (–2, 3) (–1, 2) (0, 1) (1, 0) (2, –1)

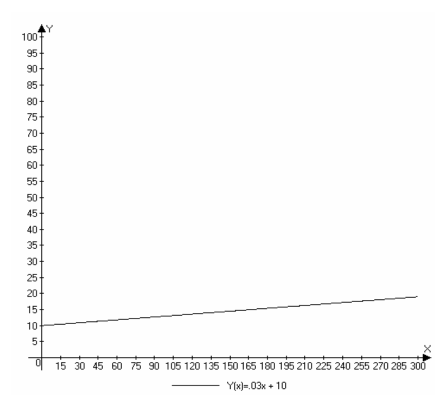
b.



6. a.

x	$y = 0.03x + 10$	(x, y)
0	$y = 0 + 10$	(0, 10)
60	$y = 1.8 + 10$	(60, 11.80)
120	$y = 3.6 + 10$	(120, 13.60)
180	$y = 5.4 + 10$	(180, 15.40)
240	$y = 7.2 + 10$	(240, 17.20)

b.



Mini Lecture 4.2
Graphing Linear Equations Using Intercepts

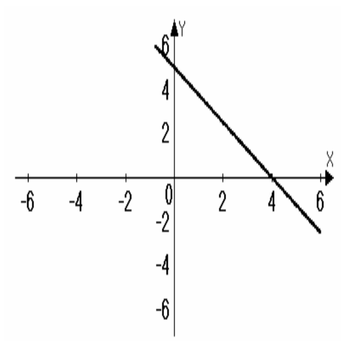
Learning Objectives:

1. Use a graph to identify *x* and *y* intercepts.
2. Graph a linear equation in two variables using intercepts.
3. Graph horizontal and vertical lines.

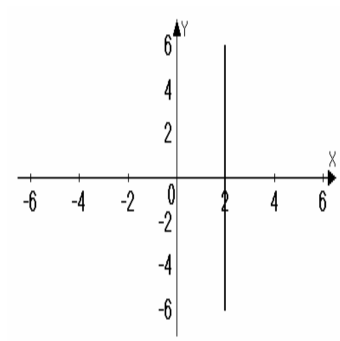
Examples:

1. Identify the *x* and *y*-intercepts of each line.

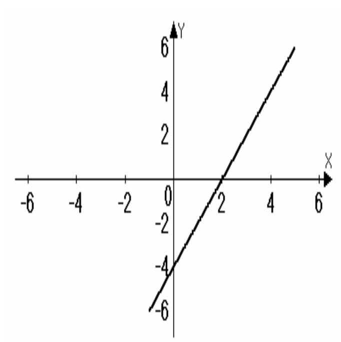
a.



b.



c.



2. Find the *x*-intercept of the graphs of each of the following equations by substituting 0 in for *y* and solving for *x*.
   a. $4x + 7y = 12$ b. $y = 3x - 3$ c. $x - 2y = -8$

3. Find the *y*-intercept of the graphs of each of the following equations by substituting 0 in for the *x* and solving for *y*.
   a. $3x + 2y = -12$ b. $y = 2x + 7$ c. $5x - y = 3$

4. Graph each of the following equations by finding the *x* and *y*-intercepts and a check point. Label the intercepts.
   a. $2x - 4y = 12$ b. $5x + 3y = -15$ c. $y = 2x + 6$

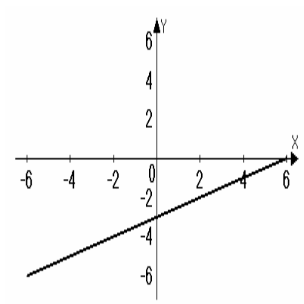
5. Graph each equation on the coordinate plane.
   a. $y + 8 = 12$ b. $x = -3$

Teaching Notes:

- Intercepts are points not just numbers.
- The *x*-intercept is the point where a graph crosses the *x* axis. The value of *y* is always zero at the *x*-intercept.
- The *y*-intercept is the point where a graph crosses the *y* axis. The value of *x* is always zero at the *y*-intercept.
- When an equation is in standard form and *a* and *b* are factors of *c*, then finding intercepts is a good method to choose for graphing.
- A table is often useful to find intercepts.
- A vertical line has no *y*-intercept, unless it is the *y*-axis ($x = 0$).
- A horizontal line has no *x*-intercept, unless it is the *x*-axis ($y = 0$).

Answers: 1. a. *x*-intercept (4,0); *y*-intercept (0,5) b. *x*-intercept (2,0); *y*-intercept (none)
c. *x*-intercept (2,0); *y*-intercept (0, –4) 2. a. (3,0) b. (1,0) c. (–8,0) 3. a. (0, –6) b. (0,7) c. (0, –3)

4. a.



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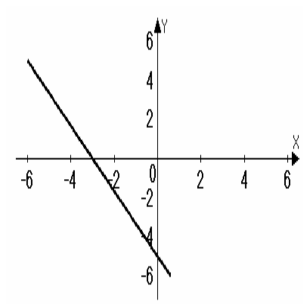

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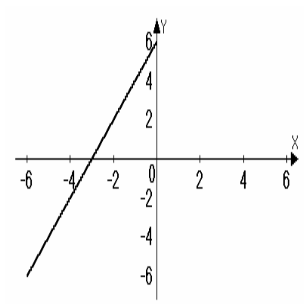
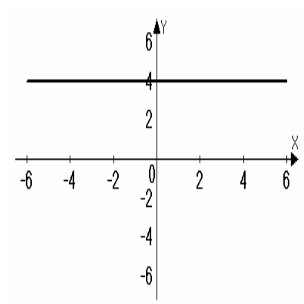


5. a.



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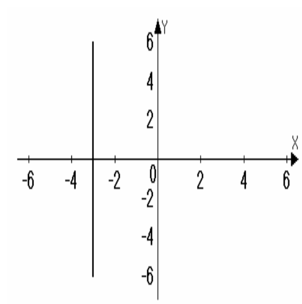


Mini Lecture 4.3
Slope

Learning Objectives:

1. Compute a line's slope.
2. Use slope to show that lines are parallel.
3. Use slope to show that lines are perpendicular.
4. Calculate rate of change in applied situations.

Examples:

1. Using the formula for slope $m = \dfrac{y_2 - y_1}{x_2 - x_1}$, find the slope of the line passing through each pair of points.
   a. (2, 4) (–3, 1)

   b. (–4, 2) (3, –1)

   c. (1, 5) (2, 5)

   d. (–8, 3) (–8, 1)

2. Determine which lines are parallel.

A (5, 2)	B (3, 4)	Slope of $\overline{AB}$ ________________.
C (3, 1)	D (5, 3)	Slope of $\overline{CD}$ ________________.
E (–3, 5)	F (–1, 3)	Slope of $\overline{EF}$ ________________.

   Line ___________ is parallel to line ____________.

3. Determine which lines are perpendicular.

S (1, 4)	T (5, 6)	Slope of $\overline{ST}$ ________________.
U (3, 2)	V (1, 1)	Slope of $\overline{UV}$ ________________.
W (1, –5)	X (0, –3)	Slope of $\overline{WX}$ ________________.

   Line ___________ is perpendicular to line ___________.

4 Property taxes have continued to increase year after year. Given that in 1980 a home's taxes were $1200 and that same home's taxes were $2300 in 2002. If *x* represents the year and *y* the real estate tax, calculate the slope and explain the meaning of your answer.

Teaching Notes:

- Slope is defined as $\dfrac{rise}{run}\left(\dfrac{horizontal\ change}{vertical\ change}\right)$.
- $m = \dfrac{y_2 - y_1}{x_2 - x_1}$, where *m* represents slope and comes from the French verb "*monter*" meaning to rise or ascend.

- Four slope possibilities:
    1. $m > 0$, positive slope, rises from left to right
    2. $m < 0$, negative slope, falls from left to right
    3. $m = 0$, line is horizontal
    4. m is undefined, line is vertical

Answers: 1.a. $\frac{3}{5}$ b. $-\frac{3}{7}$ c. 0 d. undefined 2. $\overline{AB},\ m = -1$ $\overline{\text{CD}}, m = 1$ $\overline{EF}, m = -1$; $\overline{AB} \parallel \overline{EF}$

3. $\overline{ST},\ m = \frac{1}{2}$ $\overline{UV},\ m = -\frac{1}{2}$ $\overline{WX},\ m = -2$; $\overline{ST} \perp \overline{WX}$

4. slope is $\frac{50}{1}$; taxes went up $50 per year.

Mini Lecture 4.4

The Slope-Intercept Form of the Equation of a Line

Learning Objectives:

1. Identify the slope and *y*-intercept of a line from its equation.
2. Graph lines in slope-intercept form.
3. Change an equation from standard form to slope-intercept form.

Examples:

1. Find the slope and *y*-intercept of each line with the following equations: (Write the *y*-intercept as a point.)

   a. $y = \frac{2}{3}x - 4$ b. $y = -3x + 2$ c. $y = -1$

   d. $y = \frac{1}{2}x$ e. $y = 4x - 5$ f. $y = -\frac{3}{4}x + 8$

2. Put each equation in slope-intercept form by solving for *y*. (Isolate *y*) Then name the slope and *y*-intercept.

   a. $2x + y = -6$ b. $-4x - 3y = 6$ c. $x - 2y = 8$

   d. $5y = 10x + 4$ e. $x + y = 10$ f. $3x - 4y = 7$

3. Graph each equation using the slope and the *y*-intercept.

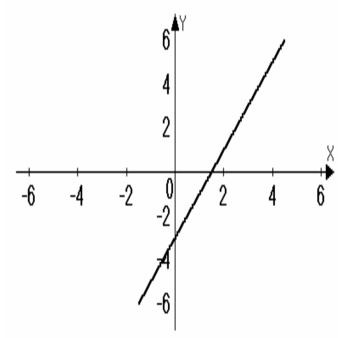
   a. $4x - 2y = 6$ b. $6y = -3x + 12$ c. $3x - y = -3$

Teaching Notes:

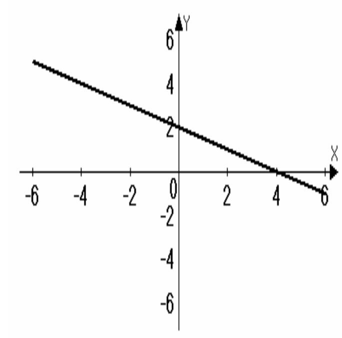
- In an equation in the form $y = mx + b$, m is the slope of line and b is the *y*-coordinate of the *y*-intercept.
- To graph: use the *y*-intercept as the starting point. Then use the slope to plot at least two more points.
- Remember, the slope must be in fraction form, $\frac{rise}{run}$. If the slope is an integer, it can be put over 1 to form a fraction.

Answers: 1. a. slope $\frac{2}{3}$; *y*-intercept (0, –4) b. slope –3; *y*-intercept (0, 2) c. slope 0; *y*-intercept (0, –1)
d. slope $\frac{1}{2}$; *y*-intercept (0, 0) e. slope 4, *y*-intercept (0, –5) f. slope $-\frac{3}{4}$; *y*-intercept (0, 8)
2. a. y = –2x – 6; slope –2; *y*-intercept (0, –6) b. $y = -\frac{4}{3}x - 2$; slope $-\frac{4}{3}$; *y*-intercept (0, –2)
c. $y = \frac{1}{2}x - 4$; slope $\frac{1}{2}$; *y*-intercept (0, –4) d. $y = 2x + \frac{4}{5}$; slope 2; *y*-intercept (0, $\frac{4}{5}$)
e. $y = -x + 10$; slope –1, *y*-intercept (0, 10) f. $y = \frac{3}{4}x - \frac{7}{4}$; slope $\frac{3}{4}$, *y*-intercept $\left(0, -\frac{7}{4}\right)$

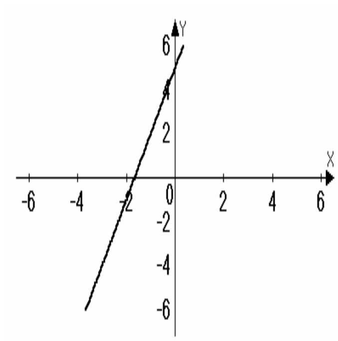
3. a.



b.



c.



Mini Lecture 4.5

The Point-Slope Form of the Equation of a Line

Learning Objectives:

1. Use the point-slope form to write equations of a line.
2. Write linear equations that model data and make predictions.

Examples:

1. Write the point-slope form and the slope-intercept form of the equation of the line with slope 3 that passes through the point (–1, 4).
2. Write the point-slope form and the slope-intercept form of the equation of the line through the points (1, 4) (–2, 3).
3. The cost of graphing calculators over time has decreased. In 1990, one particular brand sold for $110, in 2004 that same calculator sold for $82. Use the coordinates of the two points (1990, 110) (2004, 82) to write the equation in point-slope and slope-intercept form.

Teaching Notes:

- Distinguish between 2 forms of an equation of a line: point-slope (4.3) and slope-intercept (4.4).
- Point-slope equation with the slope of m and passing through point (x_1, y_1) is $y - y_1 = m(x - x_1)$.
- Use the point-slope equation when given at least one point and the slope.
- If given two points and asked to find the equation of the line, one must first find the slope, section 4.3 and then use the slope and one of the given points in the point-slope equation.

Answers: 1. $y - 4 = 3(x + 1)$; $y = 3x + 7$ 2. $y - 4 = \frac{1}{3}(x - 1)$; $y = \frac{1}{3}x + \frac{11}{3}$

3. $y - 110 = -2(x - 1990)$; $y = -2x + 4090$ or $y - 82 = -2(x - 2004)$; $y = -2x + 4090$

Mini Lecture 4.6
Linear Inequalities in Two Variables

Learning Objectives:

1. Determine whether or not an ordered pair is a solution of an inequality.
2. Graph linear inequalities in two variables.

Examples:

Determine which ordered pairs are solutions to the given inequalities.

1. $2x + 3y < 10$
   a. (–1, 4) b. (4, –1) c. (0, 3) d. (3, 2)

2. $y \geq -x + 3$
   a. (4, 7) b. (–3, 0) c. (5, 2) d. (–1, –1)

3. $4x - 2y \leq 8$
   a. (0, 2) b. (2, 0) c. (–2, –2) d. (1, –5)

Put each inequality in slope-intercept form then graph the boundary line and shade the appropriate half plane.

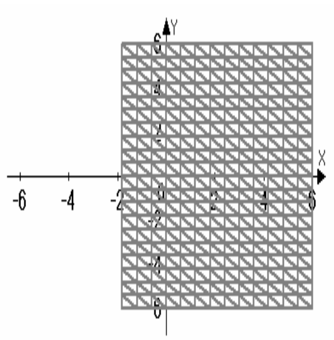
4. a. $2x + 7 \geq 3$ b. $3x - 3y > 6$ c. $y \geq \frac{1}{4}x - 3$
   d. $x < y$ e. $4x + 6y \leq -12$ f. $x \geq 4$

Teaching Notes:

- An inequality has many solutions, therefore shading must be used to show the solutions.
- The graphed line is a boundary and it divides the coordinate plane into two half planes, one of which will be shaded.
- Remind students to use a solid line if the points on the line are included in the solution (≥ or ≤).
- Remind students to use a broken or dashed line if points on the line are not included in the solution (> or <).
- Students tend to forget to reverse the inequality symbol when multiplying or dividing both sides by a negative number.
- Remind students to use a straight edge when graphing lines.

Answers: 1.no; yes; yes; no 2. yes; no; yes; no 3. yes; yes; yes; no

4. a.



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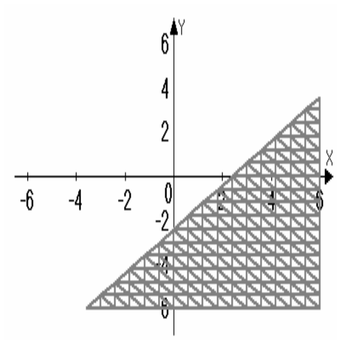
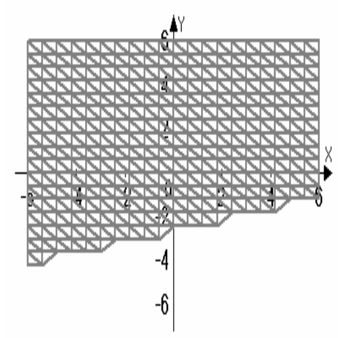


Boundary line dashed

c.



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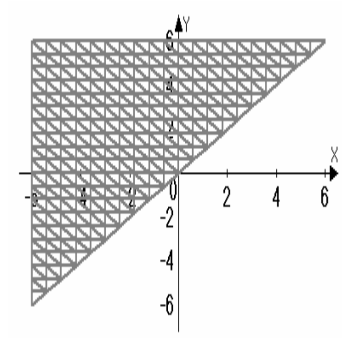
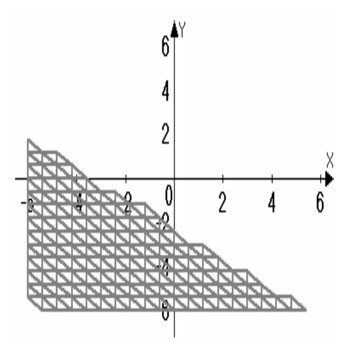


Boundary line dashed

e.



f.