

Mini Lecture 5.1

Solving Systems of Linear Equations by Graphing

Learning Objectives:

1. Decide whether an ordered pair is a solution of a linear system.
2. Solve systems of linear equations by graphing.
3. Use graphing to identify systems with no solution or infinitely many solutions.
4. Use graphs of linear systems to solve problems.

Examples:

1. Consider the system.

$$x + y = -3$$

$$2x + y = 1$$

Determine if each ordered pair is a solution of the system.

a. $(4, 7)$

b. $(4, -7)$

2. Solve the following systems by graphing. State the solution (the intersection point) as an ordered pair (x, y) or state if there is no solution, or state if there are an infinite number of solutions.

a. $2x + y = -3$

$$y = -2x - 3$$

b. $2x + y = 3$

$$3x - 2y = 8$$

c. $x + 2y = 6$

$$x + 2y = 2$$

Teaching Notes:

- When graphing a system of linear equations, there are three possible outcomes:
 1. The two lines can intersect at one point, meaning there is one solution to the system.
 2. The two lines can be parallel to one another, meaning there is no solution to the system.
 3. The two lines are identical or coincide, meaning there are infinitely many solutions to the system.
- When two lines are parallel the system is inconsistent and has no solution.
- When two lines are coinciding, they are called dependent equations and have infinitely many solutions.

Answers:

1. a. not a solution b. yes, a solution 2. a. infinitely many solutions b. $(2, -1)$ c. lines parallel, no solution

Mini Lecture 5.2

Solving Systems of Linear Equations by the Substitution Method

Learning Objectives:

1. Solve linear systems by the substitution method.
2. Use the substitution method to identify systems with no solution or infinitely many solutions.
3. Solve problems using the substitution method.

Examples:

Solve each system using the substitution method. If there is no solution or an infinite number of solutions, so state.

- | | | | |
|---------------------------------------|-----------------------------------|------------------------------------|-----------------------------------|
| 1. a. $x + y = 3$
$y = x + 5$ | b. $3x - 2y = 5$
$x = 4y - 5$ | c. $7x + 6y = -9$
$y = -2x + 1$ | d. $5x - 6y = -4$
$x = y$ |
| 2. a. $x + 3y = 4$
$x - 2y = -1$ | b. $-2x - y = -3$
$3x + y = 0$ | c. $8x - y = 15$
$3x + 4y = 10$ | d. $3x - 5y = 12$
$x + 2y = 4$ |
| 3. a. $3x + 5y = -3$
$x - 5y = -5$ | b. $2x - 4y = -4$
$x + 2y = 8$ | c. $7x - 6y = -1$
$x - 2y = -1$ | d. $2x - y = 1$
$4x + y = 8$ |
| 4. a. $6x + 3y = 1$
$y = -2x - 5$ | b. $4x - 4y = 8$
$x - y = 2$ | c. $4x - 2y = 8$
$2x - y = 4$ | d. $y = -3x + 2$
$6x + 2y = 1$ |

Teaching Notes:

- Students like to follow specific steps so give them a list of steps to use for solving systems by substitution. Begin with: Isolate a variable with a coefficient of 1 first.
- Many students think they must solve for y . Stress that it does not matter whether the variable solved for is x or y .
- Use colored pens or markers to underline in one equation what will be substituted in the other equation.
- If a graphing calculator is being used in the class, graphing on the calculator is a good way to check solutions.

Answers: 1. a. $(-1, 4)$ b. $(3, 2)$ c. $(3, -5)$ D. $(4, 4)$ 2. a. $(1, 1)$ b. $(-3, 9)$ c. $(2, 1)$ d. $(4, 0)$
3. a. $(-2, \frac{3}{5})$ b. $(3, \frac{5}{2})$ c. $(\frac{1}{2}, \frac{3}{4})$ d. $(\frac{3}{2}, 2)$ 4. a. No solution b. Infinite solutions
c. Infinite solutions d. No solution

Mini Lecture 5.3

Solving Systems of Linear Equations by the Addition Method

Learning Objectives:

1. Solving linear systems by the addition method.
2. Use the addition method to identify systems with no solution or infinitely many solutions.
3. Determine the most efficient method for solving a linear system.

Examples:

Solve the following systems by the addition method.

1. $x + y = 10$
 $x - y = 8$

2. $4x + 3y = 7$
 $-4x + y = 5$

3. $3x - y = 8$
 $x + 2y = 5$

4. $2w - 3z = -1$
 $3w + 4z = 24$

5. $4x - 5y = 8$
 $-4x + 5y = -8$

6. $2x = 5y + 4$
 $2x - 5y = 6$

Teaching Notes:

- When solving a system of linear equations there are three methods:
Graphing (5.1)
Substitution (5.2)
Addition (5.3)
- Any of the three methods will work when solving a system and produce the correct answer.
- Teach students how to determine which of the three methods is the most efficient when solving a system of equations.

Answers: 1. (9, 1) 2. $\left(-\frac{1}{2}, 3\right)$ 3. (3, 1) 4. (4, 3) 5. infinitely many solutions 6. no solution

Mini Lecture 5.4

Problem Solving Using Systems of Equations

Learning Objectives:

1. Solve problems using linear systems.

Examples:

Use variables to represent unknown quantities. Write a: Let $x =$ and $y =$ statement for each problem. (Do not solve).

1. The sum of two numbers is 14. One number is six times larger than the other. Find the two numbers.
2. Three pairs of socks and two pairs of mitten cost \$42. One pair of the same kind of socks and four pair of the mittens cost \$24. Find out how much one pair of socks and one pair of mittens cost.
3. John has \$5 bills and \$10 bills in his wallet. He has a total of \$80. He has twice as many \$5 bills as \$10 bills. How many \$5 bills and how many \$10 bills does he have?

Now, for problems 4 – 6, write a system of equations that models the conditions of each problem. (Do not solve).

- 4.
- 5.
- 6.

Solve each of the following using a system of equations.

7. The sum of two numbers is 11. The second number is 1 less than twice the first number. Find the two numbers.
8. Alexis has \$1.65 in nickels and quarters. She has 9 coins altogether. How many coins of each kind does she have?
9. Paul invested \$12,000 in two accounts. One account paid 4% interest and one account paid 5% interest. At the end of the year his money had earned \$560 in interest. How much did he invest in each account?
10. A department store receives 2 shipments of bud vases and picture frames. The first shipment of 5 bud vases and 4 picture frames costs \$62. The second shipment of 10 bud vases and 3 picture frames cost \$84. Find the cost of a vase and a picture frame.

Teaching Notes:

- Stress the importance of reading the problem several times before beginning. Reading aloud really helps.
- Have students write a Let $x =$ and $y =$ statements for each word problem before trying to write the system of equations.
- Help students look at the system they have created and determine which method of solving will work best.
- Remind students to make sure their answers make sense for the given situation.
- Try to build confidence with word problems.

Answers: 1. Let $x =$ one number; let $y =$ the other number. 2. Let $x =$ cost of 1 pair of socks; let $y =$ cost of 1 pair of mittens. 3. Let $x =$ number of \$5 bills; let $y =$ number of \$10 bills

4. $x + y = 14$

5. $3x + 2y = 42$

6. $5x + 10y = 80$

$x = 6y$

$x + 4y = 24$

$x = 2y$

7. The numbers are 4 and 7 8. 3 nickels, 6 quarters 9. \$4000 invested @ 4% and \$8000 invested @ 5%

10. bud vases \$6, picture frames \$8

