

Mini Lecture 6.1
Adding and Subtracting Polynomials

Learning Objectives:

1. Understand the vocabulary used to describe polynomials.
2. Add polynomials.
3. Subtract polynomials.
4. Graph equations defined by polynomials of degree 2.

Examples:

1. Identify each polynomial as a monomial, a binomial, or a trinomial. Give the degree of the polynomial.
 - a. $5x - 1$
 - b. $9x^2$
 - c. 8
 - d. $3x^2 - 2x + 1$
2. Add: $(5x^3 + 3x^2 - 5x + 4) + (-2x^3 + 4x^2 - 8x - 2)$
3. Add:
$$\begin{array}{r} -9x^3 + 4x^2 - 5x + 3 \\ 2x^3 + 3x - 7 \\ \hline \end{array}$$
4. Subtract: $(4x^4 + 3x^3 + 2x - 7) - (-2x^3 + x^2 - 4x + 5)$
5. Subtract: $3x^2 + 5x + 4$ from $8x^2 - 2x - 1$
6. Subtract:
$$\begin{array}{r} 10x^4 + 3x^3 - 4x^2 \\ (+ 2x^3 + 3x^2 + 4x) \\ \hline \end{array}$$
7. Find seven solutions for $y = x^2 + 2$ by completing the table of values.
 - a.

x	$y = x^2 + 2$	y
-3		
-2		
-1		
0		
1		
2		
3		

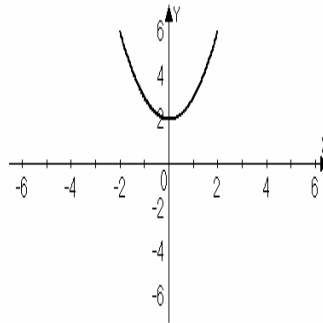
- b. Plot the ordered pairs to graph the quadratic equation $y = x^2 + 2$.

Teaching Notes:

- Vocabulary terms are very important when teaching/learning about polynomials.
Example: monomial, degree of polynomial, coefficient, exponent.
- When adding or subtracting polynomials either the horizontal or vertical format may be used.
- When using the vertical format, if a particular term is missing, leave a vacant space making sure to line up like terms in columns.
- When subtracting, make sure to change the sign of each term being subtracted to its opposite, then use rules for adding like and unlike signs.

Answers: 1. a. binomial, 2 b. monomial, 2 c. monomial, 0 d. trinomial, 2 2. $3x^3 + 7x^2 - 13x + 2$
3. $-7x^3 + 4x^2 - 2x - 4$ 4. $4x^4 + 5x^3 - x^2 + 6x - 12$ 5. $5x^2 - 7x - 5$
6. $10x^4 + x^3 - 7x^2 - 4x + 5$
7.

x	$y = x^2 + 2$	y
-3	$y = (-3)^2 + 2$	
-2	$y = (-2)^2 + 2$	
-1	$y = (-1)^2 + 2$	
0	$y = (0)^2 + 2$	
1	$y = (1)^2 + 2$	
2	$y = (2)^2 + 2$	
3	$y = (3)^2 + 2$	



Mini Lecture 6.2

Multiplying Polynomials

Learning Objectives:

1. Use the product rule for exponents.
2. Use the power to power rule for exponents.
3. Use the products-to-powers rule for exponents.
4. Multiply monomials.
5. Multiply a monomial and a polynomial.
6. Multiply polynomials when neither is a monomial.

Examples:

Simplify each expression.

- | | | | |
|-------------------------------|------------------------------------|-------------------------|---------------------------|
| 1. a. $x^5 \cdot x^2$ | b. $a^2 \cdot a \cdot a^8$ | c. $4^2 \cdot 4^4$ | d. $y \cdot y \cdot y$ |
| 2. a. $(x^4)^2$ | b. $(a^5)^5$ | c. $(4^2)^4$ | d. $(y^5)^2$ |
| 3. a. $(3a)^4$ | b. $(6x^3)^2$ | c. $(2a^2b^3c^5)^3$ | d. $(-6x^3)^4$ |
| 4. $4y^2 \cdot 3y^4$ | b. $(-3x)(-2x^3)$ | c. $(8x^2y^2)(3x^2y^4)$ | d. $(5x^3)(-3x^3y^7)$ |
| 5. a. $5a(a^2 + 2)$ | b. $3x(2x - 4)$ | c. $5y^2(y^2 - y + 2)$ | d. $2x^2y(5x^3y - 2xy^3)$ |
| 6. a. $(a + 2)(a + 8)$ | b. $(2x - 3)(3x - 2)$ | c. $(y + 7)(2y - 3)$ | d. $(2b - 5)(5b - 2)$ |
| 7. a. $(x + 2)(x^2 + 3x + 4)$ | b. $(a + 4)(a^2 - 3a + 1)$ | | |
| c. $(y - 3)(2y^2 + 3y + 5)$ | d. $(x - 2)(2x^3 - 4x^2 + 3x - 6)$ | | |

Teaching Notes:

- Have students write out exponent rules in words on one (1) page with examples for referral and study.
- Students find each rule easy as presented but the rules get jumbled when used together.
- Practice often and review.
- A very common mistake will be students multiplying bases if they are numbers – warn against this! The base stays the same when multiplying like bases.

Answers: 1. a. x^7 b. a^{11} c. 4^6 d. y^3 2. a. x^8 b. a^{25} c. 4^8 d. y^{10} 3. a. $81a^4$ b. $36x^6$ c. $8a^6b^9c^{15}$
d. $1296x^{12}$ 4. a. $12y^6$ b. $6x^4$ c. $24x^4y^6$ d. $-15x^6y^7$ 5. a. $5a^3 + 10a$ b. $6x^2 - 12x$ c. $5y^4 - 5y^3 + 10y^2$
d. $10x^5y^2 - 4x^3y^4$ 6. a. $a^2 + 10a + 16$ b. $6x^2 - 13x + 6$ c. $2y^2 + 11y - 21$ d. $10b^2 - 29b + 10$
7. a. $x^3 + 5x^2 + 10x + 8$ b. $a^3 + a^2 - 11a + 4$ c. $2y^3 - 3y^2 - 4y - 15$
d. $2x^4 - 8x^3 + 11x^2 - 12x + 12$

Mini Lecture 6.4
Polynomials in Several Variables

Learning Objectives:

1. Evaluate polynomials in several variables.
2. Know and understand polynomial vocabulary.
3. Add and subtract polynomials in several variables.
4. Multiply polynomials in several variables.

Examples:

Evaluate the following polynomial given values for the variables:

1. $2x^2 - xy + 3y^2$ a. $x = 2$ $y = -1$ b. $x = -2$ $y = -2$
2. $5x^3 + 2x^2y - 3xy - y^2$ a. $x = 1$ $y = 3$ b. $x = -2$ $y = -1$

Answer the following questions about the following polynomials:

3. $8x^4y^3 - 5xy^2 + 4x^3y - x^2y^6$ 4. $3x^5y - x^4y^3 + 2x^2y^2 + xy^3 + 4y^4$
 - a. Name the coefficient of each term.
 - b. Tell the degree of each term.
 - c. What is the degree of the polynomial?
5. Add or subtract the polynomials.
 - a. $(6x^2y + 2xy - 3xy^2) + (4x^2y - xy + 5xy^2)$
 - b. $(10x^3y^2 - 5x^2y + 2xy^2 - 3xy^3) - (4x^3y^2 - 5x^2y + 4xy^2 - 6xy)$
 - c. $(3a^4b^2 + 2a^3b - 5a^2b^3) + (5a^4b^2 + 3ab^3 + 4a^2b^3)$
 - d. $(5x^4y^3 + 3x^2y^2 + 2xy^4) - (3x^4y^4 - 3x^2y^2 - 4xy^4)$
6. Multiply the polynomials.
 - a. $(4x^2y^2)(7x^3y^3)$
 - b. $(-2xy^3)(5x^3y)$
 - c. $5x^2y^2(3x^3y^2 - 4xy^3 + 6x^2y)$
 - d. $(2x + 3y)(x - 4y)$
 - e. $(4a + 3b)(4a + b)$
 - f. $(3x + 5y)^2$
 - g. $(2x - 8y)^2$
 - h. $(x - y)(2x^2 - 3xy + y^2)$

Teaching Notes:

- It is very important to use parentheses when substituting for a variable.
- Students need to be reminded that the sign in front of a term is the sign of the coefficient of the term.
- Students will have problems with the degree of the polynomial. Remind them it is the same as the one term with the highest degree. DO NOT add all the exponents in the polynomial.

Answers: 1. a. 13 b. 16 2. a. -7 b. -55 3. a. 8, -5, 4, -1 b. 7, 3, 4, 8 c. degree 8
4. a. 3, -1, 2, 1, 4 b. 6, 7, 4, 4, 4 c. degree 7 5. a. $10x^2y + xy + 2xy^2$ b. $6x^3y^2 - 2xy^2 + 3xy^3$
c. $8a^4b^2 + 2a^3b + 3ab^3 - a^2b^3$ d. $2x^4y^3 + 6x^2y^2 + 6xy^4$ 6. a. $28x^5y^5$ b. $-10x^4y^4$
c. $15x^5y^4 - 20x^3y^5 + 30x^4y^3$ d. $2x^2 - 5xy - 12y^2$ e. $16a^2 + 16ab + 3b^2$
f. $9x^2 + 30xy + 25y^2$ g. $4x^2 - 32xy + 64y^2$ h. $2x^3 - 5x^2 + 4xy^2 - y^3$

Mini Lecture 6.5

Dividing Polynomials

Learning Objectives:

1. Use the quotient rule for exponents.
2. Use the zero-exponent rule.
3. Use the quotients-to-powers rule.
4. Divide monomials.
5. Check polynomial division.
6. Divide a polynomial by a monomial.

Examples:

1. Divide using the quotient rule:

a. $\frac{7^5}{7^2}$ b. $\frac{x^{11}}{x^7}$ c. $\frac{y^8}{y}$

2. Use the zero-exponent rule to simplify each expression.

a. 8^0 b. $(-8)^0$ c. -8^0 d. $30x^0$ e. $(30x)^0$

3. Simplify each expression using the quotient-to-powers rule.

a. $\left(\frac{a}{6}\right)^2$ b. $\left(\frac{x^4}{3}\right)^3$ c. $\left(\frac{2y^3}{z^4}\right)^4$

4. Divide the monomials.

a. $\frac{-24x^8}{6x^4}$ b. $\frac{2x^6}{6x^6}$ c. $\frac{12x^6y^4}{2x^3y}$

5. Find the quotient.

a. $\frac{-16x^8 + 12x^5 - 2x^2}{2x}$ b. $\frac{30x^6 - 25x^5 + 10x^3}{5x^3}$ c. $\frac{24x^6y^3 - 6x^4y^2 + 12x^2y}{6xy}$

Teaching Notes:

- When dividing exponential expressions with the same nonzero base, subtract the denominator exponent from the numerator exponent. Ex: $\left(\frac{a^x}{a^y}\right) = a^{x-y}$.
- Any nonzero base raised to the 0 power equals 1.
- When raising quotients to a power, raise every factor in the parenthesis to the power.

The base in the denominator \neq zero $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$.

- When dividing monomials, divide the coefficients and divide the variables. When dividing the variables, keep the variable and subtract the exponents.
- To check a division problem, divisor \cdot quotient = dividend.
- When dividing a polynomial a monomial by a monomial, divide each term by the monomial.

Answers: 1.a. 7^3 b. x^4 c. y^7 2.a. 1 b. 1 c. -1 d. 30 e. 1 3.a. $\frac{a^2}{36}$ b. $\frac{x^{12}}{27}$ c. $\frac{16y^{12}}{z^{16}}$

4.a. $-4x^4$ b. $\frac{1}{3}$ c. $6x^3y^3$ 5.a. $-8x^7 + 6x^4 - x$ b. $6x^3 - 5x^2 + 2$ c. $4x^5y^2 - x^3y + 2x$

Mini Lecture 6.6

Dividing Polynomials by Binomials

Teaching Objectives:

- Divide polynomials by binomials.

Examples:

Divide and check: (no remainders)

1. a. $\frac{x^2 + 5x + 6}{x + 3}$ b. $\frac{2a^2 - 9a - 5}{2a + 1}$ c. $(a^2 + 9a + 20) \div (a + 4)$

Divide and check: (some will have remainders)

2. a. $\frac{x^2 + 5x + 6}{x + 3}$ b. $\frac{x^2 + 5x - 6}{x + 1}$ c. $(6a^2 + 5a + 1) \div (2a + 3)$

d. $\frac{6a^3 - 13a^2 - 4a + 15}{3a - 5}$ e. $(2x^4 - 13x^3 + 16x^2 - 9x + 20) \div (x - 5)$

f. $\frac{7x^2 - 4x + 12 + 3x^3}{x + 1}$ g. $(20x^3 - 8x^2 + 5x - 5) \div (5x - 2)$

Divide. Fill in any missing terms.

3. a. $(3x^4 - 25x^2 - 20) \div (x - 3)$ b. $\frac{y^3 - 8}{y - 2}$ c. $\frac{3x^2 - 4}{x - 1}$

Teaching Notes:

1. Dividing polynomials is like long division. Most students can divide using long division. Remind about writing remainders as fractions.
2. Teach this concept by comparing to long division.
3. Students often find it helpful to cover up the second term of a binomial before dividing.
4. Using parentheses before subtracting may help reduce sign errors.
5. It is often helpful to write these steps vertically on the page when doing examples with the students:
Cover up; divide; uncover; multiply back; parentheses; subtract; bring down, cover up; divide, uncover; multiply back; parentheses; subtract; bring down... Students are able to follow the above pattern if they practice enough. Remind students that the divisor and the dividend must be in DESCENDING powers of the variable WITHOUT skipping any powers.
6. The sign errors when subtracting will be the biggest obstacle in getting the correct answer.
7. Students like to see how they can check their answers.

Answers: 1. a. $x + 2$ b. $a - 5$ c. $a + 5$ 2. a. $x + 2 + \frac{2}{x + 3}$ b. $x + 4 + \frac{-10}{x + 1}$ c. $3a - 2 + \frac{7}{a + 3}$

d. $2a^2 - a - 3$ e. $2x^3 - 3x^2 + x - 4$ f. $3x^2 + 4x - 8 + \frac{20}{x + 1}$ g. $4x^2 + 1 + \frac{-3}{5x - 2}$

3. a. $3x^3 + 9x^2 + 2x + 6 + \frac{-2}{x - 3}$ b. $y^2 + 2y + 4$ c. $3x + 3 + \frac{-1}{x - 1}$

Mini Lecture 6.7
Negative Exponents and Scientific Notation

Learning Objectives:

1. Use the negative exponent rule.
2. Simplify exponential expressions.
3. Convert from scientific notation to decimal notation.
4. Convert from decimal notation to scientific notation.
5. Compute with scientific notation.
6. Solve applied problems with scientific notation.

Examples:

1. Rewrite each expression with a positive exponent, then simplify if possible.
 - a. 3^{-2}
 - b. 4^{-3}
 - c. $(-2)^{-4}$
 - d. -2^{-4}
 - e. 5^{-1}
 - f. $\frac{2^{-2}}{5^{-3}}$
 - g. $\left(\frac{5}{7}\right)^{-2}$
 - h. $\frac{1}{4y^{-3}}$
 - i. $\frac{x^{-3}}{y^{-5}}$
2. Simplify the following, make sure all answers are written with positive exponents.
 - a. $x^{-10} \cdot x^4$
 - b. $\frac{x^2}{x^9}$
 - c. $\frac{24x^3}{8x^7}$
 - d. $\frac{100y^4}{25y^{10}}$
 - e. $\frac{(3x^3)^2}{x^{10}}$
 - f. $\left(\frac{x^6}{x^2}\right)^{-3}$
3. Write each number in scientific notation.
 - a. 3,840,000
 - b. 0.000158
4. Write each number in decimal notation.
 - a. 9.2×10^{-3}
 - b. 3.851×10^5
5. Perform the indicated computation writing final answers in scientific notation.
 - a. $(4 \times 10^4)(2 \times 10^8)$
 - b. $\frac{4.2 \times 10^8}{2 \times 10^{-2}}$
6. If 3.29×10^5 people live in your city, what is the population of your hometown in decimal notation?

Teaching Notes:

- To simplify exponential expressions remember: to remove all parentheses, no powers are raised to powers, each base occurs only once, no negative or zero exponents appear.
- If the base is any real number other than 0 and n is a negative number, then $x^{-n} = \frac{1}{x^n}$.
- A positive number is written in scientific notation when it is expressed in the form $a \times 10^n$ where $1 \leq a < 10$ and n is an integer.
- When changing scientific notation to decimal notation, move the decimal to the right " n " places if n is negative.

Answers: 1. a. $\frac{1}{3^2} = \frac{1}{9}$ b. $\frac{1}{4^3} = \frac{1}{64}$ c. $\frac{1}{(-2)^4} = \frac{1}{16}$ d. $\frac{1}{-2^4} = -\frac{1}{16}$ e. $\frac{1}{5^1} = \frac{1}{5}$

f. $\frac{5^3}{2^2} = \frac{125}{4}$ g. $\left(\frac{7}{5}\right)^2 = \frac{49}{25}$ h. $\frac{y^3}{4}$ i. $\frac{y^5}{x^3}$ 2. a. $\frac{1}{x^6}$ b. $\frac{1}{x^7}$ c. $\frac{3}{x^4}$ d. $\frac{4}{y^6}$ e. $\frac{9}{x^4}$

f. $\frac{1}{x^{12}}$ 3. a. 3.84×10^6 b. 1.58×10^{-4} 4. a. 0.0092 b. 385,100 5. a. (8×10^{12})

b. (2.1×10^{10}) 6. 329,000 people