

## Mini Lecture 7.1

### The Greatest Common Factor and Factoring by Grouping

#### Learning Objectives:

1. Find the greatest common factor.
2. Factor out the greatest common factor of a polynomial.
3. Factor by grouping.

#### Examples:

1. Find the greatest common factor for each list of terms.  
a.  $24x^3$  and  $12x^2$       b.  $-18x^2$ ,  $30x^5$ , and  $48x$       c.  $x^2y^4$ ,  $x^3y$ , and  $x^5y^7$
2. Factor the following:  
a.  $9x^2 + 27$       b.  $45x^2 + 50x^5$       c.  $18x^6 - 6x^3 + 24x^2$   
d.  $12x^2y^3 - 8x^3y^2 + 16xy^4$       e.  $10x^3y + 15x^2y^2 - 5xy$
3. Factor.  
a.  $x^2(x-4) + 3(x-4)$       b.  $x(x^2+9) + (x^2+9)$
4. Factor by grouping.  
a.  $6x^2 - 10x + 9x - 15$       b.  $2x^4 + 2x^2 - 5x^2 - 5$

#### Teaching Notes:

- To factor means to find an equivalent expression whose product gives the original polynomial.
- The greatest common factor (GCF) is an expression of the highest degree that will divide into each term of a polynomial.
- To factor by grouping: first group terms that have a common monomial factor. Next, factor out the common monomial from each group, and then factor out the remaining common binomial factor (if one exists).
- Factoring can be easily checked by multiplying the terms through distributing or FOIL (if two binomials).

Answers: 1. a.  $12x^2$    b.  $6x$    c.  $x^2y$    2. a.  $9(x^2+3)$    b.  $5x^2(9+10x^3)$    c.  $6x^2(3x^4-x+4)$   
d.  $4xy^2(3xy-2x^2+4y^2)$    e.  $5xy(2x^2+3xy-1)$    4.  $4xy^2(3xy-2x^2+4y^2)$    e.  $5xy(2x^2+3xy-1)$   
3. a.  $(x-4)(x^2+3)$    b.  $(x^2+9)(x+1)$    4. a.  $(3x-5)(2x+3)$    b.  $(x^2+1)(2x^2-5)$

**Mini Lecture 7.2**  
Factoring Trinomials Whose Leading Coefficient is 1

**Learning Objectives:**

1. Factor trinomials of the form  $ax^2 + bx + c$ .

**Examples:**

Factor these similar problems. Make an observation of the signs.

1.     a.  $x^2 + 9x + 18$      b.  $x^2 - 9x + 18$      c.  $x^2 - 3x - 18$      d.  $x^2 + 3x - 18$

Factor each polynomial. Check using the FOIL method.

- |                         |                       |                          |
|-------------------------|-----------------------|--------------------------|
| 2. $x - 2x - 24$        | 3. $a^2 - 14a - 72$   | 4. $x^2 + 2xy + y^2$     |
| 5. $a - 15a + 24$       | 6. $2y^2 + 14y + 24$  | 7. $a^2 + 8a + 16$       |
| 8. $x^2 + 10x + 25$     | 9. $a^2 - 13a - 30$   | 10. $y^2 - 8y - 20$      |
| 11. $y^2 + y - 12$      | 12. $3x^2 - 12x + 9$  | 13. $a^2 + 10ab + 16b^2$ |
| 14. $x^2 - 6xy - 16y^2$ | 15. $4y^2 - 20y + 16$ | 16. $x^2 + 2x - 15$      |

**Teaching Notes:**

- Students need as much practice as possible to become comfortable factoring.
- Stress checking the answer! Especially the “oi” (outside + inside).

Answers: 1. a.  $(x+6)(x+3)$     b.  $(x-6)(x-3)$     c.  $(x-6)(x+3)$     d.  $(x+6)(x-3)$     2.  $(x-6)(x+4)$   
3.  $(a-18)(a+4)$     4.  $(x+y)(x+y)$     5.  $(a-8)(a-3)$     6.  $2(y+3)(y+4)$     7.  $(a+4)(a+4)$   
8.  $(x+5)(x+5)$     9.  $(a-15)(a+2)$     10.  $(y-10)(y+2)$     11.  $(y+4)(y-3)$     12.  $3(x-3)(x-1)$   
13.  $(a+2b)(a+8b)$     14.  $(x-8y)(x+2y)$     15.  $4(y-4)(y-1)$     16.  $(x+5)(x-3)$

### Mini Lecture 7.3

#### Factoring Trinomials Whose Leading Coefficient is Not 1

#### Learning Objectives:

1. Factor trinomials by trial and error.
2. Factor trinomials by grouping.

#### Examples:

Factor each trinomial. Try both the trial and error method and grouping. If the trinomial is prime, so state. Show that the factorization is correct by multiplying the factors using the FOIL method.

- |                         |                         |
|-------------------------|-------------------------|
| 1. $6x^2 + 13x + 6$     | 2. $3x^2 - 4x - 15$     |
| 3. $5x^2 + x - 18$      | 4. $6x^2 + 2x + 1$      |
| 5. $7x^2 + 15x + 2$     | 6. $3x^2 - 5x - 2$      |
| 7. $3x^2 + 4x + 1$      | 8. $4x^2 + 4x - 15$     |
| 9. $18x^2 - 21x - 9$    | 10. $6x^2 + 7x + 2$     |
| 11. $8x^2 + 10x - 3$    | 12. $15x^2 - 19x + 6$   |
| 13. $6x^2 - 41x - 7$    | 14. $9x^2 + 18x + 8$    |
| 15. $3x^2 + 2x - 5$     | 16. $4x^2 + 2xy - 6y^2$ |
| 17. $6x^2 - 7xy - 3y^2$ | 18. $4x^2 + 4xy - 3y^2$ |
| 19. $6x^2 + 15x + 9$    | 20. $2x^2 + 5x - 2$     |

#### Teaching Notes:

- When factoring  $ax^2 + bx + c$ , it is important to know the sign combinations. If the polynomial is in the form  $ax^2 + bx + c$ , then the factored form is  $(+)(+)$ . If the polynomial is in the form  $ax^2 + bx - c$ , then the factored form is  $(+)(-)$  or  $(-)(+)$ .
- When using the trial and error method in factoring  $ax^2 + bx + c$ , ALWAYS look for the greatest common factor first. If there is a GCF, factor it out first, then continue by finding 2 factors that equal  $ax^2$   $(\underline{\quad})(\underline{\quad})$ . Then find two factors whose product equals  $c$   $(\underline{\quad})(\underline{\quad})$ . Check the sum of the outside and inside product because it must equal  $bx$   $(\underline{\quad}\underline{\quad})(\underline{\quad}\underline{\quad})$ . If no combination exists, the polynomial is prime.
- When using the grouping method to factor  $ax^2 + bx + c$ , if  $a \neq 1$ , first multiply the leading coefficient,  $a$ , and the constant,  $c$ . Then find two factors of  $ac$  whose sum is  $b$ . Rewrite the middle term,  $bx$ , as a sum or difference using the two factors found. Use the grouping method discussed in 7.1.

Answers: 1.  $(3x+2)(2x+3)$  2.  $(3x+5)(x-3)$  3.  $(5x-9)(x+2)$  4. prime 5.  $(7x+1)(x+2)$   
6.  $(3x+1)(x-2)$  7.  $(3x+1)(x+1)$  8.  $(2x+5)(2x-3)$  9.  $3(3x+1)(2x-3)$  10.  $(3x+2)(2x+1)$   
11.  $(4x-1)(2x+3)$  12.  $(5x-3)(3x-2)$  13.  $(6x+1)(x-7)$  14.  $(3x+4)(3x+2)$   
15.  $(3x+5)(x-1)$  16.  $2(2x+3y)(x-y)$  17.  $(3x+y)(2x-3y)$  18.  $(2x+3y)(2x-y)$   
19.  $3(x+1)(2x+3)$  20. prime

## Mini Lecture 7.4

### Factoring Special Forms

#### Learning Objectives:

1. Factor the difference of two squares.
2. Factor perfect square trinomials.
3. Factor the sum or difference of two cubes.

#### Examples:

Factor each polynomial completely.

- |    |                   |                     |                     |                      |
|----|-------------------|---------------------|---------------------|----------------------|
| 1. | a. $a^2 - 16$     | b. $x^2 - 169$      | c. $a^2 - b^2$      | d. $4x^2 - 9y^2$     |
| 2. | a. $a^2 + 4a + 4$ | b. $y^2 - 10y + 25$ | c. $x^2 + 16x + 64$ | d. $4a^2 - 20a + 25$ |
| 3. | a. $x^3 - 1$      | b. $y^3 + 27$       | c. $8a^3 + 27b^3$   | d. $16x^3 - 2$       |
|    | e. $64 - y^3$     | f. $a^6 + b^6$      | g. $5x^3 - 5y^3$    | h. $a^3 + 125$       |

Factor completely. (Some may be prime.)

- |    |                      |                 |                     |                    |
|----|----------------------|-----------------|---------------------|--------------------|
| 4. | a. $16x^2 + 18y^2$   | b. $x^4 - 81$   | c. $a^2 + 12a + 36$ | d. $25x^2 - 16y^2$ |
|    | e. $3x^2 + 18x + 27$ | f. $125x^3 - 8$ | g. $x^5 + 8x^2$     | h. $27x^2 - 12$    |

#### Teaching Notes:

- Students must spend time learning to identify these special products.
- The difference of squares and perfect square trinomials seems to come fairly easily, but the cubes take more practice and simply must be memorized.
- A PST (**P**erfect **S**quare **T**rinomial) always results in BS (**B**inomial **S**quare or **B**inomials **S**ame).

Answers: 1. a.  $(a + 4)(a - 4)$  b.  $(x + 13)(x - 13)$  c.  $(a + b)(a - b)$  d.  $(2x + 3y)(2x - 3y)$   
2. a.  $(a + 2)^2$  b.  $(y - 5)^2$  c.  $(x + 8)^2$  d.  $(2a - 5)^2$  3. a.  $(x - 1)(x^2 + x + 1)$   
b.  $(y + 3)(y^2 - 3y + 9)$  c.  $(2a + 3b)(4a^2 - 6ab + 9b^2)$  d.  $2(2x - 1)(4x^2 + 2x + 1)$   
e.  $(4 - y)(16 + 4y + y^2)$  f.  $(a^2 + b^2)(a^4 - a^2b^2 + b^4)$  g.  $5(x - y)(x^2 + xy + y^2)$   
h.  $(a + 5)(a^2 - 5a + 25)$  4. a.  $2(8x^2 + 9y^2)$  b.  $(x^2 + 9)(x + 3)(x - 3)$  c.  $(a + 6)(a + 6)$   
d.  $(5x + 4y)(5x - 4y)$  e.  $3(x + 3)(x + 3)$  f.  $(5x - 2)(25x^2 + 10x + 4)$  g.  $x^2(x + 2)(x^2 - 2x + 4)$   
h.  $3(3x + 2)(3x - 2)$

## Mini Lecture 7.5

### A General Factoring Strategy

#### **Learning Objectives:**

1. Recognize the appropriate method for factoring a polynomial.
2. Use a general strategy for factoring polynomials.

#### **Examples:**

Factor the polynomials. Check your factorization by multiplying.

1.  $9x^2 + 81$
2.  $4x^2 - 64$
3.  $27x^3 + 64y^3$
4.  $x^3 - 1$
5.  $81x^4 - 1$
6.  $a^2 - 18a + 72$
7.  $36a^2 + 66a + 24$
8.  $12x^3 + 18x^2 - 30x^2 - 45x$
9.  $2x^3 + 10x + x^2 + 5$
10.  $2a^2 - 8ab + 12b^2$

#### **Teaching Notes:**

- Practice, practice, practice!
- When factoring, always look for a GCF first.
- Count the number of terms in the polynomial.
- If two terms – is it difference of squares, sum of two cubes, difference of two cubes?
- If three terms – is it a perfect square trinomial?
- With a trinomial, use trial and error or grouping method.
- If four or more terms, try factoring by grouping.
- Is the polynomial prime?

Answers: 1.  $9(x^2 + 9)$  2.  $4(x + 4)(x - 4)$  3.  $(3x + 4y)(9x^2 - 12xy + 16y^2)$  4.  $(x - 1)(x^2 + x + 1)$   
5.  $(9x^2 + 1)(3x + 1)(3x - 1)$  6.  $(a - 12)(a - 6)$  7.  $6(3a + 4)(2a + 1)$  8.  $3x(2x - 5)(2x + 3)$   
9.  $(x^2 + 5)(2x + 1)$  10.  $2(a^2 - 4ab + 6b^2)$

**Mini Lecture 7.6**  
Solving Quadratic Equations by Factoring

**Learning Objectives:**

1. Use the zero product principle.
2. Solve quadratic equations by factoring.
3. Solve problems using quadratic equations.

**Examples:**

1. Solve each equation.  
a.  $(x-3)(x+2)=0$       b.  $(2x-5)(x-10)=0$       c.  $(x+4)(x+7)=0$
2. Put each equation in standard form. Make sure the leading coefficient is positive.  
a.  $4x^2 + 3x = 10$       b.  $5x = 20 - x^2$       c.  $3x^2 - 6x = 2x^2 - 11$
3. Solve each quadratic equation by factoring.  
a.  $2x^2 - 3x - 20 = 0$       b.  $a^2 - 11a + 30 = 0$       c.  $2x^2 - 5x = 12$   
d.  $5x^2 = 2x$       e.  $8x^3 - 2x^2 = 10x$       f.  $x(14 - x) = 48$   
g.  $4x^2 - 49 = 0$       h.  $3x^2 = 15x$       i.  $3a^2 + 7a - 20 = 0$

**Teaching Notes:**

- Make sure students know what the standard form of a quadratic equation is and what the letters “a”, “b”, and “c” represent.
- Students need to be able to recognize a quadratic equation by the squared term.
- Students often want to shortcut the steps. Warn them not to take shortcuts when they should set each factor equal to zero and solve.

**Answers:** 1. a.  $x = 3$   $x = -2$     b.  $x = \frac{5}{2}$   $x = 10$     c.  $x = -4$   $x = -7$     2.a.  $4x^2 + 3x - 10 = 0$   
b.  $x^2 + 5x - 20 = 0$     c.  $x^2 - 6x + 11 = 0$     3. a.  $x = -\frac{5}{2}$   $x = 4$     b.  $a = 6$   $a = 5$     c.  $x = -\frac{3}{2}$   $x = 4$   
d.  $x = 0$   $x = \frac{2}{5}$     e.  $x = 0$   $x = \frac{5}{4}$   $x = -1$     f.  $x = 6$   $x = 8$     g.  $x = -\frac{7}{2}$   $x = \frac{7}{2}$     h.  $x = 0$   $x = 5$   
i.  $a = \frac{5}{3}$   $a = -4$