Mini Lecture 9.1
Finding Roots

Learning Objectives:

1. Find square roots.
2. Evaluate models containing square roots.
3. Use a calculator to find decimal approximations for irrational square roots.
4. Find higher roots.

Examples:

Evaluate.

1. a. $\sqrt{49}$ b. $-\sqrt{100}$ c. $\sqrt{\frac{1}{36}}$ d. $\sqrt{9+16}$ e. $\sqrt{9}+\sqrt{16}$

Given the equation, $y=\frac{5}{4\sqrt{a}}$, solve for *y* given that:

2. a. $a = 9$ b. $a = 25$ c. $a = 49$

Use a calculator to approximate each expression and round to three decimal places. If the expression is not a real number, so state.

3. a. $\sqrt{30}$ b. $\sqrt{7}$ c. $\sqrt{11-1}$ d. $\sqrt{-8}$

Find the indicated root, or state that the expression is not a real number.

4. a. $\sqrt[3]{27}$ b. $\sqrt[5]{-1}$ c. $\sqrt[4]{-81}$ d. $\sqrt[5]{-32}$

   e. $\sqrt[6]{64}$ f. $-\sqrt[3]{64}$ g. $-\sqrt[4]{1}$ h. $\sqrt[5]{-32}$

Teaching Notes:

- The symbol $\sqrt{\ }$ is called the radical sign.
- The number under the radical sign is called the radicand.
- Together we refer to the radical sign and its radicand as a radical.
- The symbol $-\sqrt{\ }$ is used to denote the negative square root of a number.
- The square root of a negative number is not a real number. This also applies to any even root $\left(\sqrt[4]{-x},\ \sqrt[6]{-x},\ \sqrt[8]{-x}\ ...\right)$.
- Not all radicals are square roots.

Answers: 1. a. 7 b. –10 c. $\frac{1}{6}$ d. 5 e. 7 2. a. $\frac{5}{12}$ b. $\frac{1}{4}$ c. $\frac{5}{28}$ 3. a. 5.477 b. 2.646 c. 3.162 d. not a real number 4. a. 3 b. –1 c. not a real number d. –2 e. 2 f. –4 g. –1 h. –2

Mini Lecture 9.2
Multiplying and Dividing Radicals

Learning Objectives:
1. Multiply square roots.
2. Simplify square roots.
3. Use the quotient rule for square roots.
4. Use the product and quotient rules for other roots.

Examples:

1. Multiply using the product rule.
   a. $\sqrt{3}\cdot\sqrt{5}$ b. $\sqrt{10}\cdot\sqrt{7}$ c. $\sqrt{6}\cdot\sqrt{\frac{1}{2}}$ d. $\sqrt{5}\cdot\sqrt{7}$
2. Simplify using the product rule.
   a. $\sqrt{20}$ b. $\sqrt{90}$ c. $\sqrt{48}$ d. $\sqrt{120}$
3. Simplify. (Look for a pattern) Assume all variables represent positive number only.
   a. $\sqrt{x^2}$ b. $\sqrt{x^3}$ c. $\sqrt{x^4}$ d. $\sqrt{x^5}$
   e. $\sqrt{x^6}$ f. $\sqrt{x^7}$ g. $\sqrt{x^8}$ h. $\sqrt{x^9}$
   i. $\sqrt{x^{10}}$ j. $\sqrt{x^{11}}$ k. $\sqrt{x^{12}}$
4. Simplify.
   a. $\sqrt{32x^2}$ b. $\sqrt{80y^6}$ c. $\sqrt{75x^7}$ d. $\sqrt{45x^4y^5}$
5. Multiply. Then simplify if possible.
   a. $\sqrt{6}\cdot\sqrt{2}$ b. $\sqrt{5x}\cdot\sqrt{10x}$ c. $\sqrt{7x^2}\cdot\sqrt{8x^3}$ d. $\sqrt{15y^4}\cdot\sqrt{5y^4}$
6. Simplify.
   a. $\sqrt{\frac{9}{25}}$ b. $\sqrt{\frac{8}{81}}$ c. $\sqrt{\frac{90x^2}{169}}$ d. $\sqrt{\frac{1}{49}}$
7. Simplify.
   a. $\sqrt[3]{24}$ b. $\sqrt[5]{1}$ c. $\sqrt[4]{48x^4}$ d. $\sqrt[3]{3}\cdot\sqrt[3]{6}$

Teaching Notes:
- Have students memorize perfect square numbers through 225 and perfect cubes through 216.
- Get as much out of the radicand as possible.
- Since radicals are unfamiliar to most students, it is important they see the relationship of squaring numbers and square roots, cubing numbers and cube roots, etc.

Answers: 1. a. $\sqrt{15}$ b. $\sqrt{70}$ c. $\sqrt{3}$ d. $\sqrt{35}$ 2. a. $2\sqrt{5}$ b. $3\sqrt{10}$ c. $4\sqrt{3}$ d. $2\sqrt{30}$
3. a. x b. $x\sqrt{x}$ c. x^2 d. $x^2\sqrt{x}$ e. x^3 f. $x^3\sqrt{x}$ g. x^4 h. $x^4\sqrt{x}$ i. x^5 j. $x^5\sqrt{x}$ k. x^6
4. a. $4x\sqrt{2}$ b. $4y^3\sqrt{5}$ c. $5x^3\sqrt{3x}$ d. $3x^2y^2\sqrt{5y}$ 5. a. $2\sqrt{3}$ b. $5x\sqrt{2}$ c. $2x^2\sqrt{14x}$
d. $5y^4\sqrt{3}$ 6. a. $\frac{3}{5}$ b. $\frac{2\sqrt{2}}{9}$ c. $\frac{3x\sqrt{10}}{13}$ d. $\frac{1}{7}$ 7. a. $2\sqrt[3]{3}$ b. 1 c. $2x\sqrt[4]{3}$ d. $\sqrt[3]{18}$

Mini Lecture 9.3
Operations with Radicals

Learning Objectives:

1. Add and subtract radicals.
2. Multiply radical expressions with more than one term.
3. Multiply conjugates.

Examples:

Add or subtract as indicated.

1. a. $9\sqrt{2}+2\sqrt{2}$ b. $\sqrt{3x}-5\sqrt{3x}$
   c. $3\sqrt{8}+5\sqrt{18}$ d. $3\sqrt{27x}-8\sqrt{12x}$
   e. $2\sqrt{5}+4\sqrt{3}$

Multiply.

2. a. $\sqrt{3}\left(\sqrt{7}+\sqrt{5}\right)$ b. $\left(2+\sqrt{5}\right)\left(4+\sqrt{5}\right)$
   c. $\left(9+\sqrt{3}\right)\left(4-2\sqrt{3}\right)$ d. $\left(4+\sqrt{2}\right)\left(4-\sqrt{2}\right)$
   e. $\left(\sqrt{3}-\sqrt{5}\right)\left(\sqrt{3}+\sqrt{5}\right)$ f. $\left(\sqrt{x}+\sqrt{2}\right)^2$

Teaching Notes:

- Two or more square roots can be combined using the distributive property provided they have the same radicand.
- In some cases, radicals can be combined after they have been simplified.
- When multiplying radical expressions, distribute. This is similar to multiplying a monomial by a polynomial.
- When multiplying radical expressions use the FOIL method like multiplying binomials.
- When multiplying conjugates (expressions that involve the sum and difference of the same two terms), the FOIL method may be used or the special product formula. When using the FOIL method with conjugates the OI (outside & inside) will equal 0.

Answers: 1. a. $11\sqrt{2}$ b. $-4\sqrt{3x}$ c. $21\sqrt{2}$ d. $-7\sqrt{3x}$ e. cannot be combined
2. a. $\sqrt{21}+\sqrt{15}$ b. $13+6\sqrt{5}$ c. $30-14\sqrt{3}$ d. 14 e. –2 f. $x+2\sqrt{2x}+2$

Mini Lecture 9.4
Rationalizing the Denominator

Learning Objectives:

1. Rationalize denominators containing one term.
2. Rationalize denominators containing two terms.

Examples:

Multiply and simplify.

1. a. $\sqrt{2}\cdot\sqrt{2}$ b. $\sqrt{3}\cdot\sqrt{3}$ c. $\sqrt{4}\cdot\sqrt{4}$ d. $\sqrt{7}\cdot\sqrt{7}$
 e. $\sqrt[3]{9}\cdot\sqrt[3]{3}$ f. $\sqrt[3]{4}\cdot\sqrt[3]{2}$ g. $\sqrt[4]{4}\cdot\sqrt[4]{4}$

Rationalize each denominator.

2. a. $\sqrt{\frac{1}{2}}$ b. $\frac{2}{\sqrt{3}}$ c. $\sqrt{\frac{2}{3}}$ d. $\frac{\sqrt{2}}{\sqrt{8}}$
 e. $\frac{5}{\sqrt{5}}$ f. $\frac{2x}{\sqrt{10}}$ g. $\frac{2}{\sqrt{2x}}$ h. $\sqrt{\frac{5x}{7x}}$

State the conjugate of each of the following.

3. a. $4+\sqrt{2}$ b. $\sqrt{5}-\sqrt{3}$ c. $\sqrt{6}-3$ d. $\sqrt{7}+\sqrt{3}$

Multiply.

4. a. $\left(6+\sqrt{3}\right)\left(6-\sqrt{3}\right)$ b. $\left(\sqrt{11}+\sqrt{5}\right)\left(\sqrt{11}-\sqrt{5}\right)$ c. $\left(\sqrt{6}+2\right)\left(\sqrt{6}-2\right)$

Rationalize each denominator and write in simplest form.

5. a. $\frac{3}{\sqrt{3}-4}$ b. $\frac{5}{\sqrt{7}+\sqrt{3}}$ c. $\frac{\sqrt{2}}{\sqrt{5}+\sqrt{2}}$ d. $\frac{\sqrt{x}-2}{\sqrt{x}+2}$ e. $\frac{\sqrt{7}-\sqrt{3}}{\sqrt{7}+\sqrt{3}}$

Teaching Notes:

- Remind students of the definition of a rational number. This will help them understand the meaning of "rationalizing the denominator."
- It may be helpful to discuss the special product of $(a+b)(a-b)$ with several examples to let students "see" again what happens to the middle term when the binomials are foiled.

Answers: 1. a. 2 b. 3 c. 4 d. 7 3. 3 f. 2 g. 2 2. a. $\frac{\sqrt{2}}{2}$ b. $\frac{2\sqrt{3}}{3}$ c. $\frac{\sqrt{6}}{3}$ d. $\frac{1}{2}$ e. $\sqrt{5}$ f. $\frac{x\sqrt{10}}{5}$ g. $\frac{\sqrt{2x}}{x}$ h. $\frac{\sqrt{35}}{7}$ 3. a. $4-\sqrt{2}$ b. $\sqrt{5}+\sqrt{3}$ c. $\sqrt{6}+3$ d. $\sqrt{7}-\sqrt{3}$ 4. a. 33 b. 6 c. 2 5 a. $\frac{-3\sqrt{3}-12}{13}$ b. $\frac{5\sqrt{7}-5\sqrt{3}}{4}$ c. $\frac{\sqrt{10}-2}{3}$ d. $\frac{x-4\sqrt{x}+4}{x-4}$ e. $\frac{5-\sqrt{21}}{2}$

Mini Lecture 9.5
Radical Equations

Learning Objectives:

1. Solve radical equations.
2. Solve problems involving square root models.

Examples:

Solve each radical equation. If the equation has no solution, so state.

1. a. $\sqrt{4x+1}=3$ b. $\sqrt{3x+1}=4$

   c. $\sqrt{x+9}-2\sqrt{x}=0$ d. $\sqrt{x}+3=0$

   e. $\sqrt{4x+16}=x+5$ f. $\sqrt{x+2}=-6$

   g. $\sqrt{2x+5}-1=4$ h. $2\sqrt{x}+4=1$

   i. $\sqrt{x}-6=3$ j. $x=\sqrt{x+2}-2$

Teaching Notes:

- A radical equation is an equation in which the variable occurs in a square root, cube root, or any higher root.
- To solve a radical equation containing square roots, first arrange terms so that one radical is isolated on one side of the equation. Next, square both sides of the equation to eliminate the square root. Solve and ALWAYS check the answer in the original equation.
- There may be an extra solution(s) that does not check in the original equation. This solution(s) is/are called extraneous solutions.

Answers: 1. a. $x=2$ b. $x=5$ c. $x=3$ d. no solution e. –3 f. no solution g. 10 h. no solution i. 81 j. –2, –1

Mini Lecture 9.6
Rational Exponents

Learning Objectives:

1. Evaluate expressions with rational exponents.
2. Solve problems using models with rational exponents.

Examples:

Write each of the following in radical form first, then simplify.

1. a. $64^{\frac{1}{2}}$ b. $27^{\frac{1}{3}}$ c. $121^{\frac{1}{2}}$ d. $(-216)^{\frac{1}{3}}$ e. $81^{\frac{1}{4}}$

2. a. $64^{\frac{4}{3}}$ b. $25^{\frac{3}{2}}$ c. $125^{\frac{2}{3}}$ d. $32^{\frac{2}{5}}$ e. $-27^{\frac{4}{3}}$

Simplify.

3. a. 6^{-2} b. $36^{\frac{1}{2}}$ c. $36^{-\frac{1}{2}}$ d. 8^{-3} e. $8^{\frac{1}{3}}$ f. $8^{-\frac{1}{3}}$

4. a. $27^{-\frac{2}{3}}$ b. $100^{-\frac{3}{2}}$ c. $343^{-\frac{4}{3}}$ d. $256^{\frac{3}{4}}$ e. $64^{-\frac{1}{3}}$ f. $\left(\frac{1}{36}\right)^{-\frac{1}{2}}$

Teaching Notes:

- If a graphing calculator is being used in the class, it is **helpful** to show that $x^{\frac{1}{n}}$ is the same as the $\sqrt[n]{x}$ using number values.
- Stress to students that the denominator of a rational exponent is the index of the corresponding radical expression.
- When the numerator of a rational exponent is not 1, the numerator is the power to which the radical is raised. It is usually easier to simplify it this way, but it is possible to raise the radicand to the power instead.
- When the exponent is negative, write the base as its reciprocal, and raise to the positive power.

Answers: 1. a. $\sqrt{64}=8$ b. $\sqrt[3]{27}=3$ c. $\sqrt{121}=11$ d. $\sqrt[3]{-216}=-6$ e. $\sqrt[4]{81}=3$
2. a. $\left(\sqrt[3]{64}\right)^4 = 256$ b. $\left(\sqrt{25}\right)^3 = 125$ c. $\left(\sqrt[3]{125}\right)^2 = 25$ d. $\left(\sqrt[5]{32}\right)^2 = 4$ e. $-\left(\sqrt[3]{27}\right)^4 = -81$ 3. a. $\frac{1}{36}$ b. 6 c. $\frac{1}{6}$ d. $\frac{1}{512}$ e. 2 f. $\frac{1}{2}$ 4. a. $\frac{1}{9}$ b. $\frac{1}{1000}$ c. $\frac{1}{2401}$ d. 64 e. $\frac{1}{4}$ f. 6