Mini Lecture 10.1

Solving Quadratic Equations by the Square Root Property

Learning Objectives:

1. Solve quadratic equations using the square root property.
2. Solve problems using the Pythagorean Theorem.
3. Find the distance between two points.

Examples:

1. Solve each quadratic equation by the square root property. If possible, simplify radicals or rationalize denominators.

   a. $x^2 = 25$ b. $3x^2 = 27$ c. $3x^2 - 5 = 0$ d. $(x-2)^2 = 16$ e. $(x+4)^2 = 5$

2. Solve each quadratic equation by first factoring the perfect square on the left. Then apply the square root property. Simplify radicals, if possible.

   a. $x^2 + 6x + 9 = 64$ b. $x^2 + 8x + 16 = 11$

3. Use the Pythagorean Theorem to solve the following.

   If a ladder is angled against a home and it is 6 ft. from the base of the home and reaches 8 ft. high on the side of the home, how tall is the ladder?

4. Find the distance between the given 2 points. Express answers in simplest radical form and if necessary, round to two decimal places.

   a. $(-3, 6)$ and $(1,\ 5)$ b. $(5, -3)$ and $(9,\ -6)$

Teaching Notes:

- If $u^2 = d$ is being in an algebraic expression and d is a positive real number, then there will be exactly two solutions: $u = \sqrt{d}$ and $u = -\sqrt{d}$ or $u = \pm\sqrt{d}$.
- The Pythagorean Theorem states that the sum of the squares of the lengths of the legs of a right triangle equals the square of the length of the hypotenuse. If the lengths of the legs are *a* and *b*, and the hypotenuse has length *c*, then $a^2 + b^2 = c^2$.
- The Distance Formula is used to find the distance between two points in the rectangular coordinate system $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ given 2 points $(x_1, y_1)(x_{2,} y_2)$.

Answers: 1. a. ± 5 b. ± 3 c. $\frac{\pm\sqrt{15}}{3}$ d. 6, –2 e. $-4 \pm \sqrt{5}$ 2. a. –11 ,5 b. $-4 \pm \sqrt{11}$ 3. 10 ft. 4. a. $\sqrt{17}$ b. 5

Mini Lecture 10.2
Solving Quadratic Equations by Completing the Square

Learning Objectives:
1. Complete the square of a binomial.
2. Solve quadratic equations by completing the square.

Examples:
1. Complete the square for each binomial by adding "one-half the coefficient of the *x* term squared." Then factor the resulting perfect square trinomial.

a. $x^2 + 2x +$ __________ = __________ b. $x^2 + 8x +$ __________ = __________
c. $x^2 - 16x +$ __________ = __________ d. $x^2 + 10x +$ __________ = __________
e. $x^2 - 14x +$ __________ = __________ f. $x^2 + 3x +$ __________ = __________
g. $x^2 - 6x +$ __________ = __________

2. Solve each quadratic equation by completing the square.

a. $x^2 = 11x - 30$ b. $x^2 + 2x - 8 = 0$ c. $x^2 = 4x - 1$

d. $16x^2 = 8x + 3$ e. $x^2 + 12x + 12 = 0$ f. $x^2 + \frac{8}{3}x = 1$

g. $x^2 - 6x = 3$

Teaching Notes:
- Remind students that in order to complete the square the leading coefficient must be 1 and there must be an x^2 term and an *x* term.
- When using completing the square to solve a quadratic equation, the variable terms must be on one side of the equation and the constant term on the other side before starting the process.

Answers: 1. a. $x^2 + 2x + 1 = (x+1)^2$ b. $x^2 + 8x + 16 = (x+4)^2$ c. $x^2 - 16x + 64 = (x-8)^2$
d. $x^2 + 10x + 25 = (x+5)^2$ e. $x^2 - 14x + 49 = (x-7)^2$ f. $x^2 + 3x + \frac{9}{4} = (x + \frac{3}{2})^2$
g. $x^2 - 6x + 9 = (x-3)^2$ 2. a. 6, 5 b. 4, –2 c. $2 \pm \sqrt{3}$ d. $\frac{3}{4}, -\frac{1}{4}$ e. $-6 \pm 2\sqrt{6}$ f. $\frac{1}{3}, -3$
g. $3 \pm 2\sqrt{3}$

Mini Lecture 10.3
The Quadratic Formula

Learning Objectives:

1. Solve quadratic equations using the quadratic formula.
2. Determine the most efficient method to use when solving quadratic equations.
3. Solve problems using quadratic equations.

Examples:

1. Solve each equation using the quadratic formula.

   a. $3x^2 + 10x + 3 = 0$

   b. $x^2 = 5x + 2$

   c. $x^2 + 3x = -1$

2. Solve each equation by the <u>method of your choice.</u> Simplify irrational solutions, if possible.

   a. $2x^2 = 40$
   b. $(x+4)^2 = 8$
   c. $3x^2 - 4x + 1 = 0$
   d. $2x^2 + 7x + 4 = 0$
   e. $4x^2 + 2x - 5 = 0$

3. A football is kicked straight up from a height of 2 feet with an initial speed of 50 ft. per second. The formula $h = -16t^2 + 50t + 2$ describes the ball's height above the ground, *h*, in feet, *t* seconds after it is kicked. Using your calculator, find when the ball will hit the ground. Round to the nearest tenth of a second.

<u>Answers:</u> 1. a. $-\frac{1}{3}, -3$ b. $\frac{5 \pm \sqrt{33}}{2}$ c. $\frac{-3 \pm \sqrt{5}}{2}$ 2. a. $x = \pm 2\sqrt{5}$ b. $x = -4 \pm 2\sqrt{2}$ c. $x = \frac{1}{3}, x = 1$ d. $\frac{-7 \pm \sqrt{17}}{4}$ e. $\frac{-1 \pm \sqrt{21}}{4}$ 3. 3.2 seconds

Mini Lecture 10.4
Imaginary Numbers as Solutions of Quadratic Equations

Learning Objectives:

1. Understand the concepts: $i^2 = -1$ and $\sqrt{-1} = i$.
2. Express square roots of negative numbers in terms of *i*.
3. Solve quadratic equations with imaginary solutions.

Examples:

Write as a multiple of *i*.

1. a. $\sqrt{-64}$ b. $\sqrt{-4}$ c. $\sqrt{-10}$ d. $\sqrt{-20}$ e. $\sqrt{-40}$ f. $\sqrt{-75}$

Solve each quadratic equation using the square root property.

2. a. $(x-2)^2 = -16$ b. $(x+3)^2 = -1$ c. $x^2 + 4 = 0$

Solve each quadratic equation using the quadratic formula.

3. a. $x^2 + 4x + 7 = 0$ b. $2x^2 + 18 = 0$ c. $6x^2 + 8x = 5$

   d. $3x^2 + 16x + 5 = 0$ e. $3x^2 + x = -3$

Teaching Notes:

- When simplifying the square root of –1, it is helpful to write the "*i*" to the left of the radical so it does not look like the "*i*" is under the radical.
- Always simplify the $\sqrt{-1}$ first.
- Remind students that in order to use the square root property to solve a quadratic equation, there must be a perfect square containing the variable isolated on 1 side of the equation.
- Students need to be cautioned again about watching the signs very carefully when using the quadratic formula.

Answers: 1. a. $8i$ b. $2i$ c. $i\sqrt{10}$ d. $2i\sqrt{5}$ e. $2i\sqrt{10}$ f. $5i\sqrt{3}$ 2. a. $2 \pm 4i$ b. $-3 + i$ c. $\pm 2i$
3. a. $-2 \pm i\sqrt{3}$ b. $3i, -3i$ c. $\dfrac{-4 \pm \sqrt{46}}{6}$ d. $-\dfrac{1}{3}, -5$ e. $\dfrac{-1 \pm i\sqrt{35}}{6}$

Mini Lecture 10.5
Graphs of Quadratic Equations

Learning Objectives:

1. Understand the characteristics of graphs of quadratic equations.
2. Find a parabola's intercepts.
3. Find a parabola's vertex.
4. Graph quadratic equations.
5. Solve problems using a parabola's vertex.

Examples:

1. Consider the quadratic equation: $y = x^2 + 4x + 3$.

   a. Will the parabola open upward or downward?
   b. Complete the table of values and graph the parabola.

x	$y = x^2 + 4x + 3$	(x, y)
–4		
–3		
–2		
–1		
0		
1		
2		

   c. Find the x-intercepts.
   d. Find the y-intercepts.
   e. Find the vertex.

2. Complete the same steps a, b, c, d, e, from number 1 for the equation: $y = -x^2 - 2x - 1$.

Teaching Notes:

- The graph of quadratic equation $y = ax^2 + bx + c$, $a \neq 0$ is called a parabola. If a is positive, the parabola opens upward. If a is negative, the parabola opens downward.
- The vertex, or turning point, of the parabola is referred to as the maximum if the parabola opens downward and the minimum if the parabola opens upward.
- Parabolas are symmetric with respect to a line known as the axis of symmetry that runs through the vertex. The two halves match exactly.
- To find the x-intercepts, let $y = 0$. Then factor or use the quadratic formula to solve for x.
- To find the y-intercept, let $x = 0$ and solve for y.

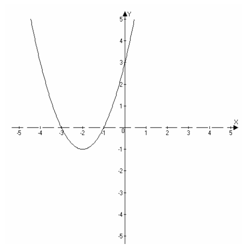
- To find the vertex of a parabola with the equation, $y = ax^2 + bx + c$, first solve for *x*. $x = -\frac{b}{2a}$. Then substitute the value of *x* into the parabola's equation and evaluate.
- After plotting the vertex and intercepts, make sure to connect with a smooth curve.

<u>Answers:</u>

1. a. up

   b.

–4	(–4, 3)
–3	(–3, 0)
–2	(–2, –1)
–1	(–1, 0)
0	(0, 3)
1	(1, 8)
2	(2, 15)



   c. *x*-intercepts (–3, 0)(–1, 0)

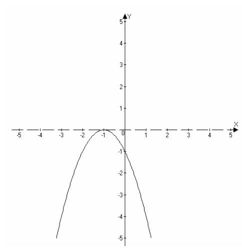
   d. *y*-intercept (0, 3)

   e. vertex (–2 , –1)

2. a. down

   b.

–4	(–4, –9)
–3	(–3, –4)
–2	(–2, –1)
–1	(–1, 0)
0	(0,–1)
1	(1, –4)
2	(2, –9)



   c. *x*-intercept (–1, 0)

   d. *y*-intercept (0 , –1)

   e. vertex (–1 , 0)

Mini Lecture 10.6
Introduction to Functions

Learning Objectives:
1. Find the domain and range of a relation.
2. Determine whether a relation is a function.
3. Evaluate a function.
4. Use the vertical line test to identify functions.

Examples:
Determine whether relation is a function. Give the domain and range for each relation.

1. a. $\{(3, 4), (4, 5), (5, 6)\}$ b. $\{(2, 7), (4, 8), (2, 10), (8, 12)\}$

c. $\{(-2, 3), (-1, 3), (2, 3), (3, 3)\}$ d. $\{(5, -2), (-3, 6), (0, -2)\}$

e. $\{(7, 0), (6, 5), (5, 7), (7, 9)\}$

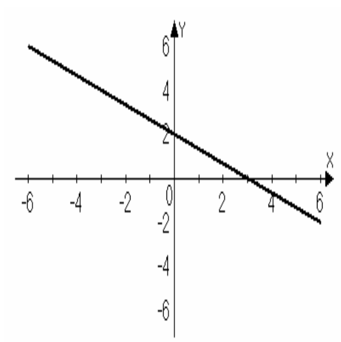
2. Evaluate the functions at the given values.

$f(x) = 3x - 5$
a. $f(3)$ b. $f(-2)$ c. $f(0)$

3. $g(x) = x^2 - x + 2$
a. $g(4)$ b. $g(0)$ c. $g(-2)$

4. $h(x) = -2x^2 + x$
a. $h(0)$ b. $h(-3)$ c. $h(1)$

5. Use the vertical line test to identify graph in which y is a function of x.

a.


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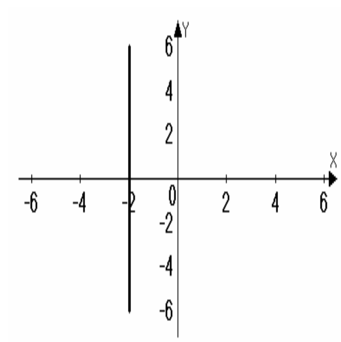
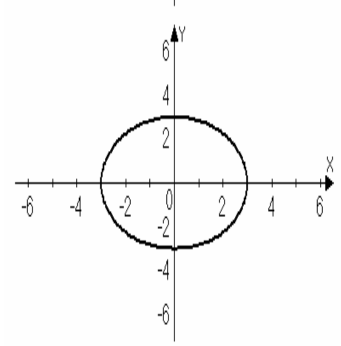


c.


Teaching Notes:

- Students should understand that every function is a relation BUT not every relation is a function.
- The domain of a relation or a function is all the *x*-values or possible *x*-values.
- The range of a relation or a function is all the *y*-values or possible *y*-values.
- A function is a relation in which **no two *x* values are the same**.
- "*f*(*x*)" and "*y*" are like math synonyms.
- *f*(*x*) indicates a function.

Answers: 1. a. function; domain {3, 4, 5}; range {4, 5, 6}
b. not a function; domain {2, 4, 8}; range {7, 8, 10, 12}
c. function; domain {–2, –1, 2, 3}; range {3}
d. function; domain }5, –3, 0}; range {–2, 6}
e. not a function; domain {7, 6, 5}; range {0, 5, 7, 9}
2. a. $f(x) - 6x + 7$ b. $f(x) = 2x^2 + 5$ 3. a. 4 b. –11 c. –5 4. a. 14 b. 2 c. 8 5. a. 0
b. –21 c. –1 5. a. function b. not a function c. not a function