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(5) $x^4 + 4x^2 = P(x)$

$x^2(x^2+4) = P(x)$

$x^2(x^2+4) = 0$

$x^2 = 0 \quad x^2+4=0$

$x^2 = -4$

$x = \pm\sqrt{-4}$

a) $x = 0$ or $x = \pm 2i$

b) $P(x) = x(x-2i)(x+2i)$

(7) $x^3 - 2x^2 + 2x = P(x)$

$x(x^2 - 2x + 2) = 0$

a) $x = 0$ or

$x^2 - 2x + 2 = 0$

$x^2 - 2x + 1 = -2 + 1$

$(x-1)^2 = -1$

$x - 1 = \pm\sqrt{-1}$

$x = 1 \pm i$

b) $P(x) = x(x-(1-i))(x-(1+i))$

$P(x) = x(x-1+i)(x-1-i)$

(9) $P(x) = x^4 + 2x^2 + 1$

$0 = x^4 + 2x^2 + 1$

$0 = (x^2+1)(x^2+1)$

$0 = (x^2+1)^2$

$\pm\sqrt{0} = x^2 + 1$

$-1 = x^2$

$\pm\sqrt{-1} = x$

a) $x = \pm i$

b) $(x-i)(x+i) = P(x)$

(11) $P(x) = x^4 - 16$

$0 = (x^2-4)(x^2+4)$

$x^2 - 4 = 0$ or $x^2 + 4 = 0$

a) $x = \pm 2$ or $x = \pm 2i$

b) $P(x) = (x-2)(x+2)(x-2i)(x+2i)$

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(13) $P(x) = x^3 + 8$

$0 = (x+2)(x^2 - 2x + 4)$

$x+2 = 0$ or $x^2 - 2x + 4 = 0$

$\boxed{x = -2}$ so $x = \dfrac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(4)}}{2(1)}$

$= \dfrac{-(-2) \pm \sqrt{-12}}{2}$

$= \dfrac{(-2) \pm 2i\sqrt{3}}{2}$

$\boxed{x = 1 \pm i\sqrt{3}}$

b)

$P(x) = (x+2)\left(x - (1 - i\sqrt{3})\right)\left(x - (1 + i\sqrt{3})\right)$

$\boxed{P(x) = (x+2)(x - 1 + i\sqrt{3})(x - 1 - i\sqrt{3})}$

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(15)

$$P(x) = x^6 - 1 = (x^3 - 1)(x^3 + 1)$$

$$0 = (x-1)(x^2+x+1)(x+1)(x^2-x+1)$$

$x - 1 = 0$ | $x^2 + x + 1 = 0$ | $x + 1 = 0$ | $x^2 - x + 1 = 0$

$\boxed{x = 1}$ | $\left(x + \frac{1}{2}\right)^2 = \frac{-3}{4}$ | $\boxed{x = -1}$ | $\left(x - \frac{1}{2}\right)^2 = \frac{-3}{4}$

$x = -\frac{1}{2} \pm \frac{\sqrt{-3}}{2}$ | $x = \frac{1}{2} \pm \frac{\sqrt{-3}}{2}$

$\boxed{x = -\frac{1}{2} \pm \frac{i\sqrt{3}}{2}}$ | $\boxed{x = \frac{1}{2} \pm \frac{i\sqrt{3}}{2}}$

$$P(x) = (x-1)^{(x+1)}\left[x - \left(-\frac{1}{2} - \frac{i\sqrt{3}}{2}\right)\right]\left[x - \left(-\frac{1}{2} + \frac{i\sqrt{3}}{2}\right)\right]\left[x - \left(\frac{1}{2} - \frac{i\sqrt{3}}{2}\right)\right]\left[x - \left(\frac{1}{2} + \frac{i\sqrt{3}}{2}\right)\right]$$

$$\boxed{P(x) = (x-1)^{(x+1)}\left(x + \frac{1}{2} + \frac{i\sqrt{3}}{2}\right)\left(x + \frac{1}{2} - \frac{i\sqrt{3}}{2}\right)\left(x - \frac{1}{2} + \frac{i\sqrt{3}}{2}\right)\left(x - \frac{1}{2} - \frac{i\sqrt{3}}{2}\right)}$$

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(17) $P(x) = x^2 + 25$

$0 = x^2 + 25$

$x^2 = -25$

$x = \pm 5i$ each of multiplicity one

$P(x) = (x - 5i)(x + 5i)$

(19) $Q(x) = x^2 + 2x + 2$

$0 = x^2 + 2x + 2$

$1 - 2 = (x+1)^2$

$\pm\sqrt{-1} = x + 1$

$x = -1 \pm i$
each of mult. one

$Q(x) = [x - (-1 - i)][x - (-1 + i)]$

$Q(x) = (x + 1 + i)(x + 1 - i)$

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(33) $P(x) = x^5 + 6x^3 + 9x$

$= x(x^4 + 6x^2 + 9)$

$= x(x^2 + 3)^2$

Let

$0 = x(x^2 + 3)^2$

$= x(x^2+3)(x^2+3)$

$x = 0 \qquad x^2 = -3$

$x = \pm\sqrt{-3}$

$= \pm\sqrt{3}i$

$P(x) = x(x + \sqrt{3}i)^2 (x - \sqrt{3}i)^2$

0 multiplicity one
$\sqrt{3}i$ multiplicity two
$-\sqrt{3}i$ multiplicity two

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(35) $p(x) = [x-(1+i)][x+(1-i)]$

$= x^2 + x(1-i) - x(1+i) - [(1+i)(1-i)]$

$= x^2 + x - xi - x - xi - (1-i^2)$

$= x^2 - 2xi - (1-(-1))$

$\boxed{p(x) = x^2 - 2xi - 2}$

(37) $Q(x) = (x-3)(x-2i)[x-(-2i)]$

$= (x-3)(x-2i)(x+2i)$

$= (x-3)(x^2-4i^2)$

$= (x-3)(x^2+4)$

$= x^3 + 4x - 3x^2 - 12$

$\boxed{Q(x) = x^3 - 3x^2 + 4x - 12}$

(39) $p(x) = (x-2)(x-i)(x+i)$ By conjugate zeros theorem

$= (x-2)(x^2-i^2)$

$= (x-2)(x^2+1)$

$= x^3 + x - 2x^2 - 2$

$\boxed{p(x) = x^3 - 2x^2 + x - 2}$

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(41) $R(x) = (x-1)^2\left[x-(1-2i)\right]\left[x-(1+2i)\right]$

$= (x^2-2x+1)\left[(x-1)+2i\right]\left[(x-1)-2i\right]$

$= (x^2-2x+1)\left[(x-1)^2+4\right]$

$= (x^2-2x+1)(x^2-2x+1+4)$

$= (x^2-2x+1)(x^2-2x+5)$

$= x^4 - 2x^3 + 5x^2$
$\quad - 2x^3 + 4x^2 - 10x$
$+ \qquad\qquad x^2 - 2x + 5$

$$R(x) = x^4 - 4x^3 + 10x^2 - 12x + 5$$

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(43) $T(x) = c(x-i)(x+i)[x-(1+i)][x-(1-i)]$ — constant term of 12

$= c(x^2+1)[(x-1)+i][(x-1)-i]$

$= c((x^2+1)[(x-1)^2+1]$

$= c(x^2+1)[x^2-2x+2]$

$= c[x^4-2x^3+2x^2+x^2-2x+2]$

$T(x) = c[x^4-2x^3+3x^2-2x+2]$

Constant term of 12

$\Rightarrow 2c = 12$

$c = 6$ $\therefore$

$T(x) = 6[x^4-2x^3+3x^2-2x+2]$

$\boxed{T(x) = 6x^4-12x^3+18x^2-12x+12}$

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(45) $P(x) = x^3 + 2x^2 + 4x + 8$

$= x^2(x+2) + 4(x+2)$

$= (x+2)(x^2+4)$

let $0 = (x+2)(x^2+4)$

$x = -2$ or $x = \pm 2i$